

ECONOMICS 7330—Probability and Statistics, Fall 2023

Homework 2. Due Wednesday September 6.

1. Let $X \sim U[0, 1]$. Find the distribution function of $Y = \log\left(\frac{X}{1-X}\right)$.

2. Define

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-x} & \text{if } x \geq 0. \end{cases} \quad (1)$$

(a) Show that $F(x)$ is a CDF.

(b) Find the PDF $f(x)$.

(c) Find $E[X]$.

(d) Find the PDF of $Y = X^{1/2}$.

3. Show that if the density satisfies $f(x) = f(-x)$ for all $x \in \mathbb{R}$ then the distribution function satisfies $F(-x) = 1 - F(x)$.

4. The skewness of a distribution (random variable X) with mean μ and standard deviation σ is $\text{skew} = \frac{E(X-\mu)^3}{\sigma^3}$.

(a) Show that if the density function is symmetric about μ , then $\text{skew} = 0$.

(b) Calculate skew for a random variable with density $f(x) = \lambda \exp(-\lambda x), x \geq 0$.

3. Let X be a random variable with mean μ and variance σ^2 . Show that $E(X-\mu)^4 \geq \sigma^4$. (The fourth central moment is called *kurtosis*. This is a commonly used term that you need to know.)