

ECONOMICS 7330—Probability and Statistics, Fall 2023

Homework 1. Monday August 22. Due Wednesday August 31.

1. Describe the sample space S for the following experiments.
 - (a) Flip a coin.
 - (b) Roll a six-sided die.
 - (c) Time waiting for a cab.
2. Prove that $P[A \cup B] \geq P[A] + P[B] - 1$.
3. Prove that $[\bigcup_{i=1}^{\infty} A_i]^C = \bigcap_{i=1}^{\infty} A_i^C$.
4. (Hansen 1.18). A , B , and C are three events. If $P(A \cap B | C) = P(A | C) * P(B | C)$ we say that A and B are conditionally independent given C . Consider the experiment of tossing two dice. Let $A = \{\text{First die is 6}\}$, $B = \{\text{Second die is 6}\}$, and $C = \{\text{The two dies are identical}\}$. Show that A and B are (unconditionally) independent, but A and B are dependent given C .
5. We observe the price of 2 stocks, stock A and stock B .
The probability that the price of stock A increases is 0.7 and the probability that the price of stock B increases is 0.5. We also know that the probability of the event that the price of either stock A or stock B increases is 0.9.
 - a) What is the probability that the price of stock A increases at the same time as the price of stock B increases?
 - b) What is the probability that the price of A increases if you observe that the price of stock B increases?(State clearly which formulas you used).
6. Let B be an event and $A_1, A_2, \dots, A_n$ be n mutually exclusive events. Define $A = \bigcup_{i=1}^n A_i$. Also assume $P(A_i) > 0$ and $P(B | A_i) = p$ for all i . Show that $P(B | A)$ is also equal to p . [A Venn diagram might help.]
7. Let $X \sim U[0, 1]$ (uniform distribution). Find the PDF (density) of $Y = X^2$. (Use the formula, or find the CDF first.)