ECONOMETRICS II, Fall 2018
Bent E. Sørensen

Midterm Exam II - October 29, 2018

Each sub-question in the following carries equal weight except when otherwise noted.

1. (20%)
a) Explain what a duration model is, including the definition of the survivor function and the hazard function.
b) Derive the hazard function for the exponential duration model.
c) Write down the likelihood function for a sample of observations from an exponential duration model with some incomplete spells.

2. (18%) a) Find the estimated β that minimizes $(Y - X\beta)' W (Y - X\beta)$, where Y is a $T \times 1$ vector, X a $T \times k$ vector, and W is a full rank symmetric "weighting matrix."
b) Assuming that $Y - X\beta$ has variance matrix Ω, derive the variance of the estimated β

3. (16%) In the weak-IV survey article by Murray, there is a formula that I asked you memorize. In that formula, there are 4 factors that determines the approximate bias. of 2SLS. What are these factors (4% for each).

4. (16%) a) In the following code, what is the object B_xxx calculated where there is an A:?
b) In the following code, what is the object B_yyy calculated where there is an B:?
c) Explain why you might want to use B_yyy or B_xxx under certain conditions (which?).

```
    X = [ones(T,1) x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 x13];          %exogenous
for s = 1:sim

    u1 = normrnd(0,sigma1,T,1);                                              % Residuls for equa
    u2 = normrnd(0,sigma2,T,1) + 5*u1;                                       % Residuals for equ

    y2 = beta3 + beta4*x1 + beta5*x2 + beta6*x3 + beta7*x4 + beta8*x5 + beta9*x6 + beta10*x7 +
    y1 = beta0 + beta1*y2 + beta2*x1 + u1;

    Y = [y1 y2];
    Y1 = Y(:,1);   %same as y1
```

```
    Y2 = Y(:,2);   %endogenous regressors, same as generated y2


    X_exo1 = X(:,1:2); %exogenous regressors in the reduced form
    X_OLS=[ones(T,1) Y2 x1];

       B2_hat = inv(X'*X)*X'*Y2;
       Y2_hat = X*B2_hat;
        X1_hat = [ones(T,1) Y2_hat x1];
A:     B_xxx(s,:) = inv(X1_hat'*X1_hat)*X1_hat'*Y1;

   N = length(Y2);
    Mexo = eye(N) - X*inv(X'*X)*X';                                  %projection matrix, X is
    Mexo1 = eye(N) - X_exo1*inv(X_exo1'*X_exo1)*X_exo1';
    W = [Y1 Y2]'*(Mexo)*[Y1 Y2];                                     %2x2 matrix
    W1 = [Y1 Y2]'*(Mexo1)*[Y1 Y2];   %2x2 matrix
    lambda = min( eig(inv(W)*W1 )) ;
B:    B_yyy(s,:) = inv(X_OLS'*(eye(N)-(lambda*Mexo))*X_OLS)*(X_OLS'*(eye(N)-(lambda*Mexo))*Y1)
```

5. (30%) Consider a GMM problem where you have a sample of scalar observations y_t, $(t = 1, ...T)$, which satisfies $Ey_t = h(x_t; \theta)$, where x_t is vector of observed variables and θ a K-dimensional vector of unknown parameters that you want to estimate. You further have access (for each t) to an L-dimensional vector z_t which satisfies $L > K$ and $E\{z_t*(y_t - h(x_t; \theta))\} = 0$ for a unique value θ_0.

a) Write down the formula for GMM estimator (with identity weighting matrix) of θ in terms of the variables given.
b) Using the notation of my notes., identify where f_t is calculated in the text (e.g., what is the name used in the code?), where g_T is calculated and where the criterion function is calculated.
c) Explain in words how the test for overidentifying restrictions is calculated and what is its distribution?

```
GMM CODE.
%{
    Xavier Martin G. Bautista
    Fall 2018
    Macro 3
    HW 2

    GMM_Main.m
```

```
    This replicates Hansen and Singleton (1982). Estimation is done using
    GMM. The variance matrix can be estimated using either Newey-West or
    Quadratic Spectral kernels.

    Note: Convention used is U(C) = (C^(1-gamma))/(1-gamma).
%}

%% 1. Change working directory and load data.

    close all
    clear
    clc

    addpath('D:/Xavier_Laptops/Xavier_Asus/Xavier_Classes/Fall_2018/Macro3/HW2')

    global c lag re rf n T Z

    load data

    lag = 3;                                                        % Number of lags us

    c  = data(:,1);                                                 % c(t)/c(t-1).
    re = data(:,2);                                                 % Value-weighted av
    rf = data(:,3);                                                 % T-bill rate.

    T = size(data,1);
    Z = [ones(T-lag,1) c(1:T-3)  c(2:T-2) c(3:T-1)...               % Instruments: 3 la
                      re(1:T-3) re(2:T-2) re(3:T-1)...              % value-weighted av
                      rf(1:T-3) rf(2:T-2) rf(3:T-1)];
    n = size(Z,2);                                                  % Number of instrum

    clear data

%% 2. GMM Stage 1: Identity Weighting Matrix.

    b0 = [0.5 0.5];                                                 % Initial guess of
    W  = weight(b0,0);                                              % 0 = Identity mati

    opt = optimset('FinDiffType','central','HessUpdate','BFGS');    % Use central diffe
```

```
    b1  = fminunc('gmm_obj',b0,opt,W);
    clear W


****************
function crit = gmm_xxx(guess,W)

global lag T

mom = ((sum(orth(guess),1))./(T-lag))';
crit = mom'*W*mom;

end
*********************************************
function ZXb = orth(guess)

%{
    orth.m

    This is the orthogonality condition from Hansen and Singleton (1982)
    E(z(t)*((beta*((C(t)/C(t-1))^(-gamma))*r(t)) - 1)) = 0.
%}

global c lag n rf T Z

beta  = guess(1);
gamma = guess(2);

C = repmat(c(1+lag:T),1,n);
R = repmat(rf(1+lag:T),1,n);

ZXb = Z.*((beta.*(C.^(-gamma)).*R)-1);

end
***************************************
```