ECONOMETRICS 1, Spring 2024
Bent E. Sørensen

Midterm Exam—February 19, 2024

Each sub-question in the following carries equal weight.

1. (20%) Assume that you have estimated the model

$$Y_i \;=\; \beta_1 X_{1i} \;+\; \beta_2 X_{2i} \;+\; \beta_3 X_{3i} \;+\; \epsilon_i$$

by OLS, and that the standard assumptions for OLS—inclusive of normality—hold. Assume that you used 10 observations. Let β be the column vector $(\beta_1, \beta_2, \beta_3)'$. We are interested in testing the following restriction:

$$R\beta = 1 \;,$$

where $R = (1, 1, 1)$. Assume that the $X'X$ matrix is given as

$$X'X \;=\; \begin{pmatrix} .2 & .1 & .0 \\ .1 & .2 & .0 \\ .0 & .0 & .001 \end{pmatrix} .$$

and that your estimated coefficients are

$$\hat{\beta}_1 = .5 \quad \hat{\beta}_2 = .6 \quad \hat{\beta}_3 = 3$$

and that you also found the estimated variance of the error term to be

$$\hat{\sigma}^2 = .2$$

a) Explain in detail which test you would use to test the restriction and give the formulas.
b) Find the numerical value of the test statistic using the numbers provided.

2. (30%) Assume that you have estimated the model

$$Y_i \;=\; X_i \beta \;+\; \epsilon_i$$

by OLS, and that the standard assumptions for OLS—inclusive of normality—holds. Assume that you have 5 observations of (X_i, Y_i) where the X matrix takes the values

$$X \;=\; \begin{pmatrix} 1 & 1 \\ 1 & -2 \\ 1 & 2 \\ 1 & -2 \\ 1 & 1 \end{pmatrix} .$$

Also assume that you find the residual vector $e = (1, -2, 0, 2, -1)'$, and that you estimate $\hat{\beta} = (2, 3)'$.

a) What are the implied values for the Y-vector?

b) If you construct

$$Z = 3X\hat{\beta} + \iota ,$$

where $\iota = (1, 1, 1, 1, 1)$, what is then the projection $P_Z e$ of the residual vector e on Z?

c) What is the projection of $\hat{Y}$ on ι?

3. (15%) Assume that you want to estimate the following model using quarterly data for 10 years:

$$y_t = \beta_0 + \sum_{k=1}^{3} \beta_k D_{kt} + \beta_4 x_t + \epsilon_t ,$$

where all the "OLS-assumptions" - including normality of ϵ_t - hold. The regressors D_{kt} are quarterly dummy variables, such that

$$\begin{aligned} D_{1t} &= 1 \text{ in the 2nd quarter ; 0 otherwise} \\ D_{2t} &= 1 \text{ in the 3rd quarter ; 0 otherwise} \\ D_{3t} &= 1 \text{ in the 4th quarter ; 0 otherwise} \end{aligned}$$

Now assume that $\bar{y} = 5$ and if we let $\bar{y}_j$; $j = 2, 3, 4$ denote the average of the y-values in the kth quarter, assume that

$$\begin{aligned} \bar{y}_2 &= 4 , \\ \bar{y}_3 &= 2 , \\ \bar{y}_4 &= 0 . \end{aligned}$$

Also assume that $\bar{x} = 0$ and that x_t is orthogonal to D_k ; $k = 1, 2, 3$.

Based on the given information, find the values of the OLS-estimates $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2$ and $\hat{\beta}_3$.

4. (15%) Assume that you want to estimate the model

$$Y_i = \beta_1 X_{1i} + \beta_2 X_{2i} + \epsilon_i ,$$

where X_1 and X_2 are orthogonal regressors.

Assume that *all* the assumptions for OLS to be efficient holds, but you accidentally estimate the model

$$Y_i = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \epsilon_i .$$

Assume that a regression of X_3 on X_1 gives an R^2 of 0, whereas a regression of X_3 on X_2 gives an R^2 of .999.
a) This inclusion of X_3 creates a problem–what is that called and how does it affect the estimated parameters (explain how it affects the properties of the OLS estimator of both β_1 and β_2).
b) What is the expected value of the OLS estimators $\hat{\beta}_1$, $\hat{\beta}_2$, and $\hat{\beta}_3$?

5. Computer question (20%). Read the Matlab code below and answer the questions in the code.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Econometrics 1
% Spring 2024
% Midterm 1
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear;
clc;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% This code estimates the model
%
%                   y = beta0 + beta1*X1 + beta2*X2 + e
%
% using OLS and calculates other things.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Generate the data.

n = 500;                        % Sample size

X1 = randn(n,1);                % X1
X2 = randn(n,1);                % X2

X = [ones(n,1) X1 X2];          % X matrix with constant

beta = [1; 3; 2];               % True coefficients
```

```
u = randn(n,1);                  % Standard normal disturbances

y = X*beta + u;                  % Observed values of y

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Estimate the coefficents using OLS.

b = inv(X'*X)*X'*y;              % OLS estimates

% Compute the standard errors.

k = size(beta, 1);               % Number of coefficients

yhat = X*b;                      % Predicted values of Y
uhat = y - yhat;                 % Residuals

s2 = (uhat'*uhat)/(n-k);         % S Squared

vc = s2*inv(X'*X);               % Variance-Covariance Matrix

se = [sqrt(vc(1,1));...
      sqrt(vc(2,2)); ...         % Standard Errors
      sqrt(vc(3,3))];

% Compute the t-statistics.

t = b./se;                       % t-statistics
t = abs(t);                      % Absolute value of t-statistics

disp(' ')
disp('Model: y = beta0 + beta1*X1 + beta2*X2 + e')
disp(' ')

disp('Regression Results')
disp(' ')

disp('   Estimates    SE     |t-stat|')
disp([b se t])
disp('Note: OLS estimates are b0, b1 and b2 in that order.')
```

```
disp(' ')

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Question 1:
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

WHAT IS GOING ON HERE?

R = [0; 1; 1];
q = [0];

J = size(R,1);

h = R*b - q;
varm = XXXXXXXXXXXXXX;          WHAT SHOULD THIS BE?

A = (h'*inv(varm)*h)/J;

disp('xxx Test')
disp('H0: beta1 = 0 and beta2 = 0')
disp(' ')

disp('     xxx-stat    ')
disp([A])
disp(' ')

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
% Question 2: Complete the code below by filling in the comments,
% identifying what AA, BB, CC and DD compute.
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

i = ones(n,1);                  % Iota
```

```
DD = eye(n) - (1/n)*i*i';       % This computes XXXXXXXXXXXXXX.

AA = b'*X'*DD*X*b;              % This computes XXXXXXXXXXXXXX.
BB = y'*DD*y;                   % This computes XXXXXXXXXXXXXX.

CC = AA/BB;                     % This computes XXXXXXXXXXXXXX.

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```