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ECONOMICS 7330—Probability and Statistics, Fall 2024

Homework 2. Due Wednesday September 4.

1. Let $X \sim U[0,1]$. Find the distribution function of $Y = \log(\frac{X}{1-X})$.

2. Define

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-x} & \text{if } x \geq 0\,. \end{cases} \tag{1}$$

(a) Show that $F(x)$ is a CDF.
(b) Find the PDF $f(x)$.
(c) Find $E[X]$.
(d) Find the PDF of $Y = X^{1/2}$.

3. Show that if the density satisfies $f(x) = f(-x)$ for all $x \in R$ then the distribution function satisfies $F(-x) = 1 - F(x)$.

4. The skewness of a distribution (random variable X) with mean μ and standard deviation σ is skew $= \frac{E(X-\mu)^3}{\sigma^3}$.
(a) Show that if the density function is symmetric about μ, then skew $= 0$.
(b) Calculate skew for a random variable with density $f(x) = \lambda \exp -(\lambda x), x \geq 0$.

3. Let X be a random variable with mean μ and variance σ^2. Show that $E(X-\mu)^4 \geq \sigma^4$. (The fourth central moment is call *kurtosis*. This is a commonly used term that you need to know. For the standard normal the kurtosis is 3, so for a normal with variance σ^2 it is $3\,\sigma^4$ and because the normal is sort-of a standard, we say that a distribution with higher variance than that has “excess kurtosis”.)