ECONOMETRICS II, Fall 2023
Bent E. Sørensen

Final Exam. December 4. 6 questions.

1. (16%) Consider the ARMA(1,1) model

$$y_t = 0.5 y_{t-1} + u_t - u_{t-1} \, .$$

a) What is the first-order autocovariance?
b) What is the stationary distribution of y_1?

2. (14%) Consider the CDF of exponential distribution, which is commonly used for duration modeling:

$$F(t) = 1 - \exp(-t\lambda) \; .$$

(For the exponential distribution, $\mathrm{E}\, t = 1/\lambda$ and $\mathrm{Var}\, t = 1/\lambda^2$.)

a) Find the asymptotic information "matrix" analytically by taking second-order derivatives.
b) Find the asymptotic information "matrix" analytically by means of first-order derivatives.

3. (24%) a) Assume you have estimated 3 parameters, with estimates $\hat{\beta}_1 = 7$, $\hat{\beta}_2 = 9$, and $\hat{\beta}_3 = 11$, and assume that you know for sure the estimates are normally distributed and the known variance-covariance matrix is

$$\Sigma = \begin{pmatrix} 5 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 4 \end{pmatrix} .$$

a) Write down a (Wald) test statistic for $\beta_2 = 0$. (I want you to write this as a scalar, a real number, although you do not have to solve for ratios or invert matrices in the questions here.)
b) Write down a (Wald) test statistic for $\beta_2 + \beta_3 = 1$. (I want you to write this as a scalar, a real number.)
c) What is the distribution of the test statistic you wrote down? (Be clear about whether you are talking about an asymptotic test of not in this and the following.)
d) Write down a test statistic for (β_2, β_3)=(2,1). (You don't need to multiply everything out, but it has to be an expression with only numbers.)

e) What is the distribution of the test statistic you wrote down?
f) Write down a test statistic for the hypothesis $\log(\beta_1) - \beta_3^2 = 2$. (Again, I need to see the numbers.)
g) Assuming the coefficients are consistent, what is the distribution of the test statistic you wrote down?

4. (16%) Consider the model :

$$v_i = b\, w_i + c\, x_i + e_i,$$

$$w_i = d\, y_i + f\, q_i + g\, x_i + u_i\,,$$

and

$$y_i = h\, w_i + k\, v_i + p_i,$$

and where v, w, and y are endogenous random variables, x, and q are exogenous random variables, b, c, d, f, g, h and k are unknown parameters (assume they are non-zero), and e, u, and p are independent white noise terms.

a) Explain which (if any) of these equations are identified.
b) Suggest a typical consistent estimator for the equation(s) that you can estimate. (You do not need to write down any formula, but you do need to explain which instruments, if any, a valid for each equation.)

5. (15%)/ Explain what should be in the code where it says "FILL IN HERE." (What is the model and what code should be there...it does not have to be perfect Matlab notation.)

```
function [L] = logl_xx(b)
% The following is the loglikelihood function for an xx model.

global x z N

b0 = b(1);
b1 = b(2);

XB = b0*ones(size(x,1),1) + b1*x ;

L=0 ;
for i = 1:N
  if z(i) == 1
    L = L + log(normcdf(XB(i)));
  else
    L =FILL IN HERE;
  end
end

L = -L;

end
```

6. (15%)

This code estimates the simultaneous equation model

$$y_1 = \beta_0 + \beta_1 y_2 + \beta_2 x_1 + u_1 \ ,$$
$$y_2 = \beta_4 + \beta_5 y_1 + \beta_7 x_2 + u_2 \ ,$$

with $U \sim NID(0, \Sigma)$, using three stage least squares. Complete the code. You can write the formula, not using Matlab syntax, but you have to say what will do in terms of the output generated by the program. If you do not understand the code, you will not get points.)

Set the parameters.

There are 1000 observations. Set $\beta_0 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.3$, $\beta_4 = 0.2$, $\beta_5 = 0.3$, $\beta_7 = 1$, $\sigma_1 = 1.2$, $\sigma_2 = 1.1$ and $\rho = 0.05$.

```
clear
clc

N = 1000;                                   % Number of observations.

beta0 = 0.1;
beta1 = 0.5;
beta2 = 0.3;
beta4 = 0.2;
beta5 = 0.3;
beta7 = 1;
sigma1 = 1.2;
sigma2 = 1.1;
rho = 0.05;

mu = [0 0];                                 % Mean vector of U.
smat = [sigma1^2 rho*sigma1*sigma2;         % Variance matrix of U.
        rho*sigma1*sigma2 sigma2^2];

G_inv = ones(2,2);                          % G^{-1} matrix.
G_inv(2,1) = beta1;
```

```
G_inv(1,2) = beta5;
G_inv = G_inv./(1-beta1*beta5);

B = zeros(3,2);                         % B matrix.
B(1,1) = beta0;
B(2,1) = beta2;
B(1,2) = beta4;
B(3,2) = beta7;
```

Generate the data.

Generate the data, X, then draw the error terms, U, and construct Y.

```
x1 = 2 + ((1:N)'/N).*normrnd(0,1,N,1);
x2 = 3 + 0.5*x1 + normrnd(0,1,N,1);
X  = [ones(N,1) x1 x2];                % X.

u = mvnrnd(mu,smat,N);                 % U.

Y = X*B*G_inv + u;                     % Y.
Y1 = Y(:,1);
Y2 = Y(:,2);
```

Three Stage Least Squares.

Estimate the model using three stage least squares.

```
% Stage 1: OLS.

b1_ols = (X'*X)\X'*Y1;
Y1_hat = X*b1_ols;                     % Fitted Y1.

b2_ols = (X'*X)\X'*Y2;
Y2_hat = X*b2_ols;                     % Fitted Y2.

% Stage 2.
```

```
X1_2sls = [ones(N,1) Y2_hat x1];
X1X1 = X1_2sls'*X1_2sls;
X1Y1 = X1_2sls'*Y1;

b1_2sls = (X1X1)\X1Y1;                % 2SLS (eq. 1).

X2_2sls = [ones(N,1) Y1_hat x2];
X2X2 = X2_2sls'*X2_2sls;
X2Y2 = X2_2sls'*Y2;

b2_2sls = (X2X2)\X2Y2;                % 2SLS (eq. 2).

u1_2sls = Y1-X1_2sls*b1_2sls;         % Residuals, u1.
u2_2sls = Y2-X2_2sls*b2_2sls;         % Residuals, u2.


% Stage 3.

umat = [u1_2sls u2_2sls];             % Residuals, U.

%%% FILL IN HERE. %%%
```