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ECONOMICS 7330—Probability and Statistics, Fall 2022

Homework 2. Wednesday August 31. Due Wednesday September 7.

1. Let B be an event and $A_1, A_2, ..., A_n$ be n mutually exclusive events. Define $A = \bigcup_{i=1}^{n} A_i$. Also assume $P(A_i) > 0$ and $P(B|A_i) = p$ for all i. Show that $P(B|A)$ is also equal to p. [A Venn diagram might help.]

2. (In this an the following questions, use the frequencies given as probabilities.) A study of college students finds that while 70 percent of college students are male, only 50 percent of college students with an A average are male. In contrast, 15 percent of female students have an A average. Assuming these results are accurate answer the following questions.

a) Are "being a male student" and "having an A average" independent? Why?

b) What is the probability that a randomly selected student has an A average?

c) What is the probability that a randomly selected male student has an A average?

3. (20% of Midterm 1, 2005) Suppose that you consider 3 events: A: You pass the core exam. B: You get an A in statistics. C: The Astros (Houston sports team) wins the World Series. Assume that

$P(A) = 1/3$, $P(B) = 1/2$ and $P(C) = 1/5$.

Further assume that the event C is independent of both A and B (and all subsets of these). Finally, we assume that $P(A|B) = 2/3$.

a) What is the probability that you will pass both the core exam and get an A on the statistics exam?

b) What is the probability that either the Astros win or you get an A in statistics or you pass the core?

c) Assuming that a clairvoyant tells you that you will pass the core. Given that, what will be the probability that you will get an A in statistics?

d) What is the probability $P(A \bigcup C|B)$?

(The next three questions are from Hansen's book.)

4. Let $X \sim U[0,1]$. Find the PDF of $Y = X^2$.

5. Let $X \sim U[0,1]$. Find the distribution function of $Y = \log(\frac{X}{1-X})$.

6. Define

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-x} & \text{if } x \geq 0 \,. \end{cases} \tag{1}$$

(a) Show that $F(x)$ is a CDF.

(b) Find the PDF $f(x)$.

(c) Find $E[X]$.

(d) Find the PDF of $Y = X^{1/2}$.