ECONOMETRICS II, Fall 2016
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Final Exam. December 2. 7 questions.

1. (20%) a) Calculate and plot the impulse response functions for the model

$$\begin{pmatrix} x_{1t} \\ x_{2t} \end{pmatrix} = \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} + \begin{pmatrix} 1 & .5 \\ .3 & .2 \end{pmatrix} \begin{pmatrix} u_{1t-1} \\ u_{2t-1} \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 0 & .5 \end{pmatrix} \begin{pmatrix} u_{1t-2} \\ u_{2t-2} \end{pmatrix} ,$$

where the error terms are independent. If the variance of u_1 is 1 and the variance of u_2 is 2, and u_1 and u_2 are independent, calculate the variance decomposition for x_1.
b) Calculate the variance decomposition for x_1, when

$$\begin{pmatrix} x_{1t} \\ x_{2t} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} + \begin{pmatrix} 1 & .5 \\ .3 & .2 \end{pmatrix} \begin{pmatrix} u_{1t-1} \\ u_{2t-1} \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 0 & .5 \end{pmatrix} \begin{pmatrix} u_{1t-2} \\ u_{2t-2} \end{pmatrix} ,$$

where u_1 and u_2 now are independent with variance 1.

2. (15%) Assume that you have a sample of outcomes y_i, which can take values 0, 1, and 2, where the outcomes of y indicate an ordering. You have a set of regressors x_i, that may help explain the outcomes.
a) What statistical model can you use to estimate such outcomes (give one example)?
b) Write down the log likelihood function for the model.

3. (10%) The Wald test. (Assume you are testing $R(\theta) = 0$, that you have estimated $\hat{\theta}$, and you have a variance-covariance matrix Σ for $\hat{\theta}$. It is immaterial how these parameters are estimated.)
a) Write down the Wald test (assume that the function $R()$ is non-linear).
b) What do we know about the asymptotic distribution of the Wald test if θ is $k \times 1$ and $R(\theta)$ is $l \times 1$ where $k > l$, assuming the model and estimation methods are such the that standard Central Limit Theorem applies? (Extra points if you can say something about the finite sample distribution [under which assumptions can we typically say something about the finite sample distribution]?)

4. (15%) I have included the Gauss code from one of your homeworks. Where it says XXXXXXXXXXXXXXXXXXxx, I have removed two lines for calculationg the LM test. What should be in those lines?

5. (15%) Derive the Heckman correction term. Set up the model explicitly (as in the book).

6. (10%) What is the Augmented Dickey-Fuller test for testing a unit root with drift against a stationary model with trend? (What regression would you run, what is the null hypothesis? I am not asking for derivations.)

7. (15%) Consider a dynamic panel data model with fixed effects.
a) Show, using Frisch-Waugh, how you can remove the fixed effect and then estimate a "corrected" model without fixed effects by OLS.
b) Using the result from part a), explain why the OLS estimator is biased if the regressors are not strictly exogenous.

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new;
 output file = test.out reset ;
library optmum;

N = 12;

beta0 = .5; beta1=3 ; sigma1 = 1; beta2=0 ; sigma2 = 4; rho=.8 ;

x=seqa(1,1,N)/N.*rndn(N,1) ; /* x is the "regressor," could be anything, but if not cent
the estimates will not be well identified. As for OLS you would like ‘‘variation in the
which here mean that we want to observe a good spread between 0s and 1s */

z=ones(N,1)+2*x+5*rndn(N,1) ; /* z is another regressor for linear
relation ---not orthogonal to x*/

Xmat=ones(N,1)~x~z ;

Nsim =1 ; "Number of simulations:" Nsim ; /* create a matrix to
hold the estimated values */

results = zeros(Nsim,3) ;
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/*starting values*/

let sv = 1 1 1 1;

let svR = 1 1 1 ;

estim=0 ;
do while estim < Nsim ;
estim=estim+1 ;

u = sigma1*rndn(N,1);

epsilon=sigma2*rndn(N,1) ;


/*generate a linear relation */

y=beta0+beta1*x+beta2*z+epsilon ;

__output=0;

{b_ml,lmax,g,retcode} = optmum(&logl,sv);

 {b_mlR,lmaxR,g,retcode}=optmum(&loglR,svR) ;

__output=1 ;

/*results[estim,.]=b_ml' ;*/


endo ; " " ;

/*results of last estimation*/

 "calculate Hessian"; h=hessp(&logl,b_ml);
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 /*this is minus the hessian because we minimize -L*/

 "variance matrix"; vmat=inv(h);
 "OLS variance matrix" ; OLSvmat=inv(xmat'xmat).*b_ml[4] ;
 /*vmat; */

 "" ;

"beta2-hat, t-value" b_ml[3] b_ml[3]/sqrt(OLSvmat[3,3])

" " ; "P-vaLUES:" ;

pval1=2*cdftc(abs(b_ml[3])/sqrt(vmat[3,3]),N-4) ;

 "t-test:" ;  pval1;

pval2=cdfchic(2*(lmaxR-lmax),1) ;

"LR test" ; pval2;

pval3=cdfchic((b_ml[3])^2/vmat[3,3],1) ;

"Wald test" ; pval3 ;

/*restricted ML-estimator with restriction*/

b_LM=zeros(4,1) ;

b_LM[1]=b_mlR[1] ; b_LM[2]=b_mlR[2] ; b_LM[4]=b_mlR[3] ;

LMgrad=gradp(b_LM);

h_LM=hessp(&logl,b_LM);

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXxx

pval4=cdfchic(LMtest,1) ;
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 "LM test" ; pval4 ;


"***";

 /* averages and std dev of estimates */ "mean estimates"
meanc(results); "std of estimates" stdc(results) ;

" " ;

proc logl(b);
  local r,L,muvec,sigma2,beta0,beta1,beta2,beta3,beta4, i;

beta1=b[1] ;
beta2=b[2] ;
beta3=b[3] ;
sigma2=b[4] ;

L=0 ;
i=0 ;
do while i<N ;
  i=i+1 ;
    L = L  -0.5*ln(sigma2^2) - 0.5*(y[i]-beta1-beta2*x[i]-beta3*z[i])^2/sigma2^2;
endo ;
  retp(-L); /*optmum minimizes*/
endp;

proc loglR(b);
  local r,L,muvec,sigma2,beta0,beta1,beta2,beta3,beta4, i;

beta1=b[1] ; beta2=b[2] ; sigma2=b[3] ;

L=0 ; i=0 ; do while i<N ;
  i=i+1 ;
    L = L  -0.5*ln(sigma2^2) - 0.5*(y[i]-beta1-beta2*x[i])^2/sigma2^2;

endo ;
  retp(-L); /*optmum minimizes*/
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endp; output off ;
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