

Midterm Exam - October 4, 2017

Each sub-question in the following carries equal weight except when otherwise noted.

1. (25%)

a) (10%) Explain what is meant by a censored and by a truncated regression model.

b) (15%) Assume you are estimating the model

$$Y_i = aX_i + u_i ,$$

by OLS. (Here  $a$  is a scalar and we assume for simplicity that there is not intercept, in the true underlying model (not censored or truncated) the error term has mean 0.)

Assume that  $a$  is positive and the model is truncated for large values of  $Y_i$ . Assume that the data are such that the estimate of  $a$  is biased. Explain why and whether the estimate of  $a$  is upwards or downwards biased.

Further assume that you only have 3 observations:  $X' = 2, 4, 6$ . Assume in this case that the data are censored and values of  $Y$  larger than 6 is set to 6. Also assume that we know that the distribution of the innovation term  $U$  is such that it takes only the values  $-2$  and  $2$  (each with probability 0.5) and that the true value of  $a$  is 1.

Find the expected value of the OLS estimator of  $a$ .

2. (30%) On the next pages, I have reproduced two pieces of Matlab code that you used for a homework.

a) 5%) Explain what Code I is supposed to do.

b) 10%) Fill in the missing line in Code I.

c) 5%) Explain what Code II is supposed to do.

d) 10%) Fill in the missing line in Code II.

3) 25%) Consider the AR(1) model

$$X_t = \alpha + \beta X_{t-1} + u_t .$$

a) 10%) Mister Smith estimates the standard error of  $\hat{\beta}$  using the formula for OLS regression of the left-hand side on the right-hand side. Write down the formula he is using.

b) 15%) Ms Jones estimates the model by ML and finds the standard error of  $\hat{\beta}$  by calculating the formula for the asymptotic (implying that you can ignore the first observation) Hessian and plugs in the estimated parameter(s) as needed. Derive this formula, and compare to the expression you found in part a).

4) 20%)

a) Write down the latent-variable model used to derive the Probit model and derive the probability of each outcome.

b) Write down the log-likelihood function for the *Logit* model for a sample of  $N$  observations.

## Code I

```
function [L] = logl_xx(b)
% The following is the loglikelihood function for an xx model.

global x z N

b0 = b(1);
b1 = b(2);

XB = b0*ones(size(x,1),1) + b1*x ;

L=0 ;
for i = 1:N
    if z(i) == 1
        L = L + log(normcdf(XB(i)));
    else
        L =FILL IN HERE;
    end
end

L = -L;

end
```

## Code II

```
function [ L ] = logl_xx( b0 )

global x T

mean = b0(1);           % Mean.
stddev = b0(2);        % Standard deviation.
theta1 = b0(3);        % .

L = FILL IN THE CODE HERE

for t = 2:T
    L = L - 0.5*log(2*pi) - 0.5*log(stddev^2) ...
          - 0.5*(((x(t) - mean - theta1*x(t-1))^2/stddev^2));
end

L = -L;                 % Negative of logl(for minimization).

end
```