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Material that should be known for the final

The following is meant to help you prioritize and I cannot mention everything. Anything that was taught in class is fair game (and material in the text-books that was not taught is not on the exam). You may, of course, be asked to make a simple deduction that follows from what was taught in class, even if I did not do it. If in doubt, send me an email or walk by my office.

1. Matrix algebra. There are good introductions to this material in Davidson-MacKinnon and Greene (I like Greene's appendices better on this). I list some of the more important stuff below (although it is not exhaustive).

   (a) You are expected to know the basic rules about adding and multiplying etc. matrices before taking this class.

   (b) Partitioned matrices are important in econometrics, so you have to able to invert and multiply those.

   (c) A special case of writing a matrix in partitioned form is to write it as a collection of row vectors or a collection of column vectors. For the important issue of consistency of OLS, this is crucial.

   (d) You are expected to be able to find the determinant of a 2×2 matrix and matrices that are block-diagonal with 2×2 matrices or scalars along the diagonal.

   (e) You have to be able to diagonalize a symmetric matrix and you should know the role of the eigenvalues (More often, though, you will need to make a theoretical argument relying on the existence of a diagonalization, as opposed to doing it numerically). You should be able to find eigenvalue for 2×2 matrices. This includes the taking of the square root of a matrix and the square root of the inverse.

   (f) You should know about idempotent matrices and their eigenvalues (0 or 1).

2. Statistics

   (a) You should know the multivariate normal distribution and how it relates to the χ-square distribution.

   (b) You have to be comfortable taking means and variances of a stochastic vector (a vector of stochastic variables).

(c) You should (absolutely) know what happens to the mean and variance of a stochastic vector if it is multiplied by a matrix.

(d) You should be able to explain why $e'Me$ follows a χ-square distribution if M is idempotent and e is standard normal (and explain the degrees of freedom).

(e) You have to know (for testing) that if X is $N(0, \Sigma)$ then $X'\Sigma^{-1}X$ is χ-square. This follows because $\Sigma^{-.5}X$ is $N(0, I)$, you should be able to explain this, but the higher priority is to know the result for $X'\Sigma^{-1}X$ which is the multivariate equivalent of dividing by the standard error (if X is a scalar, then $X'\Sigma^{-1}X$ is $X^2/\sigma^2 = (X/\sigma)^2$, i.e., the square of standard normal.

3. Theoretical derivation of the regression coefficient (vector) and its variance.

4. Be able to show the $\hat{\beta}$ (the estimated coefficient in the linear regression model under the standard assumptions [know what those are]) is unbiased. The unbiased estimator of the error variance (be able to prove that it is unbiased).

5. Working with numerical examples—the linear model with 2 regressors will often be used in midterm/exam questions, I may give you some numbers and you should be able to find, say the coefficient and the standard errors.

6. The Frisch-Waugh (FM) theorem and applications. I may ask you to prove the FW theorem, so make sure you are comfortable working with the projection matrix $P_X = X\,(X'X)^{-1}X'$ and the residual maker $M_X = I - P_X = I - X\,(X'X)^{-1}X'$ Important applications of the FM theorem are

   (a) Regressing on a large number of dummy variables.

   (b) Showing the bias in the case of omitted (left-out) variables.

   (c) Evaluating the marginal impact of an extra regressor.

   (d) "Added value plots" (to check for outliers).

7. R^2, adjusted R^2, and partial R^2

8. The t- and F-test (know how to formulat the test of hypothesis described in words and know the equivalence of the "goodness of fit" version and the version where you directly use $R\hat{\beta} - q$ know how to prove that the F- and t-tests follow the t- and F-distributions). The Chow-test (and similar simple applications of the F-test that I may think of). Confidence intervals.

9. Functional Form (as I covered it in class: dummy variables, interactions, elasticities, semi-log, etc.)

10. Data issues: Classical measurement error, multi-collinearity

11. Asymptotics. You will need to use the Law of Large Numbers (LLN) and the Central Limit Theorem (CLT), but I did not mention the explicit version of the LLN or the CLT, so you can talk about "the" LLN, and "the" CLT.

    (a) Consistency of the OLS estimator (know the assumptions needed on $X'X$ and be able to explain that $X'\epsilon$ is a sum of independent variables so that a LNN holds).

    (b) Consistency of the variance estimator.

    (c) Convergence of the $t-$ test to a Normal test (whether the data are Normally distributed or not, as long a CLT holds).

    (d) Asymptotic χ^2-test of restrictions even if the errors are not Normally distributed (the case where they are, is of course a special case, so this implies that the standard F-test converges to the χ^2-test (and the F-distribution to the χ^2-distribution.

12. GLS. Understand that if Ω is the variance matrix, one can choose a Cholesky factorization so that $\Omega^{-1/2}$ is lower triangular and multiplying the $n'th$ row with the true error vector corresponds to calculating $x_n - E(x_n|x_{n-1}, ...x_1)$ (and scaling with the standard error). (Confer point 2e.) Therefore the elements of $\Omega^{-1/2}e$ are i.i.d., which is equivalent to $var(\Omega^{-1/2}\,e) = \Omega^{-1/2}\,var(e)\,\Omega^{-1/2'} = \Omega^{-1/2}\,\Omega\,\Omega^{-1/2'} = I$. This got a little detailed, but you can take that as a reminder that formulas for the variance of matrix times a stochastic vector are essential for OLS/GLS theory.

13. Feasible GLS. Main examples: 1) autocorrelation in residuals 2) heteroskedasticity

14. White robust variance estimator. Explain why it works (under suitable assumptions).

15. The IV estimator when there are more instruments than regressors and the special case when the number of instruments is equal to the number of regressors.

16. Explain why the IV-estimator is consistent (and list the assumptions) but not unbiased. (Note: there isn't so much to remember about the assumptions, we basically assume "what we need" in order to get consistency.)

17. Maximum Likelihood.

    (a) Be able to show that $\hat{\beta}_{OLS} = \hat{\beta}_{ML}$ under the standard assumptions plus normality and explain the relation between are standard OLS estimate of the error variance and the ML estimate of the error variance.

    (b) Also, be able to derive the (Normal) ML estimator in the case of heteroskedasticity. (I won't ask for the case of autocorrelated residuals.)

    (c) Know the Cramer-Rao lower bound—in particular, that the inverse information matrix is the asymptotic variance of the estimator.

(d) Be able to prove the information matrix equality (maybe for a particular simple likelihood function).

(e) Be able to find the ML estimator for simple distributions such as exponential, log-normal, Bernoulli.