ECON 8331 — ECONOMETRICS II. 8/26

Some useful tools that we will use.

1. Some standard results from statistics:

   - If X is a vector with variance Σ, the variance of AX is $A\Sigma A'$, to not put the transposition sign the wrong place, especially because A is not necessarily quadratic. For $A = \iota$ (a vector of ones), you get the variance of the sum of elements in X.

   - You should remember from Econometrics I that if $\text{Var}(X) = \Sigma$ there is a square root C of Σ so that $\Sigma = CC'$ and $C^{-1}\Sigma C^{-1'} = I$. There are many square root matrices and you can choose one that is upper triangular.

2. The multivariate (N-dimensional) normal distribution has density

$$\frac{1}{((2\pi)^N|\Sigma|)^{0.5}} \exp\{-0.5(X-\mu)'\Sigma^{-1}(X-\mu)\}.$$

   (Note, I often forget the π term which does not affect the ML estimator.)

- For dimension 2, we can write

$$\frac{1}{2\pi\sqrt{\sigma_1^2\sigma_2^2-\sigma_{12}^2}}*$$

$$\exp\{-\frac{1}{2\,(1-\rho^2)}\,(\frac{(x_1-\mu_1)^2}{\sigma_1^2}-\frac{2\rho(x_1-\mu_1)(x_2-\mu_2)}{\sigma_1\,\sigma_2}+\frac{(x_2-\mu_2)^2}{\sigma_2^2})\}\ ,$$

  where ρ is the correlation between x_1 and x_2. This simplifies to the product of two univariate normal distributions, if the covariance is zero (which implies, for the normal, that x_1 and x_2 are independent).

3. We also need to consider a multivariate normal distribution of form $(X', Y')'$ (this is just a way to write a partition column vector with X on top). The mean is $(\mu'_X, \mu'_Y)'$, the variance of X is Σ_X, of Y Σ_X and the covariance $E(X-\mu_X)(Y-\mu_Y)' = \Sigma_{XY}$. (Notice that we cannot just say the covariance as the dimension depends on which is the "first" variable—the covariance matrix is not in general symmetric. Although, I often do it anyway, but make sure the dimensions match.) Also notice where the transposition sign goes, if put it on X the expression is meaningless, it isn't even a legal mathematical operation in that case if the dimensions of X and Y differs.

- The mean of X conditional on Y is $\mu_X + \Sigma_{XY}\Sigma_Y^{-1}(Y - \mu_y)$.

- The conditional variance of X is $\Sigma_X - \Sigma_{XY}\Sigma_Y^{-1}\Sigma_{YX}$.

In general, the conditional density of X, if Y is observed to be Y_0 is $f(X|Y_0) = f(X, Y_0)/f(Y_0)$. We can also treat Y_0 as a random variable here, in which case we usually drop the 0. (That is how you prove the previous result, but we will not check that).

4. The probability that $X \in A$ conditional on Y is $\int_{x \in A} f(x|Y)dx$.

5. $f(x|x > a)$ is $f(x)/(1 - F(a))$.