ECONOMETRICS 1, Spring, 2016
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Final Exam - May 6, 2016

1. (20%) In the standard regression model

$$Y = X\beta + u \ ,$$

where β is a K-dimensional vector of parameters, derive the test-statistic for $R\beta = q$, (where R is a $J \times K$ constant matrix and q a fixed vector of dimension J), and find its distribution under the standard assumptions (state these) including normality of u. You may use directly the distribution of $\hat{\sigma}^2$ from class, and that $\hat{\sigma}^2$ is independent of the OLS estimator $\hat{\beta}$.

2. (20%) Consider again the standard regression model

$$Y = X\beta + u \ ,$$

where β is a K dimensional vector of parameter, but now assume that $Var(u) = \Omega$.
a) Derive the formula for the GLS estimator under the usual assumptions (except of course that the errors are not i.i.d. now). You can take the result for the OLS estimator as given and start from there.
b) Under the same assumptions, assume you estimate β using the standard OLS estimator $\hat{\beta}_{OLS}$. Find (derive) the mean and variance of $\hat{\beta}_{OLS}$ under these conditions.

3. (10%) Consider the model

$$Y_i \ = \ \beta_1 X_{1i} \ + \ \beta_2 X_{2i} \ + \ \epsilon_i \ \ ; \ i = 1, ..., N \ .$$

Assume that $\sum X_{1i}^2$=1 and $\sum X_{2i}^2$=1. Further assume that all variables have mean 0. Let $M_k = (I - X_k(X_k'X_k)^{-1}X_k')$; for $k = 1, 2$ where X_k is the column vector $(X_{k1}, ..., X_{kN})'$.
Assume that you estimate the model

$$Y_i \ = \ \gamma_1 X_{1i} \ + \ \gamma_2 (M_1 X_{2i}) \ + \ \epsilon_i \ .$$

by OLS and the variance Var($\hat{\gamma}_1$)=1 and variance Var($\hat{\gamma}_2$)=1000.
Next assume that you estimate the model

$$Y_i \ = \ \alpha_1 (M_2 X_{1i}) \ + \ \alpha_2 X_{2i} \ + \ \epsilon_i \ .$$

What is the variance of $\hat{\alpha}_2$?

PLEASE TURN OVER

4. (10%) Assume that you want to estimate the model

$$Y_i \;=\; \beta_1 X_{1i} \;+\; \beta_2 X_{2i} \;+\; \epsilon_i \;,$$

and that you know already that the true value of $\beta_1 = 7$.
Assume that the "OLS-assumptions," including normality holds.
Explain how you can estimate β_2 efficiently while imposing the true value of β_1.

5. (15%) You want to estimate the model

$$y_i = \alpha_0 + \alpha_1 x_i + u_i$$

by maximum likelihood. Assume that the variance of the error term is $var(u_i) = \gamma\, i$ for some constant γ. Derive the maximum likelihood estimators for α_1 and γ.

6. (15%) In the standard regression model

$$Y = X\beta + u \;,$$

show that the residual vector e from an OLS regression satisfies $X'e = 0$

7. (10%) Assume that you want to estimate the model

$$Y_i \;=\; \beta_1 X_{1i} \;+\; \beta_2 X_{2i} \;+\; \epsilon_i \;,$$

where X_1 and X_2 are orthogonal regressors.

Assume that you know that the variance $\mathrm{Var}(\epsilon_i) = X_{1i} + 3\ln(X_{2i})$. This is a breakdown of one of the "standard OLS-assumptions."
a) What is the name for the problem?
b) How does it affect the results from OLS estimation: are the $\hat{\beta}$-coefficients biased? are they consistent? is the t-statistic (as usually calculate under the standard assumptions) t-distributed?
c) Suggest an efficient estimator.