ECONOMICS 7344, Spring 2022

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HOMEWORK 4. Due Friday April 22.

1. (20% of final, Summer 2010). Assume that there are two states of the economy next year, "good" and "bad," each with probability 0.5. In the good state aggregate consumption grows 4% and in the bad state it grows 0%. Now consider assets D and E. For these we know the payouts. For D the payout is 5 in the bad state and 15 in the good state, while for E the payout is 5 in the bad state and 5 in the good state. Use the CCAPM as it was derived in the handout. The safe rate of return is 1%.
a) What would be the prices of assets D and E?
b) What would be the returns (you can give gross or net, but state which) of assets D and E?

2. (10% of final 2010) Consider the CAPM-model. Assume the safe rate of interest is 10%, the mean return to the market portfolio is 20% and the variance of the return to the market portfolio is 0.02. Now consider assets D and E. For these we know the distribution of the pay-outs. For D the payout is normally distributed with mean 100 and variance 10, while for E the payout is normally distributed with mean 200 and variance 40. Assume the covariance of the payout to asset D with the market return is 1 while the covariance of payout to asset E with the market return is 2.
What would be the prices of assets D and E?

The following questions are in the "Obstfeld-Rogoff" note, so you probably do not want to start on this before class Monday.

3. (Partial equilibrium.) Consider an agent with income ("output" in Obstfeld-Rogoff) $Y_1 = 10$, $Y_2^A = 16$, and $Y_2^B = 6$, where A and B are states of the world with $\pi^A = 0.5$ and $\pi^B = 0.5$. Assume $p^A = p^B$, $r = 10\%$ and the discount rate is $\beta = \frac{1}{1+r}$.

a) Assume the agent has quadratic utility and that the agent can trade in Arrow-securities for both state A and state B. Does the "PIH-relation" $C_1 = EC_2$ hold?

b) Find C_2^A/C_2^B.

c) How many units of each Arrow-security does the agent purchase and how many units of the period 1 good? (this can be a negative number so "purchase" may mean sell.)

Now assume that the agent has utility function $U(C) = -\frac{1}{3}C^{-3}$.

d) Find C_2^A/C_2^B. (Give the intuition for why it does or does not change from the answer in part b). [This is probably a hard question]).

e) Find C_1.

f) Now assume $\frac{p^A}{p^B} = \frac{2}{3}$. Now find C_1 and C_2^S for $S = A, B$ and check if $C_1 = EC_2$.