Bent E. Sørensen

ECONOMICS 7344 – MACROECONOMIC THEORY II, Spring 2023

Homework 2, Wednesday March 29. Due April 5.

1. Assume that y_t follows the AR(2) process

$$y_t = \mu + a y_{t-1} + b y_{t-2} + e_t \quad (*)$$

where e_t is white noise with variance σ_e^2.

a) (8%) Find the mean and variance of y_t assuming that y_t is stationary. (The importance of this question is in practicing the manipulation of the AR(2) equations, most people will not remember the actual formulas.)

2. A common model in economics is one where a typical agent's log wage is a sum of a random walk and independent white noise.

Define y_t such that

$$y_t = y_{t-1} + u_t,$$

where u_t is white noise with variance σ_u^2, and define

$$w_t = y_t + e_t,$$

where e_t is white noise with variance σ_e^2.

Econometricians has estimated this model by matching the moments of empirical wages to the theoretical moments of this model. w_t is not stationary (it is not stable so it cannot be stationary), so econometricians instead finds the moments of Δw_t.
Find the variance, and the autocovariances of order one and two for Δw_t.

3. Assume that an agent's wage income follows the AR(1) process

$$y_t = \mu + \beta y_{t-1} + e_t \quad (*)$$

where e_t is white noise with variance σ_e^2 and $\beta < 1$.

Assume the agent's wage was 100$ period 0.
a) What is the agent's expected wages in period t (for any $t > 0$)?
b) If the discount rate $\delta = \frac{1}{1+r}$ is 0.9 percent, what is the discounted (conditional) expected value of all future income ($\Sigma_{t=0}^{\infty} \delta^t E_0 y_t$)? (Hint: use the formula for geometric sums in δ times β. It takes a few more steps to get to that for the mean term.)

4. (From the January 2011, make-up core exam.) A consumer lives for 2 periods and earns $Y_1 = 20\$$, in period 1, and in period 2 he or she earns $Y_2^a = 20\$$ with probability 1/2 (state a) and $Y_2^b = 30\$$ with probability 1/2 (state b). The consumer starts with 0 assets and maximizes

$$U(C_1) + \frac{1}{1.10} E_1 U(C_2) ,$$

where

$$U(C) = 100C - \frac{1}{2} C^2 .$$

Assume that the safe rate of interest is 10 percent.

A) (5%) Let B denote the amount lent in period 1 (or, equivalently, the amount of a safe bond bought). Assuming that the agent only have access to a safe bond, find B and consumption in each period (for period 2, that means the consumption plan listing consumption in state a and state b.)

For the next question, assume the rate of interest on the bond (lending) is 0 percent and the consumer maximizes

$$U(C_1) + E_1 U(C_2) .$$

(These changes are just to simplify calculations.)

B) (15%) Now assume that a stock (equity) exists besides the safe bond. Let the amount of equity bought be S (it can be negative). Assume that the stock has a (net) rate of return of 0% if state a occurs [meaning that agent gets back the principal] and 100% if state b occurs. Find B and S and the implied consumption plan. (Note: the question is set up with "extreme" values to make the algebra easier, so the solution may also be "extreme." Also note, that for the PIH negative values of consumption are valid. If you are running out of time, most points will be accrued when you write down the equations that determines the answer.)