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## **The Empirical Implications of Theoretical Models (EITM): A Framework for Methodological Unification\***

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## **Abstract**

We provide a framework for the Empirical Implications of Theoretical Models (EITM) initiative. The objective of EITM is to encourage political and social scientists to test empirical models that are directly connected to a formal model. As scholars merge formal and empirical analysis they minimize non-falsifiable research practices and lay the foundation for social scientific cumulation. Our EITM framework involves three steps. The first step is for researchers to unify theoretical mechanisms and applied statistical concepts. Step two is to develop measurable devices (“analogues”) for these mechanisms and concepts. The final step is to unify the analogues. The significance of the EITM framework is that it encourages scholars to use a set of plausible facts or axioms and then model them in a rigorous mathematical manner to identify causal relations that explain empirical regularities.

# 1 Introduction

## 1.1 Background and Justification

On July 9th and 10th, 2001, the Political Science Program of the National Science Foundation (NSF) convened a Workshop to seek ways to improve technical-analytical proficiency in Political Science. This workshop, part of the planning process involved with NSF's Empirical Implications of Theoretical Models initiative (hereafter EITM), was conducted to suggest constructive approaches to foster linkage between formal analysis and empirical modeling.<sup>1</sup>

At the workshop it was acknowledged that a schism existed between those who engage in formal modeling that focuses on quantifying abstract concepts mathematically, and those who employ empirical modeling which emphasizes applied statistics. As a consequence, a good deal of research in political science is competent in one technical area, but lacking in another. This problem is manifested in research involving formal modeling with substandard (or no) empirical tests *or* applied statistical modeling without formal clarity. Such impaired competency contributes to a failure to identify the proximate causes explicated in a theory and, in turn, increases the difficulty of achieving a meaningful increase in scientific knowledge.

Some political scientists have written about the scientific shortcomings associated with disjointed quantitative work. For example, Morton (1999) discusses these issues in the following terms:

*As the use of methodological techniques in political science has advanced, researchers have found that often their empirical study leads to more questions, questions that need theoretical input. However, because little existing theory is relevant or because the well-developed theory that does exist seems unconnected to the empirical issues, typically the response is to use more sophisticated methods or data gathering to answer the questions without reference to a fully developed theory. But these new methods often lead to still more questions, which in turn result in the use of more sophisticated methods to gather or analyze the data. The connection to theory seems to get lost in the methodological discussion. Rarely do researchers take the empirical results and rework the theoretical framework that began the discussion (p. 3).*

These concerns were also shared by the 2001 NSF EITM Workshop. In diagnosing the sources for this methodological status quo, the NSF EITM Report concluded:

*There are at least two reasons for this state of research competency. One is that rigorous formal and empirical training is a somewhat recent development in political science.*

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<sup>1</sup> Formal analysis refers to deductive modeling in a theorem and proof presentation or computational modeling that requires the assistance of simulation. Applied statistical analysis involves data analysis using statistical tools. Throughout this paper we will use the words *analysis* and *modeling* interchangeably.

*Another is that there are significant obstacles in the current political science training environment. The first obstacle is time. Students who desire training in both formal and empirical modeling will take longer to get a Ph.D. and most graduate programs do not have the resources to support students for more than four or five years. Consequently, students take the sequence of formal or empirical modeling classes but seldom both sequences... The second obstacle to establishing formal and empirical modeling competency centers on the training itself. The economics discipline is illustrative. Economics graduate students are required to take one full year (usually) of mathematics for economists. This mathematical (and quantitative) approach is reinforced in substantive courses which typically are taught as an analytic science in a theorem-proof mode.*

These forces have contributed to three harmful but common applied statistical practices exist: data mining, garbage cans, and the use of statistical weighting and patching (“omega matrices”).<sup>2</sup> These applied statistical practices lack overall robustness as they obscure fundamental specification error. To see how this occurs we summarize each practice (Granato and Scioli (2004)):

- 1. Data Mining:** This practice involves putting data into a statistical package with minimal theory. Regressions (likelihoods) are then estimated until either statistically significant coefficients or coefficients the researcher likes are found. This *step-wise* search is not random and has little relation to identifying causal mechanisms (see Lovell (1983); Denton (1985)).
- 2. Garbage Cans:** This practice, related to data mining, is where a researcher includes, in a haphazard fashion, a plethora of independent variables into a statistical package and gets “significant” statistical results somewhere. Researchers who use garbage can models usually pay little attention to potential confounding factors that could corrupt statistical inferences. Efforts to identify an underlying causal mechanism are also few and far between (Achen (2005)).
- 3. Omega Matrices:** Data mining and garbage-can approaches virtually are guaranteed to break down statistically. The question is what to do when these failures occur. There are elaborate ways of using (error) weighting techniques *to correct* model misspecifications or to use other statistical patches that influence [5.4. \(b\)](#). In almost any intermediate econometric textbook one finds a section that has the Greek symbol: Omega ( $\Omega$ ) (see Johnston and DiNardo (1997: 189)). This symbol is representative of the procedure whereby a researcher weights the data that are arrayed (in matrix form) so that the statistical errors, ultimately the standard error noted above,

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<sup>2</sup> Note that we think formal modeling can sometimes contribute to noncumulation as well. Formal models can fail to incorporate empirical findings that would assist in providing a more accurate depiction of the relations that are specified. This results in modeling efforts that yield inaccurate predictions or do not fit findings. In fact, data may contradict not just a model's results but also its foundational assumptions. The problem is not just the unreal assumptions, for one way to build helpful models is to begin with stylized assumptions, test the model's predictions, and then modify the assumptions consistent with a progressively more accurate model of reality. Yet, these follow-up steps are too often not taken or left unfinished --- with the result being a model that does little to enhance understanding and advance the discipline.

are altered and the t-statistic is manipulated. In theory, there is nothing wrong with knowing the Omega matrix for a particular statistical model. The standard error(s) produced by an Omega matrix should only serve as a check on whether inferences have been confounded to such an extent that a Type I or Type II error has been committed. Far too often, however, researchers treat the Omega weights (or alternative statistical patches) as the result of a true model. This attitude hampers scientific progress because it uses a model's mistakes to obscure flaws.<sup>3</sup>

We can also evaluate these current practices in relation to how they fail to contribute to a modeling dialogue between theory and test (see Figure 1). What we see is that the process of prediction and validation are never directly applied. Instead, the empirical test(s) remains in a loop or dialogue with itself. An iterative process of data mining, garbage cans, and the use of statistical patches (Omega matrices) replaces prediction and validation.

**(Figure 1 About Here)**

Even scholars who are sensitive to establishing robustness in their applied statistical results find the tools available are inadequate when used without a formal counterpart. For example, augmenting the applied statistical tests with *Extreme Bounds Analysis* (EBA) (Leamer (1983)) provides a check on parameter stability, but the test is ex-post and does not allow for ex-ante prediction.<sup>4</sup> This should not be surprising when one considers the effects of previously unspecified covariates in this procedure. Each time an applied statistical model is respecified the entire model is subject to change. All predictions are fragile in that sense, but without apriori use of equilibrium conditions (e.g., stability conditions) in a formal model, the parameter changes in a procedure such as EBA are of unknown origin.<sup>5</sup>

Because current applied statistical practices fail to develop formal models that analyze these interactions, we add the emphasis on modeling behavior. There is more than enough academic research that can aid in this break with current methodological practice. The Cowles Commission, for example, has a venerable record on establishing conditions in which structural parameters can be identified.<sup>6</sup> It was the Cowles Commission that explored the differences between structural and reduced-form parameters. Conditions for identifiability were introduced to aid in this differentiation. This work is now part of standard texts in econometrics.<sup>7</sup>

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<sup>3</sup> An example of this practice is the "correction" for serial correlation (see Mizon (1995)).

<sup>4</sup> We will use the word *inference* to refer to a parameter in a regression or likelihood (b). We use the word *prediction* to refer to a model's forecast of a dependent variable (c). For a technical treatment of these two concepts see Engle, Hendry, and Richard (1983).

<sup>5</sup> See Sala-i-Martin (1997) for an alternative viewpoint on the use of EBA.

<sup>6</sup> Created in the 1930s, the Cowles Commission was designed "to foster the development of logical, mathematical, statistical methods of analysis for application in economics and related social sciences." (See <http://cowles.econ.yale.edu/about-cf/about.htm>). Research associated with the Cowles Commission includes (but is not limited to): Cooper (1948), Haavelmo (1943, 1944), Hood and Koopmans (1953), Klein (1947), Koopmans (1945, 1949, 1950), Koopmans and Reiersol (1950), Marschak (1947, 1953), and Vining (1949).

<sup>7</sup> Along with their work on structural parameters, the Cowles Commission also gave formal and empirical specificity to issues such as exogeneity and policy invariance (Morgan (1990), Heckman (2000: 46)).

Nevertheless, these contributions have been marginalized and the situation today is one in which so-called technical work is only loosely connected to the fundamental scientific consideration of falsifiability. The reliance on statistically *significant* results means nothing when the researcher makes little attempt to identify the precise origin of the parameters in question. Absent this identification effort, it is not evident where the model is wrong. In a different sense, current practices are getting ahead of themselves: we first need to establish some means of falsifying our models before anything else.

In this paper, we describe an EITM framework to redirect methodological practice so that models and parameters have identifiable origins that allow for their falsifiability. The EITM framework addresses these issues through the unification of formal and empirical analysis. This framework takes advantage of the mutually reinforcing properties of formal and empirical analysis to address the challenge(s) above.

Another attribute of this framework is the emphasis on concepts that are quite general and integral to many fields of research but that are seldom modeled and tested in a direct way. EITM places emphasis on finding ways to model human behavior and action and, thereby, aids in creating realistic representations that improve upon simple socio-economic categorization. Numerous social science disciplines focus a good deal of research effort on the interactions between agent behavior and public policies.

This paper is organized as follows. In the next section, we discuss the issues and components of the EITM framework. Section 3 provides two research applications of the EITM framework: “macropartisanship” and economic voting. The final section summarizes these results and provides some concluding comments on how graduate training will need to evolve to provide the necessary formal and empirical tools to apply the EITM framework to a variety of research areas.

## **2 EITM: A Framework for Methodological Unification**

To reverse current methodological emphasis --- and to build a cumulative science of politics --- we present a framework that builds on prior quantitative research. The framework has three objectives. The first is to use the EITM framework to support a cumulative scientific process geared toward finding a causal mechanism. The ability of a researcher to parse out specific causal linkages among the many factors is fundamental to the scientific enterprise. When using quantitative tools, a model that links both formal and empirical approaches alerts researchers to outcomes when specific conditions are in place. It is also one of the best ways to determine an identified “causal” relation.

A second objective is to promote interdisciplinary interactions. The EITM framework we present is based on the original work of the Cowles Commission --- a group of quantitatively

inclined economists. The contributions of the Cowles Commission rest, in part, on a scientific vision that involved merging formal and applied statistical analysis.<sup>8</sup>

Objective three is to extend the Cowles Commission approach. The Cowles methodology created new research aimed at determining valid inference through the emphasis of the properties identification and invariance. For *identification*, rules (i.e., rank and order conditions) were devised so that an equation of a model could reveal one and only one set of parameters consistent with both the model and the observations (see, for example, Koopmans (1949)). A second issue involved the *invariance* of a (structural) relation.<sup>9</sup> If an underlying mechanism is constant in the past and future, then the path of the relevant variable(s) will be predictable from the past, apart from random disturbances (see, for example, Marschak (1947, 1953)).

With these three objectives in mind, the EITM framework uses what we know about theoretical and applied statistical concepts, provide a rigorous basis for these concepts through the use of their respective analogues, and then merge these theoretical analogues with the applied statistical analogues.<sup>10</sup> A *concept* can be thought of as an abstract or general idea inferred or derived from specific instances. An *analogue* can be thought of as a device in which a concept is represented by continuously variable --- and measurable --- quantities. What would develop is a road map for others to modify, correct, or follow. More importantly, one would be providing a transparent linkage between a theory and test. This is not to say the model is correct. Instead, what you would be doing is meeting a minimal requirement that the theory and test are related and, therefore, falsifiable.

## 2.1 Steps in the EITM Framework

Both formal and applied statistical modeling have important attributes that aid in falsification and, ultimately, scientific cumulation. Formal models, for example, force clarity about

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<sup>8</sup> The basis for this linkage was the idea that random samples were governed by some latent and probabilistic law of motion (Haavelmo (1944), Morgan (1990)). Further, this view meant that formal models, when related to an applied statistical model, could be interpreted as creating a sample draw from the underlying law of motion. A well-grounded test of a theory could be accomplished by relating a formal model to an applied statistical model and testing the applied statistical model. The Cowles methodology was seen, then, as a valid representation and examination of underlying processes in existence.

<sup>9</sup> We adopt Heckman's (2000: 59) terminology below:

*Structural causal effects are defined as the direct effects of the variables in the behavioral equations...When these equations are linear, the coefficients on the causal variables are called structural parameters (emphasis added), and they fully characterize the structural effects.*

Heckman also notes there is some disagreement about what constitutes a structural parameter. The disagreement centers on whether one uses a linear model, a non-linear model or, more, recently a fully parameterized model. In the latter case, structural parameters, can also be called “deep” to distinguish between “the derivatives of a behavioral relationship used to define causal effects and the parameters that generate the behavioral relationship” (p. 60).

<sup>10</sup> Theoretical concepts center on social, behavioral, political, and economic phenomena. Throughout this paper we will use the term *formal* and *theoretical* synonymously.

assumptions and concepts; they ensure logical consistency, and they describe the underlying mechanisms that lead to outcomes (see Powell (1999: 23-39)). On the other hand, applied statistical models provide generalizations and rule out alternative explanations through multivariate analysis. Applied statistics assist in distinguishing between causes and effects, allow for reciprocal causation, and also the relative size of the effects.

Below we discuss how both methods of analysis can also discourage scientific cumulation. While scientific cumulation is based on many things involving inference and prediction we narrow our focus to a widely used test indicator --- the t-statistic --- which is defined as the ratio  $b/s.e.(b)$ . Since we accept the Cowles Commission's focus on structural parameters to demonstrate specific cause and effect, we place particular emphasis on the numerator  $(b)$  as opposed to practices that emphasize the denominator  $(s.e.(b))$ . We argue that falsifiability influences scientific cumulation and the identification of  $(b)$  is consistent with attaining falsifiability.

These steps are meant to be suggestive. We hasten to add that the practice of linking formal and applied statistical models need not include explicit behavioral components. What we are arguing is consistent with efforts to preserve structural parameters. Our framework points to an alternative where formal and applied statistical models reveal mutually reinforcing properties.

This EITM framework is summarized as follows:

**1. Unify Theoretical Mechanisms and Applied Statistical Concepts:** Given that human beings are the agents of action, mechanisms should reflect overarching social and behavioral processes. Examples include (*but are not limited to*): decision making; bargaining; expectations; learning; and social interaction. It is also important to find an appropriate statistical concept to match with the theoretical concept. Examples of applied statistical concepts include (*but are not limited to*): persistence; measurement error; nominal choice; and simultaneity.

**2. Develop Behavioral (Formal) and Applied Statistical Analogues:** To link concepts with tests, we need analogues. An analogue is a device in which a concept is represented by continuously variable --- and measurable --- quantities. Examples of analogues for the behavioral (formal) concepts such as decision making, expectations, and learning include (*but are not limited to*): decision theory (e.g., utility maximization); conditional expectations procedures; and adaptive and Bayesian learning procedures. Examples of applied statistical analogues for the applied statistical concepts of persistence, measurement error, nominal choice, and simultaneity include (respectively): autoregressive estimation; error-in-variables regression; discrete choice modeling; and multi-stage estimation (e.g., two-stage least squares).

**3. Unify and Evaluate the Analogues:** The third step unifies the mutually reinforcing properties of the formal and empirical analogues. There are various ways to establish the linkage. For example, when researchers assume citizens (voters) or economic agents are rational actors who



make decisions to maximize their own payoffs, a common analogue is utility (or profit) maximization. With this theoretical analogue in place, the other consideration is to determine the appropriate statistical concept and analogue to test the theoretical relationship. Consider a basic Downsian model of voting. Voters decide to vote for one of the parties to maximize their utilities (e.g., decision theory). This theoretical concept/analogue can be unified with the applied statistical concept, nominal choice, and its analogue, discrete choice modeling.

### 3 EITM in Practice

We now demonstrate the EITM framework using published research. The first example, concerning the “stickiness” of party identification involves linking the behavioral concept of expectations with the applied statistical concept of persistence. The second example examines how the behavioral concepts of expectations and uncertainty, when linked to the applied statistical concept of measurement error, create distinct hypotheses in the economic voting literature.

#### 3.1 Example 1: Macropartisanship

An important debate in political science centers on the persistence of party identification, or what has been termed *macropartisanship* (see Erikson, MacKuen, and Stimson (2002: 109-151)). We use Clarke and Granato's (2004) example of an EITM formulation. They assume that political campaign advertisements influence the public. They also argue that the persistence of rival political parties political advisors to target and influence (through the use of political advertisement resources) rival party voters, reduces the well known persistence in macropartisanship. As a consequence, shocks to macropartisanship can either be amplified or die out quickly depending on the rival political advisor's actions.

##### 3.1.1 Step 1: Relating Behavioral and Applied Statistical Concepts: Expectations and Persistence

Clarke and Granato (2004) relate agent expectations and the persistence of agent behavior. They demonstrate how a rival political strategist can use campaign advertisements to influence aggregate persistence in party identification. Their model comprises three equations. Each citizen ( $i$ ) is subject to an event ( $j$ ) at time ( $t$ ). They then aggregate across individuals and events so the notation will only have the subscript  $t$ .

$$M_t = a_1 M_{t-1} + a_2 E_{t-1} M_t + a_3 F_t + u_{1t} \tag{1}$$

$$F_t = b_1 F_{t-1} + b_2 A_t + u_{2t} \tag{2}$$

$$A_t = c_1 A_{t-1} + c_2 (M_t - M^*) + c_3 F_{t-1} \tag{3}$$

The first equation (1) specifies what influences aggregate party identification ( $M_t$ ). The variable  $M_{t-1}$  accounts for the possibility of persistence. Citizens also have an expectation of what portion of the population will identify with the party ( $E_{t-1}M_t$ ). They assume that, in forming their expectations, citizens use all available and relevant information (up to time  $t-1$ ) as specified in this model (rational expectations). Clarke and Granato further assume that party identification depends on how favorably a citizen views the national party ( $F_t$ ). Finally, party identification can be subject to unanticipated stochastic shocks (realignment) ( $u_{1t}$ ) where  $u_{1t} \sim N(0, \sigma_{u_1}^2)$ . The relations are assumed to be positive --  $a_1, a_2, a_3 \geq 0$ .

Equation (2) represents citizens' impression ("favorability") of a political party ( $F_t$ ). In this model, favorability is a linear function of the lag of favorability ( $F_{t-1}$ ) and an advertising resource variable ( $A_t$ ). There are many ways to measure political advertising resources.  $u_{2t}$  is a stochastic shock that represents unanticipated events (uncertainty), where  $u_{2t} \sim N(0, \sigma_{u_2}^2)$ . The parameter  $b_1 \geq 0$ , while  $b_2$  depending on the tone and content of the advertisement.

Equation (3) presents the contingency plan or rule that (rival) political advisors use. Clarke and Granato argue that political advisors track their previous period's advertising resource expenditures ( $A_{t-1}$ ) and react to that period's favorability rating for the (rival) national party ( $F_{t-1}$ ). Political advisors also base their current expenditure of advertisement resources on the degree to which macropartisanship ( $M_t$ ) approximates a prespecified and desired target ( $M^*$ ). Ideally, political advisors want  $(M_t - M^*) = 0$ . The parameters  $c_1$  and  $c_3$  are positive. The parameter  $c_2$  is countercyclical ( $-1 \leq c_2 \leq 0$ ): it reflects a political advisors' willingness to increase or conserve their advertising resources depending on whether macropartisanship is above (decrease advertising) or below (increase advertising) the target.

### 3.1.2 Step 2: Formal and Applied Statistical Analogues: Conditional Expectations and Autoregressive Estimation<sup>11</sup>

The reduced form for macropartisanship is determined by substituting (3) into (2). Note that there is an autoregressive component ( $\Theta_1 M_{t-1}$ ) in the reduced form for macropartisanship:

$$M_t = \Theta_0 + \Theta_1 M_{t-1} + \Theta_2 E_{t-1} M_t + \Theta_3 A_{t-1} + \Theta_4 F_{t-1} + \varepsilon_t^* \quad (4)$$

where:  $\Theta_0 = (1 - b_2 c_2)^{-1} (b_2 c_1 Y^*)$ ,  $\Theta_1 = (1 - b_2 c_2)^{-1} a_1$ ,  $\Theta_2 = (1 - b_2 c_2)^{-1} a_2$ ,  $\Theta_3 = (1 - b_2 c_2)^{-1} (b_2 c_1)$ ,  $\Theta_4 = (1 - b_2 c_2)^{-1} (b_1 + b_2 c_3)$ , and  $\varepsilon_t^* = (1 - b_2 c_2)^{-1} (u_{1t} + u_{2t})$ .

The system is now simplified to a model of macropartisanship that depends on lagged

<sup>11</sup> In this example, the formal analogue is conditional expectations modeling. The applied statistical analogue is an autoregressive process. See the Appendix for the technical details for the behavioral and applied statistical analogues.

macropartisanship and also the conditional expectation at time  $t-1$  of current macropartisanship. This lagged dependent variable is the analogue for *persistence*. Note that the prior values of advertising and favorability may also have an effect.

Because (4) possesses a conditional expectations operator we must make it a function of other variables (not operators). In this example, “closing the model” and finding the rational expectations equilibrium (REE) involves taking the conditional expectation at time  $t-1$  of equation (4) and then substituting this result back into equation (4):

$$M_t = \Pi_1 + \Pi_2 M_{t-1} + \Pi_3 A_{t-2} + \Pi_4 F_{t-2} + \xi_t' \quad (5)$$

Equation (5) is the minimum state variable (MSV) solution (McCallum, 1983) for macropartisanship.<sup>12</sup> Macropartisanship ( $M_t$ ) also depends on its past history, the autoregressive component ( $M_{t-1}$ ).

### 3.1.3 Step 3: Link the Formal and Applied Statistical Analogues

The persistence of macropartisanship ( $\Pi_2$ ) is now shown to depend on the persistence and willingness of rival political advisors to maintain a rival macropartisanship target ( $c_2$ ). In other words, the EITM linkage is the MSV with the AR(1) component in (5). This can be shown by examining the reduced form AR(1) coefficient expression  $\Pi_2$ :

$$\Pi_2 = \frac{a_2 + b_2 c_2 (c_2 + b_2 + b_2 c_2)}{1 - b_2 c_2 - a_2} \quad (6)$$

We take the derivative of (6) with respect to ( $c_2$ ) and find the following relation:

$$\frac{\partial \Pi_2}{\partial c_2} = \frac{b_2 (a_2 + (1 - a_2) A)}{(1 - a_2 - b_2 c_2)^2}, \quad (7)$$

where  $A = (b_1 + c_1 + b_2 c_2)$ . Given the assumptions about the signs of the coefficients in the model, the numerator is positive as long as  $a_2 < 1$ . Therefore, under these conditions, we know that the relationship is positive ( $\frac{\partial \Pi_2}{\partial c_2} > 0$ ).

(Figure 2 About Here)

The relationship between  $c_2$  and  $\Pi_2$  is demonstrated in Figure 2. We use the following values:  $a_1 = a_2 = b_1 = b_2 = c_1 = c_3 = 0.5$ . The parameter  $c_2$  ranges from 0.0 to -1.0. As we vary the value of  $c_2$  between 0.0 and -1.0, we find that the persistence (autocorrelation) in

<sup>12</sup> Note:  $\Pi_1 = \frac{a_1}{1 - \theta_2} - (\frac{a_2}{1 - \theta_2} - \frac{a_2}{1 - \theta_2} b_2) c_2 F^*$ ,  $\Pi_2 = \frac{a_2}{1 - \theta_2} + (\frac{a_2}{1 - \theta_2} + \frac{a_2}{1 - \theta_2} b_2) c_2$ ,  $\Pi_3 = (\frac{a_3}{1 - \theta_2} + \frac{a_3}{1 - \theta_2} b_2) c_2$ ,  $\Pi_4 = \frac{a_4}{1 - \theta_2} c_2 + \frac{a_4}{1 - \theta_2} (b_1 + b_2 c_2)$ , and  $\xi_t' = \frac{\theta_2}{1 - \theta_2} u_{2t} + \sigma_t'$ .

macropartisanship ( $\Pi_2$ ) --- all things equal --- is zero when  $c_2 = -0.8$ . On the other hand, macropartisanship becomes highly autoregressive ( $\Pi_2 \rightarrow 1.0$ ) when rival political advisors fail to react ( $c_2 \rightarrow 0.0$ ) to deviations from their prespecified target. *The conclusion from this model is that negative advertisements from rival political parties can influence the persistence of their opponents national party identification.*

## 3.2 Example 2: Economic Voting

It is often the case that over time many parameters drift and also exhibit periods of volatility. Constant parameters are not necessarily the expected state of affairs. When expectations and uncertainty analogue(s) are linked to a measurement error analogue (error-in-variables regression) it can be shown that parameters in the regression reflect the increasing variance (i.e., uncertainty) of variables of interest.

### 3.2.1 Step 1: Relating Behavioral and Applied Statistical Concepts: Expectations, Uncertainty, and Measurement Error

Starting with Kramer (1983) and continuing with work that includes (but is not limited to) Alesina and Rosenthal (1995), Suzuki and Chappell (1996), and Lin (1999) there is a substantial quantitative literature on how the economy affects voting behavior. For our purposes we demonstrate how the combined work of Alesina and Rosenthal and Suzuki and Chappell have in their own way contributed constituent parts to the study of economic voting.

The uncertainty that voters face can ideally be tested using the EITM framework. The EITM relation can be found in the following way: when expectations and uncertainty analogue(s) are linked to a measurement error analogue (error-in-variables regression) it can be shown that parameters in the regression reflect the increasing variance (i.e., uncertainty) of the variables of interest. In other words, as expectations and uncertainty analogues change, the testable economic effect on voting change as well.

### 3.2.2 Step 2: Formal and Applied Statistical Analogues<sup>13</sup>

We start with a formal model that has the behavioral concepts of expectations and uncertainty. Alesina and Rosenthal (1995) provide such a formal model (pp. 191-195). Their model of economic growth is based on an expectations augmented aggregate supply curve:

$$y_t = y^n + \gamma(\pi_t - \pi_t^e) + \varepsilon_{y,t} \quad (8)$$

where  $y_t$  represents the rate of economic growth (GDP growth) in period  $t$ ,  $y^n$  is the natural economic growth rate,  $\pi_t$  is the inflation rate at time  $t$ , and  $\pi_t^e$  is the expected inflation rate at time  $t$  formed at time  $t - 1$ .

With voter inflation expectations established we now turn to the concept of uncertainty. Let us assume that voters want to determine whether to attribute credit or blame for economic growth ( $y_t$ ) outcomes to the incumbent administration. Yet, voters are faced with the uncertainty of

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<sup>13</sup> The formal analogue(s) are: conditional expectations modeling, recursive projections, and the law of iterated projections (or expectations). The applied statistical analogue is error-in-variables regression.

determining what part of the economic outcomes is due to incumbent “competence” (i.e., policy acumen) or simply good luck.

Equation (9), the shock in equation (8), can be modified to present the analogue for uncertainty:

$$a_t = \eta_t + \xi_t \quad (9)$$

The variable  $a_t$  represents the shock that is comprised of the two unobservable characteristics noted above --- competence or good luck.<sup>14</sup> The first, represented by  $\eta_t$ , reflects “competence” that can be attributed to the incumbent administration. The second, symbolized as  $\xi_t$ , are shocks to growth that are beyond administration control (and competence). Both  $\eta_t$  and  $\xi_t$  have zero mean with variance(s)  $\sigma_\eta^2$  and  $\sigma_\xi^2$  respectively.

Note also that competence can persist and *support* reelection. This feature can be characterized as an MA(1) process:

$$\eta_t = \mu_\eta + \rho\eta_{t-1}, \quad \rho \in (0, 1] \quad (10)$$

where  $\mu_\eta$  is  $iid(0, \sigma_\eta^2)$ . The parameter  $\rho$  represents the strength of the persistence. The lag or lags allow for retrospective voter judgments.

If we reference equation (8) again, let us assume that voters' judgments include a general sense of the average rate of growth ( $\bar{y}^n$ ) and the ability to observe actual growth ( $y_t$ ). Voters can evaluate their difference ( $y_t - \bar{y}^n$ ). Equation (8) also implies that when voters predict inflation with no systematic error (i.e.,  $\pi_t^e = \pi_t$ ), the result is non-inflationary growth that does not adversely affect real wages.

Next, we tie economic growth performance to voter uncertainty. We formalize how economic growth rate deviations from the average can be attributed to administration competence or fortuitous events:

$$y_t - \bar{y}^n = a_t = \eta_t + \xi_t \quad (11)$$

Equation (11) shows that when the actual economic growth rate is greater than its average or “natural rate” (i.e.,  $y_t > \bar{y}^n$ ), then  $a_t = \eta_t + \xi_t > 0$ . Again, the voters are faced with uncertainty in distinguishing the incumbent's competence ( $\eta_t$ ) from the stochastic economic shock ( $\xi_t$ ). However, because competence can persist, voters use this property to make forecasts and give greater or less weight to competence over time.

To demonstrate this behavioral effect, substitute equation (10) in (11):

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<sup>14</sup> This following modification is commonly referred to as a “signal extraction” or measurement error problem.

$$\mu_t + \xi_t = \mathcal{Y}_t - \mathcal{Y}^n - \rho\mu_{t-1} \quad (12)$$

We now determine the optimal estimate of competence,  $\eta_{t+1}$ , when the voters only see  $\mathcal{Y}_t$ . We demonstrate this result by making a one-period forecast of equation (10) and then solve for its expected value (conditional expectation) at time  $t$ :

$$E_t(\eta_{t+1}) = E_t(\mu_{t+1}) + \rho E(\mu_t | \mathcal{Y}_t) = \rho E(\mu_t | \mathcal{Y}_t), \quad (13)$$

where  $E_t(\mu_{t+1}) = 0$ . Using this analogue for expectations in equation (13), we see that competence,  $\eta_{t+1}$ , can be forecasted by predicting  $\mu_{t+1}$  and  $\mu_t$ . Since there is no information available for forecasting  $\mu_{t+1}$ , voters can only forecast  $\mu_t$  based on observable  $\mathcal{Y}_t$  (at time  $t$ ) from equation (12).

### 3.2.3 Step 3: Link the Formal and Applied Statistical Analogues

Using the method of recursive projection and equation (12) we illustrate how the behavioral analogue for expectations is linked to the empirical analogue for measurement error (an error-in-variables “equation”):<sup>15</sup>

$$E_t(\eta_{t+1}) = \rho E(\mu_t | \mathcal{Y}_t) = \rho \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\xi^2} (\mathcal{Y}_t - \mathcal{Y}^n - \rho\mu_{t-1}), \quad (14)$$

where  $0 < \rho \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\xi^2} < 1$ . Equation (14) shows that voters can forecast competence using the difference between  $\mathcal{Y}_t - \mathcal{Y}^n$ , but also the “weighted” lag of  $\mu_t$  (i.e.,  $\rho\mu_{t-1}$ ).

In equation (14), the expected value of competence is *positively* correlated with economic growth rate deviations. Voter assessment is filtered by the coefficient,  $\frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\xi^2}$ , which represents a proportion of competence that voters are able to interpret and observe. The behavioral implications are straightforward. If voters interpret that the variability of economic shocks come solely from the incumbent's competence (i.e.,  $\sigma_\xi^2 \rightarrow 0$ ), then  $\frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\xi^2} \rightarrow 1$ . On the other hand, the increase in the variability of uncontrolled shocks,  $\sigma_\xi^2$ , confounds the observability of incumbent competence since the signal coefficient  $\frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\xi^2}$  decreases. Voters assign less weight to economic performance in assessing the incumbent's competence.

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<sup>15</sup> The traditional applied statistical view of measurement errors is that the correlated signs of the measurement errors among independent variables can lead to inappropriate signs for regression coefficients. This is exactly what EITM and methodological unification accomplish. The theory --- the formal model --- implies an applied statistical model with measurement error. Consequently, one can examine, with a unified approach, the joint effects and identify the cause. Applied statistical tools cannot untangle conceptually distinct effects on a dependent variable.

To sum up, Alesina and Rosenthal provide an EITM connection between equations (8), (10) and their empirical tests. *They link the behavioral concepts --- expectations and uncertainty --- with their respective analogues (conditional expectations and measurement error) and devise a signal extraction problem.* While the empirical model resembles an error-in-variables specification, testable by dynamic methods such as rolling regression (Lin (1999)), instead they estimate the variance-covariance structure of the residuals.

## 4 Summary and Discussion

In this paper we have described the weaknesses of current research practices and how their nonfalsifiable nature do not contribute to a modeling dialogue. The failure to tie theories to tests and provide meaningful feedback means scientific cumulation suffers. In response to this status quo --- as it pertains to quantitative methodology --- we devised the EITM framework. As part of the NSF's EITM initiative this framework involves emphasis on *developing behavioral and applied statistical analogues and linking these analogues.*

The EITM framework *will require* some changes in graduate training. *Siloed* training either in formal or applied statistical training will have to give way to a pedagogical emphasis in not only learning some basics in each but thinking of ways to link them.

There are numerous research areas in the political and social sciences where the EITM framework has been applied. A sample and the attendant formal and empirical tools required would include:

- **The Political Economy of Macro Policy:** The EITM linkage is the relation between the behavioral concept of expectations with the empirical concept of persistence. Granato and Wong's (2006) real wage contract model demonstrates a linkage between monetary policy, the public's inflation expectations, and inflation persistence. The tools required to establish this relation include difference equations (various orders), their solution procedures, and stability conditions. Along with these relevant formal modeling tools is a discussion of the empirical estimation and the properties of autoregressive processes.
- **Political Parties and Political Representation:** One well researched area centers on when and why voters choose one party over the others based on the relative political positions of parties. The work of Kedar (2005), Merrill and Grofman (1999), Groseclose (2001), Ansolabehere, Snyder, and Stewart (2001), and Adams, Merrill and Grofman (2005) are just a few examples. The tools required to fit within the EITM framework involve the linkage of decision theoretic models with discrete outcomes.
- **Voter Turnout:** The EITM linkage is the behavioral concept of learning combined with the empirical concept of discrete choice (see Hetherington (2001), Plutzer (2002), and Gerber, Green, and Shachar (2003)). Other examples include Achen (2006), Dhillon and Peralta (2002), and Shachar and Nalebuff (1999). In these latter papers, the authors develop a Bayesian learning



model and link it to a discrete choice empirical model. The tools required include the theory of Bayesian learning and an introduction to discrete choice statistical models (logit and probit).

• **International Conflict and Cooperation**: The EITM linkage includes the behavioral concept of bargaining and strategic interaction combined with the empirical concept of discrete choice. Signorino's (1999) work on Quantal Response Equilibrium (QRE) --- a technique used to merge Game Theory and discrete choice models. The tools needed are elements of Game Theory, discrete choice modeling, and how these inform QRE.

Our framework is certainly not the final word on linking formal and applied statistical models. Ultimately, what EITM means is a clean break from current practices such as data mining, garbage cans, and Omega matrices. The old mindset of treating applied statistical problems as simply requiring statistical patches will need to give way to viewing these *nuisances* as having a theoretical basis. With that new viewpoint a researcher can take advantage of the mutually reinforcing properties of both formal and applied statistical analysis, and test the identified parameter-specific linkage against actual data.

## References

Achen, C. H. 2005. "Let's Put Garbage-Can Regressions and Garbage-Can Probits where They Belong". *Conflict Management and Peace Science* 22: 327-339.

Achen, C. H. 2006. "Expressive Bayesian Voters, their Turnout Decisions, and Double Probit: Empirical Implications of a Theoretical Model". Typescript. Princeton University.

Alesina, A., and H. Rosenthal. 1995. *Partisan Politics, Divided Government, and the Economy*. New York: Cambridge University Press.

Ansola-behere, S. D., J. M. Snyder, Jr., and C. Stewart III. 2001. "Candidate Positioning in U.S. House Elections". *American Journal of Political Science* 45: 136-159.

Clarke, H., and J. Granato. 2004. "Autocorrelation: From Practice to Theory". In K. Kampf-Leonard (ed.), *Encyclopedia of Social Measurement*. San Diego: Academic Press.

Cooper, G. 1948. "The Role of Econometric Models in Economic Research". *Journal of Farm Economics* 30: 101-116.

Denton, F. T. 1985. "Data Mining as an Industry". *The Review of Economics and Statistics* 67: 124-127.

Dhillon, A., and S. Peralta. 2002. "Economic Theories of Voter Turnout". *The Economic Journal* 112: 332-352.

Engle, R. F., D. F. Hendry, and J. F. Richard. 1983. "Exogeneity". *Econometrica* 51: 277-304.

- Erikson, R. S., M. B. MacKuen, and J. A. Stimson. 2002. *The Macro Polity*. New York: Cambridge University Press.
- Evans, G. W., and S. Honkapohja. 2001. *Learning and Expectations in Macroeconomics*. Princeton: Princeton University Press.
- Ezekiel, M. 1938. "The Cobweb Theorem". *Quarterly Journal of Economics* 52: 255--280.
- Gerber, A., D. Green, and R. Shachar. 2003. "Voting May Be Habit Forming". *American Journal of Political Science* 47: 540-550.
- Granato, J., and F. Scioli. 2004. "Puzzles, Proverbs, and Omega Matrices: The Scientific and Social Significance of Empirical Implications of Theoretical Models (EITM)". *Perspectives on Politics* 2: 313-323.
- Granato, J., and M. C. S. Wong. 2006. *The Role of Policymakers in Business Cycle Fluctuations*. Cambridge: Cambridge University Press.
- Groseclose, T. 2001. "A Model of Candidate Location When One Candidate Has a Valence Advantage". *American Journal of Political Science* 45: 862-886.
- Haavelmo, T. 1943. "The Statistical Implications of a System of Simultaneous Equations". *Econometrica* 11: 1-12.
- Haavelmo, T. 1944. "The Probability Approach in Econometrics". *Econometrica* 12: S1-115.
- Heckman, J. 2000. "Causal Parameters and Policy Analysis in Economics: A Twentieth Century Retrospective". *Quarterly Journal of Economics* 115: 45-97.
- Hetherington, M. 2001. "Resurgent Mass Partisanship: The Role of Elite Polarization". *American Political Science Review* 95: 619-631.
- Hood, W. C., and T. C. Koopmans., eds. 1953. *Studies in Econometric Method*. Cowles Commission Monograph No. 14. New York: John Wiley & Sons.
- Johnston, J. J., and J. DiNardo. 1997. *Econometric Methods*. New York: McGraw-Hill.
- Kedar, O. 2005. "When Moderate Voters Prefer Extreme Parties: Policy Balancing in Parliamentary Elections". *American Political Science Review* 99: 185-199.
- Klein, L. R. 1947. "The Use of Econometric Models as a Guide to Economic Policy". *Econometrica* 15: 111-151.
- Koopmans, T. 1945. "Statistical Estimation of Simultaneous Economic Relations". *Journal of the American Statistical Association* 40: 448-466.

- Koopmans, T. 1949. "Identification Problems in Economic Model Construction". *Econometrica* 17: 125-144.
- Koopmans, T., ed. 1950. *Statistical Inference in Dynamic Economic Models*. Cowles Commission Monograph No. 10. New York: John Wiley & Sons.
- Koopmans, T., and O. Reiersol. 1950. "The Identification of Structural Characteristics". *The Annals of Mathematical Statistics* 21: 165-181.
- Kramer, G. 1983. "The Ecological Fallacy Revisited: Aggregate Versus Individual-Level Findings on Economics and Elections and Sociotropic Voting". *American Political Science Review* 77: 92-111.
- Leamer, E. 1983. "Let's Take the 'Con' Out of Econometrics". *American Economic Review* 73: 31-43.
- Lin, T. M. 1999. "The Historical Significance of Economic Voting, 1872—1996". *Social Science History* 23: 561--91.
- Lovell, M. C. 1983. "Data Mining". *The Review of Economics and Statistics* 65: 1-12.
- Marschak, J. 1947. "Economic Structure, Path, Policy, and Prediction". *American Economic Review* 31: 81-84.
- Marschak, J. 1953. "Economic Measurements for Policy and Prediction". In *Studies in Econometric Method*, eds. W. C. Hood and T. G. Koopmans. New York: Wiley.
- McCallum, B. T. 1983. "On Nonuniqueness in Linear Rational Expectations Models: An Attempt at Perspective". *Journal of Monetary Economics* 11: 134-168.
- Merrill, S., III., and B. Grofman. 1999. *A Unified Theory of Voting: Directional and Proximity Spatial Models*. Cambridge: Cambridge University Press.
- Mizon, G. 1995. "A Simple Message to Autocorrelation Correctors: Don't". *Journal of Econometrics* 69: 267-288.
- Morgan, M. 1990. *The History of Econometric Ideas*. Cambridge: Cambridge University Press.
- Morton, R. B. 1999. *Methods and Models: A Guide to the Empirical Analysis of Formal Models in Political Science*. New York: Cambridge University Press.
- Muth, J. F. 1961. "Rational Expectations and the Theory of Price Movements". *Econometrica* 29: 315-335.
- Plutzer, E. 2002. "Becoming a Habitual Voter". *American Political Science Review* 96: 41-56.

Powell, R. 1999. *In the Shadow of Power*. Princeton: Princeton University Press.

Sala-i-Martin, X. 1997. "I Just Ran Two Million Regressions". *American Economic Review* 87: 178-183.

Sargent, T. 1987. *Macroeconomic Theory*. Orlando, Florida: Academic Press.

Shachar, R., and B. Nalebuff. 1999. "Follow the Leader: Theory and Evidence on Political Participation". *American Economic Review* 89: 525-547.

Signorino, C. 1999. "Strategic Interaction and the Statistical Analysis of International Conflict". *American Political Science Review* 93: 279-297.

Suzuki, M., and H. W. Chappell Jr. 1996. "The Rationality of Economic Voting Revisited". *Journal of Politics* 58: 224-236.

Vining, R. 1949. "Methodological Issues in Quantitative Economics: Koopmans on the Choice of Variables to be Studied and of Methods of Measurement". *Review of Economics and Statistics* 31: 77-86.

## **APPENDIX 1: Notes on Analogues for Example 1**

### **An Expectations Analogue: The Minimum State Variable (MSV) Solution Procedure**

Modeling expectations has a long history, ranging from the application of cobweb, adaptive, and rational analogues of expectations.<sup>16</sup> Here we demonstrate the cobweb model and relate it to a rational expectations equilibrium (REE).<sup>17</sup> These procedures are used to reach the result in equation (5).

A REE imposes a consistency condition that each agent's choice is a best response to the choices made by others (Evans and Honkapohja (2001: 11)). Furthermore, the REE rests on a strong assumption that agents use all available information in the model to formulate their predictions.

We first show how to solve models with expectations. One way is to simply substitute for the expectations operator  $(E_t)$ . A second, more general, way is to use *the method of undetermined coefficients*.

To demonstrate these procedures consider the cobweb model:

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<sup>16</sup> Adaptive expectations can be defined as a model making agent's expectations of a variable a geometric weighted average of past values of that variable. Rational expectations can be defined as a mathematical expectation of a variable conditional on all available variables observable at some point in time. See Appendix 2 for details.

<sup>17</sup> The original cobweb model is a reduced form of supply and demand in an isolated market (see Ezekiel (1938) but also Muth (1961)). It represents a single competitive market where a time lag exists in production. The model describes an adjustment process that on a price/quantity or supply/demand graph spirals toward equilibrium.

$$y_t = \alpha + \beta E_{t-1} y_t + \gamma w_{t-1} + s_t, \quad (15)$$

where  $y_t$  is an endogenous variable which can be thought of as any variable that agents are interested in forecasting,  $E_{t-1} y_t$  is the expectation of  $y_t$  formed at the end of time  $t - 1$ ,  $w_{t-1}$  is a vector of exogenous observables and  $s_t \sim iid(0, \sigma_s^2)$  is a stochastic error term.

The method of undetermined coefficients works in the following way. Assuming that the REE of equation (15) takes the following expression:

$$y_t = \Pi_0 + \Pi_1 w_{t-1} + s_t, \quad (16)$$

We take a conditional expectation  $E_{t-1}$  in equation (16), we have:

$$E_{t-1} y_t = \Pi_0 + \Pi_1 w_{t-1}.$$

Substitute this result into equation (15):

$$y_t = \alpha + \beta (\Pi_0 + \Pi_1 w_{t-1}) + \gamma w_{t-1} + s_t,$$

and collect terms:

$$y_t = (\alpha + \beta \Pi_0) + (\gamma + \beta \Pi_1) w_{t-1} + s_t. \quad (17)$$

The expression in equation (17) is identical to that of equation (16), so the coefficients of equation (16) should be equivalent to equation (17), such that:

$$[\text{From equation 16}] \Pi_0 = \alpha + \beta \Pi_0 \quad [\text{From equation (17)}]$$

$$[\text{From equation 16}] \Pi_1 = \gamma + \beta \Pi_1 \quad [\text{From equation (17)}]$$

and solve for  $\Pi_0$  and  $\Pi_1$ :

$$\Pi_0 = \frac{\alpha}{1-\beta}$$

$$\Pi_1 = \frac{\gamma}{1-\beta}. \quad (18)$$

The REE equation is found by substituting the  $\Pi$ 's back into equation (16):

$$y_t = \frac{\alpha}{1-\beta} + \frac{\gamma}{1-\beta} w_{t-1} + s_t. \quad (19)$$

Equation (19) is called the rational expectations equilibrium (REE) where  $y_t$  does not depend on its expectations ( $E_{t-1} y_t$ ), but only other exogenous variables ( $w_{t-1}$ ) and a stochastic error term ( $s_t$ ). Equation (19) has the same type of identity as equation (5).

## Persistence Analogue: AR Processes

Consider the case of a simple first-order autoregressive (AR(1)) process:

$$y_t = \phi y_{t-1} + \varepsilon_t \quad (20)$$

where  $\varepsilon_t$  is a white-noise stochastic error process and  $\text{cov}(\varepsilon_t, \varepsilon_{t-k}) = 0$ , for all  $t, k$ . Expression (20) is a component in both equations (4) and (5) in the form of the variable the lagged dependent variable,  $M_{t-1}$ .

To show how the AR(1) process captures the concept of persistence in (20) consider the following. We know that one of the attributes of time series analysis is that point estimates for the immediate effect can be used to determine the long-run or steady state effect. The identity for the dynamic multiplier can be derived when there are lags in the dependent variable. This provides the persistence and eventual decay of the initial effect from a given variable or variables.

Lag operators are important in this demonstration since they provide short-hand notation indicating how many lags of a particular variable are used in a model. The lag operator is usually represented by the letter “ $L$ ”. Consider the variable  $y_t$ . We express the first- and second-order lags of this variable below, respectively:

$$Ly_t = y_{t-1}$$

$$L^2 y_t = L(Ly_t) = Ly_{t-1} = y_{t-2}$$

This generalizes to:

$$L^p = y_{t-p}$$

and

$$L^j(L^i y_t) = y_{t-i-j}$$

To see the convenience of the lag operators, we can first derive the expression of  $y_t$  --- which depends on a series of  $\varepsilon_t$  for all  $t$  --- by the method of substitution. First, take equation (20) and lag it one period:

$$y_{t-1} = \phi y_{t-2} + \varepsilon_{t-1} \quad (21)$$

Substituting equation (21) into equation (20), we have:

$$\begin{aligned}
y_t &= \phi(\phi y_{t-2} + a_{t-1}) + a_t \\
y_t &= \phi^2 y_{t-2} + a_t + \phi a_{t-1}
\end{aligned} \tag{22}$$

where  $y_t$  depends on  $y_{t-2}$  and two stochastic errors,  $a_t$  and  $a_{t-1}$ . If we repeatedly take time periods backward in equation (21) and substitute it into equation (22), we have:

$$\begin{aligned}
y_t &= \phi^n y_{t-n} + a_t + \phi a_{t-1} + \dots + \phi^n a_{t-n} \\
y_t &= \phi^n y_{t-n} + \sum_{i=0}^n \phi^i a_{t-i}
\end{aligned} \tag{23}$$

where  $n$  represents the number of backward time periods. If we move the time period back to the initial period, that is,  $n = t$ , then we can derive the expression of  $y_t$  which depends on its initial value  $y_0$  and a series of stochastic shocks from equation (23):

$$y_t = \phi^t y_0 + \sum_{i=0}^t \phi^i a_{t-i} \tag{24}$$

However, if the series of  $y_t$  does not have an initial value, we can move the time period backward indefinitely (i.e.,  $n \rightarrow \infty$ ), we have:

$$y_t = \lim_{n \rightarrow \infty} (\phi^n y_{t-n} + \sum_{i=0}^n \phi^i a_{t-i}) = \sum_{i=0}^{\infty} \phi^i a_{t-i} \tag{25}$$

where  $\lim_{n \rightarrow \infty} (\phi^n y_0) = 0$  for  $|\phi| < 1$ . Equation (25) shows that  $y_t$  depends on an infinitely weighted sum of stochastic errors.

The lag operator can conveniently derive the process of  $y_t$  which is equivalent to equation (25). From equation (20), we can apply the lag operator:

$$y_t = \phi L y_t + a_t \tag{26}$$

Then, we can solve for the expression of  $y_t$  as:

$$(1 - \phi L) y_t = a_t$$

therefore,

$$y_t = \left( \frac{1}{1 - \phi L} \right) a_t = \sum_{i=0}^{\infty} \phi^i a_{t-i}$$

where  $\frac{1}{1+\phi L} = \sum_{j=0}^{\infty} \phi^j L^j = 1 + \phi L + \phi^2 L^2 + \dots + \phi^n L^n$ , for  $|\phi| < 1$ . As long as the process is stationary (i.e.,  $|\phi| < 1$ ), the effect of shocks gets less pronounced the further into the past one goes.<sup>18</sup>

The expression,  $1/(1 + \phi L)$ , which makes use of the lag operator, tells us the accumulated effect. What we want to know is not only the point estimate, the initial effect, but also how the effect accumulated. Consider, for example, the following AR(1) model (the constant and additional independent variables are dropped for convenience):

$$y_t = \phi y_{t-1} + \theta I_t + \tilde{\epsilon}_t,$$

where  $I_t$  is the intervention and  $|\phi| < 1$ . A clue on how long the effect lasts is the size of  $\phi$ . If there is a one-time intervention at time  $t = T$  (i.e.,  $I_{t=T} = 1$  and  $I_{t \neq T} = 0$ , for all  $t$ ) and no stochastic shock ( $\tilde{\epsilon}_t = 0$  for all  $t$ ), one can show the accumulated effect by using the lag operator:

$$y_t = \left( \frac{\theta}{1-\phi L} \right) I_t + \left( \frac{\theta}{1-\phi L} \right) \tilde{\epsilon}_t, \tag{28}$$

$$y_t = \frac{\theta}{1-\phi}, \tag{29}$$

for  $|\phi| < 1$ , where  $\phi L I_{t=T} = \phi L 1 = \phi$ . Equation (29) shows that the intervention can have an ever-lasting persistent effect as  $\phi \rightarrow 1$ . On the other hand, as  $\phi \rightarrow 0$ , the effect decays very rapidly. In summary, the more persistent the effect (i.e.,  $\phi$  is large), the larger the total effect (i.e.,  $\frac{\theta}{1-\phi}$  is large).

## APPENDIX 2: Notes on Analogues for Example 2

### Expectations and Uncertainty: The Tools and Components that Build the Behavioral Analogue

<sup>18</sup> A stationary process is a stochastic process in which the distribution of the random variables is the same for any value of the variable parameter. In this example, the variable parameter is *time*. The reason for this reduction in persistence is the  $\phi^j$  “weight” that is attached to each lag:

$$\sum_{j=0}^{\infty} \phi^j L^j = 1 + \phi L + \phi^2 L^2 + \dots + \phi^n L^n \tag{27}$$

where  $\phi^0 L^0 = 1$ . We can also represent this series in a more compact way. Simply multiply (27) by  $(1 - \phi L)$ . The result is:

$$(1 - \phi L) \sum_{j=0}^{\infty} \phi^j L^j = (1 - \phi L)(1 + \phi L + \phi^2 L^2 + \dots + \phi^n L^n) = 1 - \phi^{n+1} L^{n+1}.$$

As the sample size gets progressively larger and  $n \rightarrow \infty$ , we know  $\lim_{n \rightarrow \infty} \phi^{n+1} L^{n+1} = 0$  for  $|\phi| < 1$ . Therefore, we have the following result:

$$(1 - \phi L) \sum_{j=0}^{\infty} \phi^j L^j = 1, \text{ and therefore,}$$

$$\sum_{j=0}^{\infty} \phi^j L^j = \frac{1}{1-\phi L} \text{ for } |\phi| < 1.$$



The tools that allow us to link an error-in-variables regression with (14) are more complicated than in the first example. The discussion below is based in part on Sargent (1987: 223-238). The behavioral (formal) analogues for expectations and uncertainty center on *rational (conditional) expectations* modeling. The rational expectations hypothesis can be characterized as individuals forming their subjective expectations of a variable ( $y^e$ ) consistent with an objective mathematical expectation of ( $y$ ) that is conditional on all available information ( $\Omega$ ). Formally, rational expectations can be defined as a mathematical expectation (projection) of the probability distribution of  $y$  given a set of information,  $\Omega = \{x_0, x_1, \dots, x_n\}$ :

$$y^e = P(y|\Omega). \quad (30)$$

Assuming that the mathematical expectations are formed in a linear expression:

$$P(y|\Omega) = a_0 + a_1x_1 + a_2x_2 + \dots + a_nx_n = \sum_{i=0}^n a_i x_i, \quad (31)$$

where  $x_0 = 1$ . Given the orthogonality principle where  $E[(y - P(y|\Omega))x_i] = 0$  for all  $i$ , we have the following normal equations:<sup>19</sup>

$$\begin{bmatrix} Ey \\ Eyx_1 \\ Eyx_2 \\ \vdots \\ Eyx_n \end{bmatrix} = \begin{bmatrix} 1 & Ex_1 & Ex_2 & \dots & Ex_n \\ Ex_1 & Ex_1^2 & Ex_1x_2 & \dots & \\ Ex_2 & Ex_1x_2 & \ddots & \ddots & \\ \vdots & \vdots & & \ddots & \\ Ex_n & & & & Ex_n^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}. \quad (32)$$

From (32), we can calculate the vector of  $a$ 's:

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} = [Ex_i x_j]^{-1} [Eyx_i]. \quad (33)$$

Now apply the formulation above to the following linear expression and apply the least squares rule for  $n = 1$ :

$$P(y|1, x_1) = a_0 + a_1x_1$$

For equation (32), we have:

$$\begin{bmatrix} Ey \\ Eyx_1 \end{bmatrix} = \begin{bmatrix} 1 & Ex_1 \\ Ex_1 & Ex_1^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix},$$

<sup>19</sup> Based on the orthogonality principle,  $a_i$  can be uniquely determined. See Sargent (1987: 223-226).

where in normal equation form the result is:

$$a_0 = E y - a_1 E x_1,$$

and then applying the least squares rule:

$$a_1 = \frac{E(y - E(y))(x_1 - E x_1)}{E(x_1 - E x_1)^2} = \frac{\sigma_{xy}}{\sigma_{x_1}^2},$$

where  $\sigma_{xy}$  is the covariance between  $x_1$  and  $y$ , and  $\sigma_{x_1}^2$  is the variance of  $x_1$ .

## Relating the Behavioral Analogues to the Empirical Analogue for Measurement Error: Error-In-Variables Regression

The linkage between conditional expectations with the law of iterated expectations has a natural relation with error-in-variables regression (measurement error). There are many examples of this “EITM-like” linkage and they generally fall under the umbrella of *signal extraction* problems. Example 2 is a signal extraction problem where there is uncertainty over attributing growth performance to incumbent competence or good luck.

Signal extraction problems can be expressed in the following way. Suppose an agent wants to estimate a random variable “ $s$ ” but only “sees” the variable  $x$ :

$$x = s + n,$$

where  $Esn = 0$ ,  $E s^2, E n^2 < \infty$ ,  $E s = E n = 0$ .

Therefore using a projection operator and applying the least squares rule we have:

$$P(s|1, x) = a_0 + a_1 x,$$

and

$$a_1 = \frac{E(xs)}{E x^2} = \frac{E[(s+n)s]}{E(s+n)^2} = \frac{E s^2}{E s^2 + E n^2},$$

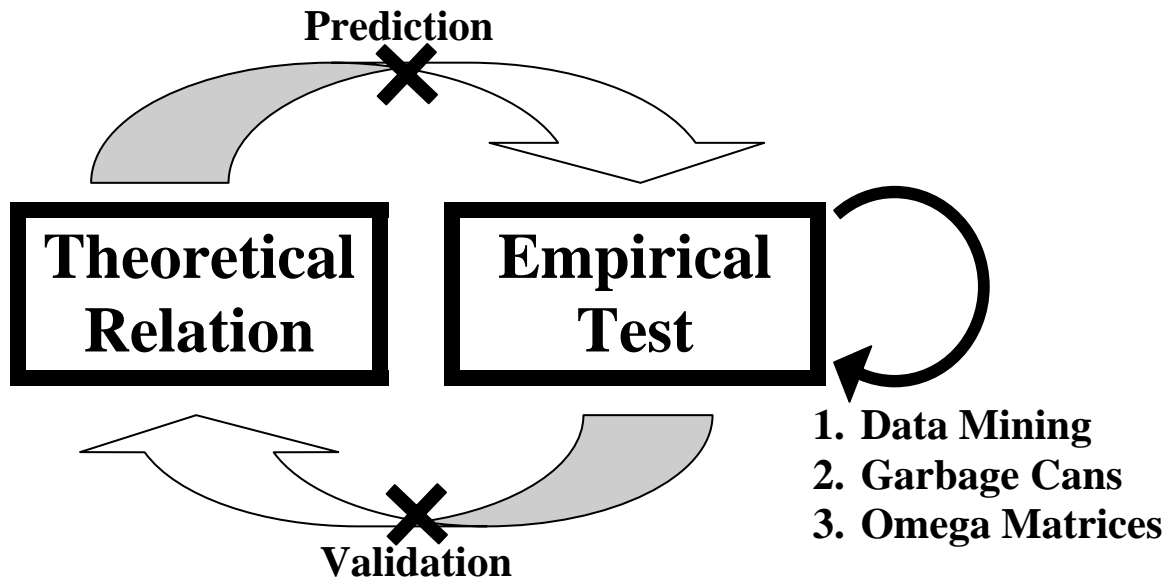
but

$$a_0 = 0.$$

The result is that there can be a behavioral basis for an error-in-variables regression (see Johnston and DiNardo (1997: 153-155)). The identity below is in the same form as equation (14):

$$P(s|1, x) = \frac{E s^2}{E s^2 + E n^2} x = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_n^2} x.$$

**Figure 1**



**Figure 2: The Relation Between  $\Pi_2$  and  $c_2$**

