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Efficient Markets.

Efficient markets. In many financial textbooks the Efficient Market Hypothesis (EMH) is treated prominently (see Chapter 2 of the Campbell, Lo, MacKinlay text or any undergraduate finance book on investment). "Efficient" financial markets should probably price assets correctly—reflecting expected future earnings and allowing for a reasonable reward for accepting uncertain returns ("risk premia"). Campbell, Lo, MacKinlay consider the issue of prices verses future dividends; but this is an area that is much harder than one should think, mainly because it is hard to know what are the discount rates that agents may use for future earnings.

So the EMH refers to the simpler question of whether asset prices are *predictable*. The simple logic is that if agents could predict that an asset would increase in value, this could not be an equilibrium (everyone would buy, increasing the price instantly until it would equal the expected future price). Starting with Eugene Fama of the University of Chicago, financial economists specify 3 forms of the EMH. The **weak form** version of the EMH states that no abnormal profits can earned from trading in past patterns of returns to a stock. Another way of stating the weak form EMH is that all information contained in historical prices are fully reflected in current prices. There are people publishing books about so-called *technical analysis*. Technical analysis is basically the search for recurrent patterns in a stock's price. The weak form EMH says that one can not make money from technical analysis. The **semi-strong** form of the EMH states that no publicly available information is useful for predicting asset returns and the **strong form** form of the EMH states that no information (public or not) is useful for forecasting returns. The strong form is not likely to be true and cannot be tested.

Example 1: One might think that a stock that does well today will also do well (or maybe less well) tomorrow - this would happen if positive information that become available about a firm only gradually affects the market price. How do we test this? We run an autoregression of the form

$$r_{it} = \alpha_i + \beta_1 r_{it-1} + ... + \beta_l r_{it-l} + u_{it} \ .$$

If the weak form EMH is true the $\beta_l = ... = \beta_1 = 0$, which you can test with an F-test. Most often you will include only one or two lagged variables in the regression In other words, the weak form EMH states that the return of any stock (or portfolio) should be unpredictable "white noise."
If the return series is white noise this means that the expected value of the stock *price* tomorrow is equal to the stock price today (when we correct for the safe return). Strictly speaking we should talk about whether the stock price is a martingale but you will often hear that EMH implies that stock prices are random walks (these are just technicalities that I mention for completeness, you can use either term). The random walk model takes the form

$$p_{it} \ = \ \alpha_i + p_{it-1} + u_{it} \ , \quad (*)$$

where p_{it} is the price of asset i at time t (corrected for dividends). The mean α_i captures the safe rate of interest and a risk premium.[1] Often you will see the random walk hypothesis stated as

$$p_{it} \;=\; p_{it-1} + u_{it} \;,$$

(i.e., without any drift term), which is reasonable for daily data since daily safe rate of interest (and the risk premium) is very small. The book "A Random Walk Down Wall Street", by Burton Malkiel has a lot of discussion about efficient market and the name of the book is a pun on the random walk model. One can perform tests for EMH directly on the stock prices rather than looking at returns, but this is complicated by "unit-root" econometric issues so we often prefer to look at returns.

If prices are random walks, then the expected value of today's price is yesterday's price, i.e. $E\{p_{it}\} = p_{it-1}$, which approximately implies that $E\{\ln p_{it} - \ln p_{it-1}\} = 0$. Since

$$E\left\{\ln p_{it} - \ln p_{it-1}\right\} = E\left\{\ln\left(p_{it}/p_{it-1}\right)\right\} = E\left\{\ln\left(1 + \frac{p_{it} - p_{it-1}}{p_{it-1}}\right)\right\} = E\left\{\ln\left(1 + r_{it}\right)\right\} \approx E\, r_{it} \;,$$

it follows that prices being random walks is equivalent to expected returns being 0. (More precisely, this should be all be done in terms of excess returns.) Typically, this is tested by estimating autoregressive models and testing if the coefficients to the lagged variables are zero. When researchers test the weak-form EMH they will typically start by calculating autocorrelations and see if they are close to 0, and then move on to estimate an autoregressive model and test if the coefficients are zero.

Example: "Filter Rules." Some have argued that one can make excess profit from following a rule like: Buy a stock when it increases more than (say) 1% from its previous low and sell it when it decreases more than (say) 0.5% from its previous high. Any such mechanical rule is called a filter rule. The way to test this is to identify such periods from the data, calculate what the return would have been to an investor following such a strategy and compare to the normal return on the stock. The autoregressive model is strictly speaking a simple filter rule. Academic research has focussed on rather simple models (compared to what you find in many books about technical analysis). But the real difference between researchers and charlatans is whether supposedly money-making filter rules are examined using statistic (econometric) tools or not. Many book are published on technical analysis and they are all nonsense (if you found a pattern in the data that allowed you to make money, you wouldn't tell anybody!)

So far I have followed the older simpler finance literature. As academic economists, we would depart from the Euler Equation.

[1] α_i is called the *drift* since you can solve the equation "recursively." Note that the equation holds for all t so by recursive substitution you get $p_{it} \;=\; t\alpha_i + p_{i0} + u_{i1} + u_{i2} + ... + u_{it}$, so the expected value of the price goes up as a line with slope α_i.

Pricing a payoff with the Euler Equation

If we consider the payoff to an asset that you can buy today and which pays off tomorrow then we can find the price using the Euler equation. Here I assume a 2 period model.

Consider an asset i with a payoff tomorrow of PO_i. The net return to a dollar investment in asset i is now

$$r_i = \frac{PO_i}{P_i} - 1 \qquad (a) \ ,$$

where P_i is today's price. Assume that the correlation of PO_i with tomorrow's marginal utility $U'(c_{t+1}$ is known. Then the Euler equation becomes

$$U'(c_t) \ = \ \beta E\{U'(c_{t+1})\frac{PO_i}{P_i}\} \ ,$$

where $\beta = 1/(1+\rho)$ is the discount factor. This determines the price:

$$P_i \ = \ E\{\beta\frac{U'(c_{t+1})}{U'(c_t)}PO_i\} \ ,$$

or, if we define,

$$m_t = \beta\frac{U'(c_{t+1})}{U'(c_t)}$$

we have

$$(*) \quad P_i \ = \ E\{m_t PO_i\}$$

for any asset i. As Cochrane points out in his text, most asset pricing models have the form (*) for different definitions of m_t which is often called a "pricing kernel" in more abstract treatments in finance. Very often m_t has an interpretation as an approximation to the change in aggregate marginal utility.

The 2-by-2 case. We will stress the simplest case with 2 periods and 2 outcomes in period 2, in order to build up understanding (I will likely ask several "2-by-2 questions" on the exam).

Pricing a pay-off using the Euler equation (and assuming exogenous consumption) in the 2 period, 2 states-of-the-world case.

A consumer lives for 2 periods and consumes C_1, in period 1, and in period 2 he or she consumes C_2^a in state a and C_2^b in state b.
You want to find the price P_i of an asset that has a payoff in period 2 of PO_i^a in state a and PO_i^b in state b. Assume that the conditional probability of state a and the conditional probability of state b is Pr^b (we often leave the word "conditional" out, but in this case it is implicit that it is conditional probabilities). As always use the Euler equation

$$U'(C_1) \ = \ \beta E\{U'(C_2)(1+r_{t+1})\} \ ,$$

which becomes

$$U'(C_1) \;=\; \beta\,[Pr^a U'(C_2^a)\frac{PO_i^a}{P_i} + Pr^b U'(C_2^b)\frac{PO_i^b}{P_i}\,] \;.$$

Now solve for P_i:

$$P_i \;=\; \beta\;[\,Pr^a\,\frac{U'(C_2^a)}{U'(C_1)}\,PO_i^a + Pr^b\,\frac{U'(C_2^b)}{U'(C_1)}\,PO_i^b\,] \;.$$

If you fully understand that equation, you basically understand the full Lucas asset pricing model (with more periods, more agents, more states-of-the-world, the math may look more complicated, but the intuition will not change).

Observe:

- The price is proportional to the discount factor. The price is how much of period 1 good to hand over for the right to a random period 2 good, so the less you value the future the less you want to pay).
- The price is proportional to the payout in period 2 (meaning that if you, say, double *both* PO_i^a and PO_i^b, the price of the asset will double. This is obvious: you pay twice as much for twice as much.
- If an asset only has a payout in state a then the price is proportional to the probability that state a will happen (of course, same for b). An example of an asset that pay out in one state of the world is a lottery ticket. If you buy two tickets, you double the probability of getting the payout and, of course, you pay twice as much.

The price of the asset is also determined by relative scarcity. Because we assume that people's choices are described by strictly concave utility functions, agents will always want to use assets to transfer goods to the period and/or the state-of-the-world where consumption is lowest (of course still taking the previous points into accounts). [A note on the word "want:" The Lucas asset pricing model is about equilibrium prices, you cannot get the price that you want when buying anything. However, prices reflect demand and therefore people's "desires."] Regarding relative scarcity, we can observe:

- If consumption is relatively low in period 1 prices a lower. The intuition is that we prefer to consume more in period 1 relative to period 2 in this situation. The math is that the marginal utility $U'(C_1)$ is high which makes the right-hand side of the equation small.
- If consumption is relatively low in state a compared to state b. You will pay more for an asset that has a relatively high payoff in state a. The intuition is again scarcity and the math is that the payoff to state a are weighed by $U'(C_2^a)$ which is large if C_2^a is low.

Let us denote the real gross (net) return to the safe asset by R_t^f (r_t^f) where we sometimes drop the t subscript indicating that the safe interest is not time-varying. In the real world the interest will typically vary over time but the real interest rate is a lot less variable than, say, stock market returns and we therefore often assume it constant for notational simplicity. The safe asset satisfies the Euler equation, which in the "kernel notation" becomes:

$$1 = E_t\{m_{t+1}R_{i,t+1}\} \quad in \;\; particular \quad 1 = E_t\{m_{t+1}\}R^f \ .$$

(The price of an asset with payout equal to the gross return is 1), so the implication is that for any asset i,

$$E_t\{m_{t+1}(R_i - R^f)\} \;=\; 0 \ .$$

So theoretical economists agrees that excess returns should have expectation 0, but *only* after adjusting the return with the "stochastic discount factor" m_t. At short horizons m_t can typically be assumed to equal 1, so in this sense the Euler Equation agrees with the semi-strong EMH. But even at short horizons an asset with a very high correlation with m_t should not have $E_t\{(R_i - R^f)\} \;=\; 0$. In order to calculate correlations, one obviously need to have specific model for m_t.

Note that because the safe rate of interest satisfies

$$R^f \;=\; 1/E_t\{m_t\}$$

and using that $E\{XY\} = EX\,EY + Cov(X,Y)$ we have that

$$0 \;=\; Em_{t+1}E\{R_i - R^f\} + Cov(R_i m_{t+1})$$

which implies that

$$E\{r_i - r^f\} = -R^f\,Cov(r_i m_{t+1}) \ .$$

So the existence of a stochastic discount factor implies that excess returns are a function of the covariance of returns with the kernel. Several important models of asset pricing centers around such a relation.