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ECONOMICS 7330 – Probability and Statistics, Fall 2024

Homework 10. Due Wednesday November 13.

1. Consider the exponential model with density

$$f(t_i) = \frac{1}{\lambda_i} \exp(-t_i/\lambda_i)\,,$$

where λ_i is the mean (in a major application of the exponential distribution it is the expected waiting time for individual i). We assume we have a sample of N persons. We cannot estimate a parameter for each person, but we can estimate the effect of covariates by assuming $\lambda_i = x_i\beta$, where we now have a limited number of parameters as the dimension of β is the number of covariates ("regressors") included.
i) Find the Score and Hessian (or what Hansen calls "Likelihood Hessian" which is multiplied by –1). Note that when λ_i varies with i you need to find the expectation for each$i$ taking t_i as a random variable. The expression of course takes the same form for each i apart of the x_i that depends on i. Also assume the Law of Large numbers hold. (Note that the parameter is now β. In general, β is a vector, but you could start by solving the univariate case.)
ii) Use this to write down the asymptotic distribution for $N(\hat{\beta} - \beta)$.

2. Consider the normal model, with its mean a function of individual-specific covariates (regressors). Here there is just one regressor. The ML-estimator maximizes the log-likelihood (suppressing the π term that does not affect maximum:

$$\Sigma_{i=1}^N - 0.5\log(\sigma^2) - 0.5\frac{(y_i - \mu_i)^2}{\sigma^2}\,,$$

where $\mu_i = X_i\beta$. (We take the X values as constant throughout.) The parameter vector now is $\theta' = (\beta, \sigma^2)$. I did the following in class, but you may well be asked to do it in the exam without your notes, so you should do it without looking at the notes.
i) Find the ML estimator. (Of β and σ^2. We already found $\hat{\beta}$ in the handout.)
ii) Find the score (a 2-dimensional vector).
iii) Find the Hessian and the asymptotic variance.

3. (Hansen Exercise 13.2.) Take the Poisson model with parameter λ. We want a test for $H_0 : \lambda = 1$ against $H_1 : \lambda \neq= 1$.
(a) Write down the expression for the Wald test. .
(b) Find the likelihood ratio statistic