ECONOMETRICS II, Fall, 2023
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Econometrics II. Midterm Exam 2—October 30, 2023

Each sub-question in the following carries equal weight.

1. (30%) Consider the bivariate model

$$(*) \quad y_i \,=\, \alpha \,+\, \kappa z_i \,+\, \beta x_i \,+\, u_i \,,$$

and

$$(**) \quad z_i \,=\, \omega \,+\, \gamma x_i \,+\, \delta w_i \,+\, v_i \,,$$

for a sample of agents, $i = 1, ..., N$, where y_i and z_i take values 0 or 1. x_i and w_i are exogenous regressors, the error terms u_i and v_i are mean zero and normally distributed and uncorrelated with regressors in their respective equation and satisfies the standard conditions (no autocorrelation/heteroskedasticity). Assume that u and v are uncorrelated.

a) Are the coefficients in an OLS regression of (**) biased or unbiased? Explain why.

For the next two sub-questions assume that u and v are correlated.
b) Are the coefficients in an OLS regression of (*) now biased or unbiased? Explain why.
c) Are the coefficients in an OLS regression of (**) biased or unbiased? Explain why.
d) Explain how you estimate the variance and covariances of the error terms.

For the next two sub-questions, assume you know the variance and covariances of the estimates.
e) How would you test $\hat{\delta} = \hat{\gamma}$. (Put down a formula and state its distribution. You should be able to answer this even without getting the previous sub-questions right.)
f) How would you test the composite hypothesis $\hat{\gamma} = \hat{\kappa}$, $\hat{\delta} = \hat{\beta}$?

2. (10%) Write down the probit model for 3 ordered outcomes. Provide an explicit example where this model is suitable (we do not need a lot of details, but we do need to see the logic of it).

3. (15%) Consider the system of equations

$$y_{it} = \beta x_{it} + e_{it}$$

$$w_{it} = \gamma z_{it} + u_{it} \,,$$

for $i = 1, ..., N$ and $t = 1, ..., T$. Assume that the regressors are exogenous and the error terms independent of the regressors with mean zero and constant variance.

a) Assume that $E\{e_{it}u_{it}\} \neq 0$ with error terms independent across individuals and across time. Write down the efficient GLS estimator using Kronecker products (be explicit about what is in any matrices).
b) Now assume we only estimate the first equation for y, but $e_{it}e_{is} = \gamma$ while observations for different agents are independent. Write down the efficient GLS estimator using Kronecker products (be explicit about what is in any matrices and how you have ordered the data).

c) Now assume we only estimate the first equation for y, but $e_{it}e_{jt} = \kappa$ while observations for different time periods are independent. Write down the efficient GLS estimator using Kronecker products (be explicit about what is in any matrices and how you have re-ordered the data).

4. (15%) a) When is it a problem to estimate the model

$$y_{it} = \mu_i + \beta y_{it-1} + e_{it}\,,$$

by OLS even if $E y_{it-1} e_{it} = 0$?
b) Explain in detail what the issue is.

5. (30%) Matlab question.
a) What model does this code simulate?
b) Write down the estimator as indicated where it say A: in the code using variables already generated.
c) Write down the estimator as indicated where it say B: in the code. What variables do you need to first generate?

Set the parameters.

There are 1000 observations. Set $\beta_0 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.3$, $\beta_4 = 0.2$, $\beta_5 = 0.3$, $\beta_7 = 1$, $\sigma_1 = 1.2$, $\sigma_2 = 1.1$ and $\rho = 0.05$.

```
clear
clc

N = 1000;                                       % Number of observations.

beta0 = 0.1;
beta1 = 0.5;
beta2 = 0.3;
beta4 = 0.2;
beta5 = 0.3;
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```
beta7 = 1;
sigma1 = 1.2;
sigma2 = 1.1;
rho = 0.05;

mu = [0 0];                              % Mean vector of U.
smat = [sigma1^2 rho*sigma1*sigma2;      % Variance matrix of U.
        rho*sigma1*sigma2 sigma2^2];

G_inv = ones(2,2);                       % G^{-1} matrix.
G_inv(2,1) = beta1;
G_inv(1,2) = beta5;
G_inv = G_inv./(1-beta1*beta5);

B = zeros(3,2);                          % B matrix.
B(1,1) = beta0;
B(2,1) = beta2;
B(1,2) = beta4;
B(3,2) = beta7;
```

Generate the data.

Generate the data, X, then draw the error terms, U, and construct Y.

```
x1 = 2 + ((1:N)'/N).*normrnd(0,1,N,1);
x2 = 3 + 0.5*x1 + normrnd(0,1,N,1);
X  = [ones(N,1) x1 x2];                 % X.

u = mvnrnd(mu,smat,N);                  % U.

Y = X*B*G_inv + u;                      % Y.
Y1 = Y(:,1);
Y2 = Y(:,2);

b1_ols = (X'*X)\X'*Y1;
Y1_hat = X*b1_ols;                      % Fitted Y1.

b2_ols = (X'*X)\X'*Y2;
```

```
Y2_hat = X*b2_ols;                     % Fitted Y2.

X1_hat = [ones(N,1) Y2_hat x1];

A:  WRITE a consistent estimator estimator for the $y_1$ equation here using the variables de

B: WRITE a consistent estimator for the $y_2$ equation here
(you need to explain more for this one than for the previous one).
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