ECONOMICS 7330, Fall 2025
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Final Exam, December 8, 2025—6 questions (100 points). All sub-questions carry equal weight.

1. (20%) Consider a random variable X, which is uniformly distributed on the interval $[0, Y]$, where Y is a discrete random variabe with the distribution

Y	P(Y)
Y=1	0.5
Y=2	0.3
Y=4	0.2

i) Write down the distribution of $E(X|Y)$ (that is, the values and their probabilities).
ii) Find the mean of X.
iii) Find $P(X > 1)$.
iv) Are the events $X > 1$ and $Y = 2$ independent?

2. (24%) Given a joint discrete probability distribution with

$$P(x, y) \;=\; \frac{x + y}{32}\;,\; x = 1, 2\,;\; y = 1, 2, 3, 4\,.$$

Find:

a) $P(x)$, the marginal probability function for X.

b) $P(y)$, the marginal probability function for Y.

c) $P(X < Y)$.

d) $P(Y = 3X)$.

e) The distribution of $X + Y$.

f) $P(X \leq 4 - Y)$.

3. (15%) Consider a sample of N independent trivariate variables $(X, Y, Z)'$ with density

$$f(x, y, z) = \kappa e^{-[(x-1)^2+(y-2)^2+3z]}\,.$$

a) Are x, y, and z mutually independent?
b) What is the marginal distribution of (x, y)?
c) What is the value of κ?

4. (12%) Assume that $Z = (Z_1, Z_2, Z_3, Z_4)'$ is a vector normally distributed random variable with mean μ and variance-covariance matrix Σ, where

$$\Sigma = \begin{pmatrix} 3 & 0 & 1 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 1 & 6 & 2 \\ 0 & 1 & 2 & 4 \end{pmatrix} \quad \text{and} \quad \mu = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \end{pmatrix}$$

a) What is the conditional mean of Z_3 given Z_2?
b) What is the conditional mean of $(Z_3 + Z_4)$ given $(Z_1, Z_2)'$?

5. (15%) Assume that $\sqrt{N}(\hat{\theta}_1, \hat{\theta}_2)'$ converges in distribution to a vector normally distributed random variable with mean $(1, 1)'$ and variance-covariance matrix Σ.

$$\Sigma = \begin{pmatrix} 4 & 2 \\ 2 & 9 \end{pmatrix}$$

a) What is the asymptotic distribution of $\sqrt{N}\,(\hat{\theta}_1^2 - 1)$? (It is implicit here that $\hat{\theta}$ depends on N and N goes to infinity.)
b) If you have 90 observations and rely on the asymptotic distribution as an approximation. What would be the standard error that you would use for $\hat{\theta}_1$?
c) What is the asymptotic distribution of the vector $\sqrt{N}[2\log\hat{\theta}_1), \theta_1\hat{\theta}_2^2]'$?

4. (14%) Assume that a random variable X is uniformly distributed on the interval [2, 4].
a) If $h(x) = 7 + 3\,x$, what is the distribution of the random variable $Y = h(X)$?
b) If $h(x) = e^x$, what is the distribution of the random variable $Y = h(X)$?
You have to be explicit about both the density for Y and the support (the area where the density for Y non-zero).