PARTIAL DIFFERENTIAL EQUATIONS PRELIMINARY EXAMINATION

November 15, 2004

The examination is based on a year-long graduate course on the theory of Partial Differential Equations, and emphasizes the tools of analysis used to study existence, uniqueness and qualitative behavior of solutions. A good reference text is the book used in Math 6326-7 (2003-4):

> Lawrence C Evans, *Partial Differential Equations*, American Mathematical Society, 1998. (See `http://math.berkeley.edu/~evans` for list of errors in text.)

Students will be examined on the material in chapters 1, 2, 3, 5, 6 and 7: examples of protoype equations, the definition of characteristics and their importance in PDE; basic techniques of separation of variables (from chapter 4); the definition and properties of Sobolev spaces, the theory of second-order elliptic equations, the Sobolev space approach to second-order linear parabolic and hyperbolic equations and linear semigroup theory.

In addition, students should be familiar with elementary concepts, such as separation of variables (the advanced undergraduate text by Strauss [7] is an excellent reference). This text also contains examples of derivation of PDE from physical models, as does Weinberger's text, [8]. (See [8] also for separation of variables.)

Some important concepts, such as the general definition of characteristic normals, are not given in Evans's text. A good reference for characteristics is Renardy and Rogers, [6]; distributions are found in [7] and [6]; the Lumer-Phillips theorem is in [6]. The other references give more detail on some topics, or an alternate approach.

TOPICS

1. Examples and derivation of PDE: definitions of linear and nonlinear equations; order of an equation; defintions of initial and boundary value problems

2. Classification of PDE; symbol of a partial differential operator; timelike directions and hyperbolicity; potential, heat, wave and transport equations; characteristics
3. Representation of solutions by separation of variables; d'Alembert's formula for the wave equation; the Poisson integral formula for Laplace's equation; the heat kernel
4. Solution of the transport equation
5. Mean-value identities for Laplace's equation
6. Solution of the wave equation by spherical means
7. Energy identities and inequalities; Green's identities
8. Solution of nonlinear first-order PDE by the method of characteristics; application to Hamilton-Jacobi equations and scalar conservation laws; weak solutions; Legendre transforms; Lax-Oleinik formula; Riemann's problem
9. Elementary distribution theory
10. Definitions of Hölder spaces and Sobolev spaces; weak derivatives; approximation by smooth functions; traces; Sobolev inequalities; Poincaré inequalities; dual spaces (H^{-1})
11. Existence of weak solutions to second-order elliptic equations by the Lax-Milgram theorem; energy estimates; interior regularity; weak and strong maximum principles
12. Weak solutions of parabolic and hyperbolic equations in Sobolev spaces $L^2(0,T;H_0^1(\Omega))$ by Galerkin approximation
13. Weak solutions of linear symmetric hyperbolic systems by viscous regularization
14. Introduction to semigroup theory; the Hille-Yosida and Lumer-Phillips theorems

References

[1] R. A. Adams. *Sobolev Spaces.* Academic Press, New York, 1975.

[2] R. Courant and D. Hilbert. *Methods of Mathematical Physics*, volume II. Wiley-Interscience, New York, 1962.

[3] P. R. Garabedian. *Partial Differential Equations, Second Edition.* Chelsea, New York, 1986.

[4] F. John. *Partial Differential Equations*. Springer-Verlag, New York, 1982.

[5] P. D. Lax. The formation and decay of shock waves. *American Mathematical Monthly*, 79:227–241, 1972.

[6] M. Renardy and R. C. Rogers. *An Introduction to Partial Differential Equations*. Springer-Verlag, New York, 1993.

[7] W. A. Strauss. *Partial Differential Equations: An Introduction*. Wiley, New York, 1992.

[8] H. F. Weinberger. *A First Course in Partial Differential Equations*. Wiley, New York, 1965.