ECONOMETRICS 1, Spring, 2024
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Final Exam—April 29, 2024

1. (20%) Consider the model

$$Y_i \;=\; \beta_1 X_{1i} \;+\; \beta_2 X_{2i} \;+\; \epsilon_i \;\; ; \; i = 1, ..., N \; .$$

For simplicity assume that all variables have mean 0. Let $(X_1' X_1) = 1$, $(X_2' X_2) = 4$, and $X_1' X_2 = 0$, where X_k is the column vector $(X_{k1}, ..., X_{kN})'$.
If you estimate the model

$$Y_i \;=\; \gamma_1 X_{1i} \;+\; \gamma_2 X_{2i} \;+\; \epsilon_i \; .$$

by OLS and find the estimated standard error of $\hat{\gamma}_1$ is 2.
i) What is the estimated variance of the residual?
ii) What is the estimated standard error of $\hat{\gamma}_2$?

2. (10%) Assume that you want to estimate the model

$$Y_i \;=\; \beta_1 X_{1i} \;+\; \beta_2 X_{2i} \;+\; \epsilon_i \; ,$$

and that you know already that the true value of $\beta_1 = 7$.
Assume that the "OLS-assumptions," including normality holds.
Explain how you can estimate β_2 efficiently while imposing the true value of β_1.

3. (20%) You want to estimate the model

$$y_i = \alpha_0 + \alpha_1 x_i + u_i$$

by maximum likelihood. Assume that the variance of the error term is $var(u_i) = \gamma\, i$ for some constant γ.
i) Derive the maximum likelihood estimators for α_0 and α_1.
ii) Write down the likelihood ratio test for α_1=0.

4. (20%) Assume that you estimated the model

$$Y_i \;=\; \beta_1 X_{1i} \;+\; \beta_2 X_{2i} \;+\; \beta_3 X_{3i} \;+\; \epsilon_i \; ,$$

where the variables have mean 0. Assume $(Y'Y) = 4$ and the R^2 if this regression is 0.5.
i) What is the value of $\hat{e}'\hat{e}$?
Assume you set $(\beta_2, \beta_3) = (0, 0)$ and estimate the model under that constraint.
ii) Suggest a test for the validity of the constraint and state its distribution?

iii) Now you are told that the number of observations $N = 100$ and that the R^2 of the restricted model is 0.25. Write down a test for $(\beta_2, \beta_3) = (0, 0)$ using the information that you are given and state its distribution (here you need to put in actual real numbers).

5. (30%) i) What model is estimated in Matlab code below?
ii) Fill in where it says FILL IN HERE below (two places). You do not have to use correct Matlab notation, but you have to use the terms calculated so it is clear that you know what they are.
iii) At the bottom of the program there are two GLS-type estimators. What are they called and how do they differ?

```
clear;

consumption = xlsread('consumption.xls');
income = xlsread('income.xls');
interest = xlsread('interest.xls');

% Transpose the row vectors into column vectors.

consumption = consumption';
income = income';
interest = interest';

logc = log(consumption);
logy = log(income);

clag = logc(1:size(logc,1)-1,1);
ylag = logy(1:size(logy,1)-1,1);


logc = logc(2:size(logc,1));
logy = logy(2:size(logy,1));

c = logc - clag;
y = logy - ylag;

r = interest(2:size(interest,1));

X = [ones(size(c,1),1) r y];

b = inv(X'*X)*X'*c;
```

```
n = size(c,1);
p = size(b,1);

e = c - X*b;
sigma2 = (e'*e)/(n-p);
vmat = sigma2*inv(X'*X);

SE = [sqrt(vmat(1,1)); sqrt(vmat(2,2)); sqrt(vmat(3,3))];
tstat = XXXX FILL IN HERE XXXX

pval = 2*(1 - tcdf(abs(tstat),n-p));

% Display the results in a table.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

e_t = e(2:18);
e_t_1 = e(1:17);

m = size(e_t,1);

E_t_1 = [ones(m,1) e_t_1];

f = size(E_t_1,2);

rho = inv(E_t_1'*E_t_1)*E_t_1'*e_t;

v = e_t - E_t_1*rho; %new residuals
omega_invrt = eye(n);
omega_invrt(1,1) = sqrt(1 - rho(2)^2);

for i = 1:n-1
    omega_invrt(i+1,i) = -rho(2);
end

omega_invrt = omega_invrt*(1/sigma2_e);

omega_inv = (omega_invrt'*omega_invrt);

b_gls = XXX FILL IN HERE XXXX;
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```
vmat_gls = inv(X'*omega_inv*X);

SE_gls = [sqrt(vmat_gls(1,1)); sqrt(vmat_gls(2,2)); sqrt(vmat_gls(3,3))];
tstat_gls = b_gls./SE_gls;

pval_gls = 2*(1 - tcdf(abs(tstat_gls),n-p));

disp(' ')
disp('-------------------------------------------------------------------')
disp(' ')

disp('Part B')
disp(' ')

disp('Model: c(t) - c(t-1) = b0 + b1r(t) + b2(y(t) - y(t-1)) + e(t)')
disp(' ')

disp('GLS Regression Results')

disp('   Estimates    SE     |t-stat|   P-value')
disp([b_gls SE_gls tstat_gls pval_gls])
disp('Note: GLS estimates for b0, b1 and b2 in that order.')

disp(' ')

omega_inv2 = omega_inv(2:18,2:18);

X2 = X(2:18,:);
c2 = c(2:18,:);

b_gls2 = inv(X2'*omega_inv2*X2)*X2'*omega_inv2*c2;

vmat_gls2 = inv(X2'*omega_inv2*X2);

SE_gls2 = [sqrt(vmat_gls2(1,1)); sqrt(vmat_gls2(2,2)); ...
    sqrt(vmat_gls2(3,3))];
tstat_gls2 = b_gls2./SE_gls2;

pval_gls2 = 2*(1 - tcdf(abs(tstat_gls2),n-p-1));
```