

Real Analysis Preliminary Examination Syllabus, May 2006

1. **METRIC SPACES:** Metrics, metric topology, open and closed sets, continuity, Cauchy condition and completeness, compact sets.
2. **MEASURES:** sigma-algebras, Borel sets, outer measures, Caratheodory theorem, Lebesgue measure, Borel and Lebesgue-Stieltjes measures on the real line, signed measures, Jordan decomposition, total variation of a measure, absolute continuity, Lebesgue and Radon-Nikodym, product measures, Littlewood's three principles.
3. **FUNCTIONS and INTEGRATION:** measurable functions, integration of non-negative functions, integration of complex and real-valued functions, convergence theorems for integrals (e.g., Fatou, monotone, dominated convergence), convergence in measure, a.e. convergence
4. **FUNCTION THEORY:** Bounded variation, absolute continuity, differentiation, fundamental theorem of calculus.
5. **BASICS of FUNCTIONAL ANALYSIS and the L^p SPACES:** Normed and Banach spaces, Hahn-Banach theorem, Baire Category theorem, Open Mapping Theorem, Closed Graph Theorem, Principle of Uniform Boundedness, L^p spaces and their duals, completeness, convergence, density, $C(X)$ spaces and their duals, Riesz Representation theorems, weak topologies, convergence of nets.

References

1. Real Analysis, 3rd Edition, H. L. Royden, Macmillan & Co.
2. Principles of Mathematical Analysis, 3rd Edition, Walter Rudin, McGraw Hill Publ.
3. Measure Theory, Paul R. Halmos, Spring Graduate Texts in mathematics, Mew York, 1950.
4. Real Analysis: Modern Techniques and their Applications, Gerald B. Folland, John Wiley and Sons,