RICARDIAN EQUIVALENCE

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Borrowing limits and Ricardian equivalence

- Does the timing of taxes matter ?
- How the timing of taxes interacts with restrictions on the ability of households to borrow ?
  - Two settings:
    - (1) an infinite horizon economy with an infinitely lived representative agent
    - (2) an infinite horizon economy with a sequence of one-period-lived agents, each of whom cares about its immediate descendant.

Infinitely Lived Agent Economy

- $\sum_{t=0}^{\infty} \beta^t u(c_t)$ (10.2.1) $\beta \in (0,1)$, $u(.)$ is str. incr., str. conc., twice diff.
- Inada Condition: $\lim_{c \to 0} u'(c) = +\infty$
- $\exists$ single risk-free asset & R>1
- Budget Constraint: $c_t + R^{-1} b_{t+1} \leq y_t + b_t$ (10.2.2)
- b_0 is given, $R\beta = 1$
- $\{y_t\}_{t=0}^{\infty}$ is nonstochastic, nonnegative endowment sequence where $\sum_{t=0}^{\infty} \beta^t y_t < \infty$

Infinitely Lived Agent Economy

- Two restrictions on asset holdings $\{b_t\}_{t=0}^{\infty}$:
  - $b_t \geq 0\,, \quad \forall\, t \geq 0$
  - $b_t \geq \widetilde{b_t}$
    - $\widetilde{b_t} = -\sum_{j=0}^{\infty} R^{-j}\, y_{t+j}$
    - $\lim_{T\to\infty} R^{-T} b_{t+T} = 0$

Optimal consumption/savings decision when $b_{t+1} \geq 0$

- Choose $\{c_t, b_{t+1}\}_{t=0}^{\infty}$ to maximize $\sum_{t=0}^{\infty} \beta^t u(c_t)$, given b_0
- subject to $c_t + R^{-1} b_{t+1} \leq y_t + b_t$ (10.2.2)

  FOC:
  - $u'(c_t) \geq \beta R u'(c_{t+1}), \forall\, t \geq 0$
  - $u'(c_t) > \beta R u'(c_{t+1}) \rightarrow b_{t+1} = 0$
- The optimal consumption plan depends on the $\{y_t\}$ path

Examples

Example 1: The consumer is never borrowing constrained.

- $b_0 = 0$, $\{y_t\}_{t=0}^{\infty} = \{y_h, y_l, y_h, y_l, \ldots\}$, $y_h > y_l > 0$
- PV of the household's endowment:

$$\sum_{t=0}^{\infty} \beta^t y_t = \sum_{t=0}^{\infty} \beta^{2t}(y_h + \beta y_l) = \frac{y_h + \beta y_l}{1 - \beta^2}$$

- PV of the annuity value of $\bar{c}$:

$$\frac{\bar{c}}{1-\beta} = \frac{y_h+\beta y_l}{1-\beta^2}$$

- Solution: $c_t = \bar{c}$, $\forall\ t \geq 0$

$$b_{t+1} = \frac{(y_h - y_l)}{1+\beta}$$

Examples

Example 2: The consumer is borrowing constrained the first period.

- $b_0 = 0$, $\{y_t\}_{t=0}^{\infty} = \{y_l, y_h, y_l, y_h, \ldots\}$, $y_h > y_l > 0$
- The optimal plan: $c_0 = y_l$, $b_1 = 0$

from period 1 onward, the solution is the same as in example 1.

Example 3: The consumer is always borrowing constrained.

- $b_0 = 0$, $y_t = \lambda^t$, $1 < \lambda < R, \lambda\beta < 1$
- Solution where $b_t \geq 0$: $c_t = \lambda^t$, $b_t = 0, \forall\, t \geq 0$

Optimal consumption/savings decision when $b_{t+1} \geq \widetilde{b_{t+1}}$

Example 4:

- $b_0 = 0,\ y_t = \lambda^t$
- PV of the household's endowment:

$$\sum_{t=0}^{\infty} \beta^t \lambda^t = \frac{1}{1 - \lambda\beta}$$

- Constant consumption level $\hat{c}$:

$$\frac{\hat{c}}{1-\beta} = \frac{1}{1-\lambda\beta}$$

- Solution: $b_t = \frac{1-\lambda^t}{1-\beta\lambda}$, $b_t > \widetilde{b_t} = -\lambda^t/(1-\beta\lambda)$

Optimal consumption/savings decision when $b_{t+1} \geq \widetilde{b_{t+1}}$

Example 5:

- $b_0 = 0, y_t = \lambda^t, \lambda < 1, \lambda\beta < 1$
- Solution: $c_t = \hat{c}$ even if the $b_t \geq 0$ is imposed

Government

- $\sum_{t=0}^{\infty} \beta^t u(c_t)$ (10.2.1), $\beta \in (0,1)$, $u\,(.)$ is str. incr., str. conc., twice diff.
- $\{g_t\}_{t=0}^{\infty}$: does not appear in the utility function, $\{\tau_t\}_{t=0}^{\infty}$: lump-sum taxes
- Government's Budget Constraints:

$$B_t + g_t = \tau_t + R^{-1} B_{t+1} \text{ (10.3.1)}$$

  - By solving the budget constraint forward we reach to an intertemporal constraint:

$$B_t = \sum_{j=0}^{\infty} R^{-j} \left(\tau_{t+j} - g_{t+j}\right) \text{ (10.3.2)}$$

  - For $t \geq 0$, $\lim_{T \to \infty} R^{-T} B_{t+T} = 0$

Government: Effect on household

- Consumer's Tax Adjusted Budget Constraint: $c_t + R^{-1}b_{t+1} \leq y_t - \tau_t + b_t$ (10.3.3)
  - Solve forward:

$$b_t = \sum_{j=0}^{\infty} R^{-j}\,(c_{t+j} + \tau_{t+j} - y_{t+j})\ (10.3.4)$$

  - By setting $c_t = 0$ we obtain natural debt limit:

$$\tilde{b}_t \geq \sum_{j=0}^{\infty} R^{-j}\,(\tau_{t+j} - y_{t+j})\ (10.3.5)$$

Government:
Effect on household

- Definition: Given initial conditions (b_0, B_0), an *equilibrium* is a household plan $\{c_t, b_{t+1}\}_{t=0}^{\infty}$ and a government policy $\{g_t, \tau_t, B_{t+1}\}_{t=0}^{\infty}$ such that (a) the government policy satisfies the government budget constraint (10.3.1), and (b) given $\{\tau_t\}_{t=0}^{\infty}$, the household's plan is optimal.

- Proposition 1: Suppose that the natural debt limit prevails. Given initial conditions (b_0, B_0), let $\{\bar{c}_t, \bar{b}_{t+1}\}_{t=0}^{\infty}$ and $\{\bar{g}_t, \bar{\tau}_t, B_{t+1}\}_{t=0}^{\infty}$ be an equilibrium. Consider any other tax policy $\{\hat{\tau}_t\}_{t=0}^{\infty}$ satisfying

$$\sum_{t=0}^{\infty} R^{-t}\, \hat{\tau}_t = \sum_{t=0}^{\infty} R^{-t}\, \bar{\tau}_t \quad (10.3.6)$$

Government: Effect on household

- Then $\{\bar{c}_t, \hat{b}_{t+1}\}_{t=0}^{\infty}$ and $\{\bar{g}_t, \hat{\tau}_t, \hat{B}_{t+1}\}_{t=0}^{\infty}$ is also an equilibrium where:

$$\hat{b}_t = \sum_{j=0}^{\infty} R^{-j} \left(\bar{c}_{t+j} + \hat{\tau}_{t+j} - y_{t+j}\right) \quad (10.3.7)$$

$$\hat{B}_t = \sum_{j=0}^{\infty} R^{-j} \left(\hat{\tau}_{t+j} - \bar{g}_{t+j}\right) \quad (10.3.8)$$

Government: Effect on household

- Proof:
  - 1st Point of the Proposition: The same consumption plan $\{\bar{c}_t\}_{t=0}^{\infty}$, but adjusted borrowing plan $\{\hat{b}_{t+1}\}_{t=0}^{\infty}$, solve the household's optimum problem under the altered government tax scheme. Under the natural debt limit, household faces a single intertemporal budget constraint (10.3.4). At time 0:

$$b_0 = \sum_{t=0}^{\infty} R^{-t}\,(c_t - y_t) + \sum_{t=0}^{\infty} R^{-t}\,\tau_t$$

  - We construct the adjusted borrowing plan $\{\hat{b}_{t+1}\}_{t=0}^{\infty}$ (by solving the 10.3.3 forward to obtain 10.3.7) which satisfies the adjusted natural debt limit in every period.

Government: Effect on household

- 2nd Point of the Proposition: The altered government tax and borrowing plans continue to satisfy the government's budget constraint. At time 0:

$$B_0 = \sum_{t=0}^{\infty} R^{-t}\,\tau_t - \sum_{t=0}^{\infty} R^{-t}\,g_t$$

  - Under the altered tax plan with an unchanged present value of taxes, the government can finance the same expenditure plan $\{\bar{g}_t\}_{t=0}^{\infty}$
  - The adjusted borrowing plan $\{\hat{B}_{t+1}\}_{t=0}^{\infty}$ is computed in a similar way as above to arrive at (10.3.8).

Government: Effect on household

- Proposition 2: Consider an initial equilibrium with consumption path $\{\bar{c}_t\}_{t=0}^{\infty}$ in which $b_{t+1} > 0$ for all $t \geq 0$. Let $\{\bar{\tau}_t\}_{t=0}^{\infty}$ be the tax rate in the initial equilibrium, and let $\{\hat{\tau}_t\}_{t=0}^{\infty}$ be any other tax-rate sequence for which

$$\hat{b}_t = \sum_{j=0}^{\infty} R^{-j} \left(\bar{c}_{t+j} + \hat{\tau}_{t+j} - y_{t+j}\right) \geq 0$$

  for all $t \geq 0$. Then $\{\bar{c}_t\}_{t=0}^{\infty}$ is also an equilibrium allocation for the $\{\hat{\tau}_t\}_{t=0}^{\infty}$ tax sequence.

- Proof ?