

A price theory of vertical and lateral integration under two-sided productivity heterogeneity

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Abstract

We analyze the interplay between product market prices and firm boundary decisions. Heterogeneous supplier units are matched to form enterprises. Each enterprise can choose between two ownership structures – centralized ownership (integration) performs well in coordinating managerial actions but ignores private benefits; dispersed ownership (non-integration), on the other hand, overvalues private benefits but is conducive to poor coordination. A more productive unit on one side of the supplier market matches with a more productive unit on the other side following a positive assortative matching pattern. The equilibrium ownership structure is monotone, i.e., high productivity suppliers integrate while the low productivity ones stay separate. Price can be positively or negatively associated with the incidence of integration in the market, depending on how a price change affects the endogenously determined distribution of surplus. When a higher price induces less integration in the market, the industry supply may be backward-bending. Our model delivers new empirical and policy implications.

1 Introduction

There is a plethora of evidence on heterogeneity of firm productivity within an industry, which is also associated with organizational variation at the firm level (e.g. [Gibbons, 2010](#); [Syverson, 2011](#)) and endogenous sorting among firms (e.g. [Hortaçsu and Syverson, 2007](#); [Atalay, Hortaçsu, and Syverson, 2014](#)). In this paper, we analyze the interplay between product market competition and firm boundary decisions (choice of ownership structure) under ex-ante productivity heterogeneity. We intend to shed light on the following two basic sets of questions pertaining to the Organizational Industrial Organization (OIO) literature which lies at the intersection of organizational and industrial economics:

1. Does competition for heterogeneous suppliers in the input market imply that enterprises are formed following an assortative matching pattern? Do differences in productivity imply differences in organizational design?

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2. To what extent changes in the market price explain the cross-sectional variation in modes of organization and production decisions of firms? In this setting, how does market price affect aggregate output?

In a recent contribution to the literature on OIO, [Legros and Newman \(2013\)](#) introduce an incentive contracting model of firm boundary decisions where the production of a homogeneous consumption good uses the assets owned by two distinct supplier units (A and B) that are run by a manager apiece. The managers choose between two ownership structures – *non-integration* (decentralized ownership), where the managers possess the decision rights, and *integration* (centralized ownership) where the decision right is conferred on an outsider. Managerial actions are ex-ante non-contractible, thus, privately costly, and must be coordinated to boost enterprise revenue. Each ownership structure entails its own inefficiencies. Non-integration, in which greater emphasis is placed on the managers’ private benefits relative to revenue, performs poorly in coordinating managerial actions. Integration, on the other hand, facilitates perfect coordination but ignores private benefits.¹ They derive two main results: i) despite [a continuum of] input suppliers being homogeneous, the two organizational modes may co-exist in the market, and ii) higher product market price makes integration more likely, and hence, the organizationally augmented industry supply curve (OAS) is upward sloping. We build on [Legros and Newman \(2013\)](#) by introducing two-sided productivity heterogeneity and endogenous sorting between the suppliers on each side of the market.

We derive the following results. First, matching between input suppliers is positive assortative. Second, ownership structures are monotonic in the sense that high-productivity enterprises integrate, and the low-productivity ones stay separate. Third, higher product market price may increase or decrease the fraction of integrated enterprises in the supplier market. Fourth, the OAS curve can be backward-bending.

The first result holds because of the complementarity between the supplier units. In particular, we show that the *generalized increasing difference* (GID) condition holds. Proving GID in our context is not straightforward because the Pareto frontiers of some enterprises may be non-concave (and non-differentiable) due to the choice between the two ownership structures. The second result holds because higher productivity enterprises generate higher revenue, and hence, the opportunity cost of staying separate (which is the foregone revenue) increases.

The intuition behind the third result is as follows. Under non-integration, firms are subject to a double moral hazard problem and surplus is imperfectly transferred (ITU) between the two suppliers using a revenue sharing contract, whereas integration gives rise to a transferrable utility (TU) setting. As is standard in double moral hazard problems, more balanced allocation of utilities improves efficiency. In this case, non-integration is more likely to dominate integration. Consider now the marginal enterprise that is indifferent between integration and non-integration. Higher market price generates higher surplus

¹For example, as discussed in [Hart and Hölmstrom \(2010\)](#), the two units may want to adopt a common standard, as in the Cisco’s acquisition of Stratacom. The benefits for the organization from a common standard can be enormous but the private benefits within the firms may decrease because of the change the new standard introduces. Moreover, there is no agreement between the firms about which “approach” should be adopted. However, agreeing on a common approach (coordination) boosts firm revenue. In the airline industry, for instance, major carriers subcontract portion of their network to regional partners. In some cases the majors own the regional partner, while in other cases the two operate as separate units and a contract governs their relationship (see [Forbes and Lederman, 2009, 2010](#)). In cases like this, the private benefits can represent, for example, job satisfaction or a way to capture different beliefs held by managers and workers about the consequences of strategic choices, such as a merger.

under both modes of organization. If more of the extra surplus is given to the side whose utility is lower then surplus sharing becomes more balanced at the higher price and non-integration at the marginal enterprise dominates. In this case a higher price is associated with less integration. How the extra surplus gets divided between the two sides at the marginal enterprise depends, as is standard in models with endogenous matching, on competition in all infra-marginal enterprises (which depends on the marginal productivities of each side and the relative measure of types).²

Holding the organizational structures in the supplier market fixed, higher market price implies higher aggregate output, because coordination under non-integration improves, and the aggregate output in the integrated enterprises is at its highest potential. But if a higher price implies less integration, then this introduces a second opposing force on aggregate output because non-integrated firms produce less output than the integrated ones. If this force is stronger, then industry output may decrease with price. In other words, the organizationally augmented supply curve (OAS) can be *backward-bending* due to the organizational restructuring a price change triggers.

2 Related literature and our contribution

The literature on OIO, which is concerned with how market structure affects firm boundaries decisions, is still in its early stages of development.³ Our paper adds to the recent contributions in this strand of literature. As we have mentioned earlier, for any given enterprise the decision to integrate depends on how the enterprise surplus is divided among the supplier units. When shares are unbalanced, non-integration performs poorly in coordinating managerial actions because incentives cannot be easily aligned, and hence, integration is a preferred choice over non-integration. This basic trade-off has been proposed by Legros and Newman's (2013) work. Coordination motives as the main driver for vertical and lateral integration has earlier been analyzed by Hart and Hölmstrom (2010), although there is no scope for ex-ante revenue sharing contract as ex-post bargaining is efficient. The role of surplus sharing in determining the choice between vertical integration and outsourcing has also been analyzed in Grossman and Helpman (2002). But their mechanism is different in the sense that large governance costs in the vertically integrated firms are balanced against costs arising from a holdup problem (as in Grossman and Hart, 1986) and search for suitable partners under outsourcing. In the equilibrium of the market with identical participants, either all firms vertically integrate or there is pervasive outsourcing. Extreme revenue share makes an equilibrium with integration more likely to occur because it generates excess demand or excess supply for intermediate inputs.

We differ from the aforementioned works in the following aspects. In Legros and Newman (2013), all units are homogeneous, the revenue shares are endogenously determined, but the utility of the B suppliers is fixed at zero. In Grossman and Helpman (2002), on the other hand, the share of intermediate input suppliers is exogenously given, which reflects the degree of input market competition, and it does

²Consider, for example, Chakraborty and Citanna (2005) who examine an occupational choice model with wealth heterogeneity, double moral hazard and matching. As in our model, the division of the gains from a match is determined by competitive forces. They show that matches are typically wealth heterogeneous, with richer individuals choosing the occupation for which incentives are more important.

³See Legros and Newman (2014) for an excellent survey. As these authors argue: "Nascent efforts at developing an OIO already suggest that market conditions or industrial structure matter for organization design. At the same time, organizational design will affect the productivity of firms, hence eventually the total industry output, the quality of products and information about this quality for consumers. Organizational design matters for consumers, hence for IO."

not interact with the degree of product market competition. As we allow supplier units to be ex-ante heterogeneous, endogenous sorting leads to an endogenous distribution of surplus. Even when [Legros and Newman \(2013\)](#) allow for ex-ante heterogeneity in firm productivities, because the matching is exogenous, all supplier units who receive offers consume their reservation payoff (zero). In light of our framework, the same utility allocation or surplus sharing cannot be part of a stable equilibrium since more productive units must receive higher utility. The endogenous distribution of surplus in our model has further implications for the association between product market price and integration. Due to endogenous sorting, the surplus division in the infra-marginal firms determines how balanced the surplus sharing is in the marginal firm, which in turn determines the choice of ownership structure of the marginal firm following a price increase. Higher surplus can generate a more even allocation of utilities in the marginal firm, which favors non-integration. Therefore in our model, a rise in price may lead to less integration, and consequently, the industry supply curve may be backward-bending.

To summarize, our contribution is that we provide a particular mechanism, which relies on two-sided heterogeneity and sorting, to illustrate how the effect of surplus sharing on firm boundary decisions manifests itself in the supplier market.⁴ In doing so, we offer a more complete picture of the interaction of market price with integration decisions and output, given that productivity heterogeneity and sorting are ubiquitous in input markets.

[Gibbons, Holden, and Powell \(2012\)](#) obtain generic heterogeneity of ownership by analyzing a rational-expectations equilibrium of price formation and endogenously chosen governance structures. They show that the informativeness of the price mechanism can induce ex-ante homogeneous firms to choose heterogeneous governance structures. [Aghion, Griffith, and Howitt \(2006\)](#) provide evidence of a U-shaped relationship between product market competition and vertical integration. Integration is more likely when competition is either soft or intense. In addition to the role of product market competition, we emphasize the significance of competition for heterogeneous supplier units in the input market in an endogenous matching framework.

[Alfaro, Conconi, Fadinger, and Newman \(2016\)](#) use changes in trade policy, e.g. tariffs, as an exogenous source of price variation. They find empirical evidence that the level of product prices do affect vertical integration decisions. [Acemoglu, Griffith, Aghion, and Zilibotti \(2010\)](#) show that technology intensity affects the likelihood of integration. They find that technology intensity of the downstream producers is positively associated with integration, while technology intensity of the upstream suppliers is negatively associated. Our model also disentangles the contributions of the two units to overall firm productivity.

In a model of managerial incentives under endogenous matching, [Alonso-Paulí and Pérez-Castrillo \(2012\)](#) analyze the choice between incentive and Codes of Best Practice (CBP) contracts. The presence of two different contracting modes gives rise to a non-concave bargaining frontier for each shareholder-manager pair, as it is the case in our model. They show that for considerable range of parameter values the equilibrium matching may be negative assortative with a high-revenue shareholder offering a CBP contract to a less efficient manager, whereas a low-revenue shareholder offers an incentive contract to lure a more efficient manager. [Macho-Stadler, Pérez-Castrillo, and Porteiro \(2014\)](#) shows the robust co-existence of two contracting modes – namely, short- and long-term contracts in a labor market where heterogeneous firms are endogenously matched with heterogeneous workers. There is positive sorting in

⁴Two-sided productivity heterogeneity and matching have also become increasingly important in the context of international trade, as there are easily available datasets with detailed information about the matching between exporters and importers (e.g. [Bernard, Moxnes, and Ulltveit-Moe, 2014](#); [Dragusanu, 2014](#); [Sugita, Teshima, and Seira, 2015](#)).

the market for short-term contracts, whereas under long-term contracts, sorting does not emerge because the gains from incentive provision by the long-term contracts are superior to the benefits of sorting under the short-term contracts. Amir, Garcia, and Knauff (2010), in the context of strategic interaction in a two-player game, considers a payoff function that is the maximum of two submodular functions. Global strategic substitutability is inherited despite the combined function is not differentiable on the action space. They argue that such kinks are possibly generated by situations when duopoly firms make R&D investment decisions, which is followed by product market competition. Our model bears some technical resemblance because we show that complementarity of each component implies global complementarity, although in our context market participants do not interact strategically.

3 The Model

3.1 Technology and matching

Consider a two-sided supplier market. On each side of the market there is a continuum of input suppliers of measure 1. Suppliers are vertically differentiated.⁵ In particular, let $J_A = [0, 1]$ be the set of “A-suppliers” on the one side of the market and $J_B = [0, 1]$, the set of “B-suppliers” on the other side. Each supplier $i \in J_A$ is assigned a type or ‘productivity’ $a = a(i) \in A$ and each $j \in J_B$ has an assigned type $b = b(j) \in B$ where the type spaces $A = [a_{min}, a_{max}]$ and $B = [b_{min}, b_{max}]$ are subintervals of \mathbb{R}_{++} . Let $G(a)$ be the fraction of A-suppliers with productivity lower than a , i.e., $G(a)$ is the cumulative distribution function of a with the associated density function $g(a) > 0$ for all $a \in A$. Similarly, let $F(b)$ be the distribution function of b with the associated density function $f(b) > 0$ for all $b \in B$.

Production of a homogeneous consumer good requires one A-supplier and one B-supplier who are matched one to one to form an ‘enterprise’. All decisions and payoffs of each enterprise will only depend on the types of the two participating units, and hence, a typical enterprise will be denoted by (a, b) . A matching is a one-to-one mapping $\alpha : B \rightarrow A$ which assigns to each $b \in B$ a type $a = \alpha(b) \in A$. Let $\beta \equiv \alpha^{-1}$ denote the inverse matching function. Such enterprises may include lateral as well as vertical relationships. The stochastic output of an enterprise (a, b) is given by:

$$\tilde{y}(a, b) = \begin{cases} z(a, b) & \text{with probability } \pi(e_A, e_B) \equiv 1 - (e_B - e_A), \\ 0 & \text{otherwise.} \end{cases}$$

The success output $z(a, b)$ can be thought of as the productivity of an enterprise (a, b) .⁶ We assume that $z(a, b)$ is twice continuously differentiable, strictly increasing in a and b , and supermodular in (a, b) ,

⁵Bloom and van Reenen (2007), using a survey from medium-sized manufacturing firms from four countries, document that management practices are heterogeneous and affect firm performance. Gibbons (2010) offers a more detailed account of various empirical studies that document persistent performance differences (PPDs). In the computer industry, computer systems manufacturers rely on networks of independent component suppliers. These suppliers are of various ‘qualities’ and produce components that are used as inputs in the production of the final product (see Fallick, Fleischman, and Rebitzer, 2006).

⁶Legros and Newman (2013) assume a quadratic probability of success of the form $\pi(e_A, e_B) \equiv 1 - (e_A - e_B)^2$. We use a linear one instead for tractability. The linear probability of success is everywhere below the quadratic one implying that non-integration is more profitable under the quadratic probability. That is why in Legros and Newman (2013) integration is never a strictly dominant organizational mode for all (a, b, u) and all P . Nevertheless, the important qualitative features of the model are not affected by the choice of linear probability.

i.e., $z_{ab}(a, b) \geq 0$.⁷ Each supplier must make a non-contractible production decision: $e_A \in [0, 1]$ by an A -supplier and $e_B \in [0, 1]$ by a B -supplier. These decisions can be made by the manager of the assets or by someone else. If the two managers coordinate their decisions and set $e_A = e_B$, then inefficiencies disappear and the enterprise reaches its full potential $z(a, b)$ with probability 1. The manager of each supplier unit is risk neutral and incurs a private cost for the managerial action. The private cost of an A unit is ce_A^2 and that of a B unit is $c(1 - e_B)^2$ with $c > 0$. Clearly, there is a disagreement about the direction of the decisions, what is easy for one is hard for the other. Also, managers with zero cash endowments are protected by limited liability, i.e., their state-contingent incomes must always be nonnegative. The importance of this assumption is that the division of surplus between the managers will affect the organizational choice.

3.2 Ownership structures and contracts

The ownership structure can be contractually determined. We assume two different options, each of which implies a different allocation of decision rights. First, the production units can remain separate firms (the *non-integration* regime, denoted by N). In this case, managers have full control over their decisions. Second, the two input suppliers can integrate, a regime denoted by I , into a single firm, giving control over managerial decisions, e_A and e_B , to a third party who always has enough cash to finance the acquisition. The third party is motivated entirely by income and incurs no costs from the managerial decisions. These costs are still borne by the managers. As argued by [Hart and Hölmstrom \(2010\)](#), integration results in an organization where less weight is placed on private benefits than under non-integration. This, however, is offset by the fact that under integration total profit, rather than individual unit profits, is maximized.

Each enterprise's revenue is publicly verifiable, and hence, ex-ante contractible. We assume that each A -supplier has all the bargaining power in an arbitrary enterprise (a, b) and makes take-it-or-leave-it offers to the B -supplier. A contract $(s, d) \in [0, 1] \times \{N, I\}$ specifies a revenue share s for the B -supplier and an ownership structure d . Consider an arbitrary enterprise (a, b) . If the members of this enterprise stay separate, then a revenue-sharing contract is simply a share s of the total revenue that accrues to the B -supplier. As we assume limited liability, the units get nothing in the case of failure.

When the two units integrate, a third party, called the headquarters, HQ , buys the assets of the A - and B -suppliers for predetermined prices in exchange for a share contract $\mathbf{s} = (s_A, s_B, s_{HQ}) \in \mathbb{R}_+^3$ with $s_A + s_B + s_{HQ} = 1$. HQ s are supplied perfectly elastically with an opportunity cost normalized to zero.

3.3 The product market

The product market is perfectly competitive where consumers and suppliers take the product price P as given. Identical consumers maximize a smooth quasi-linear utility which gives rise to a downward-sloping demand curve $D(P)$. Suppliers correctly anticipate price P when they sign contracts and make their production decisions. Define by $R(a, b) \equiv Pz(a, b)$, the revenue of an enterprise (a, b) in the event of success. We assume $0 < R(a, b) < 2c$ for all $(a, b) \in A \times B$. If $R(a, b) = 2c$, then the probability of

⁷We use subscripts to denote partial derivatives and when the subscript is a number it denotes the position of the variable with respect to which we differentiate, e.g., $\phi_2(a, b, u)$ is the partial derivative of ϕ with respect to b .

success in a non-integrated firm equals 1, and hence, integration and non-integration perform the same, and the model ceases to be predictive on that score.

3.4 Timing of events

The economy lasts for three dates, $t = 0, 1, 2$. At date 0, one A -supplier and one B -supplier match one to one to form an enterprise (a, b) . At $t = 1$, each A -supplier makes a take-it-or-leave-it contract offer (s, d) to each B -supplier. At date 1, the manager of each unit chooses e_A and e_B . Finally at $t = 2$, the revenue of each enterprise is realized and the agreed upon payments are made. We solve the model by backward induction.

3.5 Equilibrium

An equilibrium of the economy consists of a set of enterprises formed through feasible contracts, i.e., ownership structures and corresponding revenue shares, for each enterprise and a market-clearing price. Recall that there are two possible ownership structures for each enterprise – integration (I) and non-integration (N). In general, choice of ownership structures depends on the revenue share that accrues to each member of an enterprise, the output of each enterprise and the market price. An allocation for the market (α, u, v) specifies a one-to-one matching rule $\alpha : B \rightarrow A$, and payoff functions $u : B \rightarrow \mathbb{R}_+$ and $v : A \rightarrow \mathbb{R}_+$ for the B - and A -suppliers, respectively.

Definition 1 (Equilibrium) *Given the type distributions $F(b)$ and $G(a)$, an allocation (α, u, v) and a product-market price P constitute an equilibrium allocation of the economy if they satisfy the following conditions:*

- (a) **Feasibility:** *the revenue shares and the corresponding payoffs to the agents in each equilibrium enterprise are feasible given the output of the enterprise and the equilibrium price P ;*
- (b) **Optimization:** *Each A -supplier of a given type chooses optimally a B -supplier to form an enterprise, i.e., given $u(b)$ for each $b \in B$,*

$$\alpha^{-1}(a) = \operatorname{argmax}_b \phi(a, b, u(b); P), \quad (\mathcal{M}_a)$$

for each $a \in A$. The function $\phi(a, b, u(b); P)$ is the bargaining frontier or Pareto frontier of the enterprise (a, b) , which is the maximum payoff that can be achieved by a type a A -supplier given that the B -supplier of type b consumes $u(b)$ at a given market price P .

- (c) **Supplier market clearing:** *The equilibrium matching function satisfies the following ‘measure consistency’ condition. For any subinterval $[i_0, i_1] \subseteq J_A$, let $i_k = G(a_k)$ for $k = 0, 1$, i.e., a_k is the productivity of the A supplier at the i_k -th quantile. Similarly, for any subinterval $[j_0, j_1] \subseteq J_B$, let $j_h = F(b_h)$ for $h = 0, 1$. If $[a_0, a_1] = \alpha([b_0, b_1])$, then it must be the case that*

$$j_1 - j_0 = F(b_1) - F(b_0) = G(a_1) - G(a_0) = i_1 - i_0. \quad (\text{MC})$$

- (d) **Product market clearing:** *The aggregate (expected) supply in the industry $Q(P)$ is equal to the demand $D(P)$.*

Definition 1-(b) asserts that each A -supplier chooses her partner optimally. Part (c) of the above definition simply says that one cannot match say two-third of the A -suppliers to one-third of the B -suppliers because the matching is constrained to be one-to-one.

4 Equilibrium sorting and ownership structures

We proceed as follows. In Section 4.1, we derive the bargaining frontiers under N and I and establish the optimal organization for a given enterprise, as a function of the exogenously given utilities. In Section 4.2, we allow the units to match endogenously, we endogenize the utilities and we show that the equilibrium matching is positive assortative. In Section 4.3, we derive the equilibrium ownership structures in the supplier market.

4.1 Optimal ownership structure for an arbitrary enterprise

We analyze the optimal contract for an arbitrary enterprise (a, b) . We assume that the A -supplier possesses all the bargaining power in the relationship and makes take-it-or-leave-it offer to the B -supplier. We first analyze each ownership structure separately.

4.1.1 Non-integration

For the time being we suppress the argument (a, b) from the contract terms. Under this organizational structure the shares affect both the size and the distribution of surplus between the two units (imperfectly transferable utility). An optimal contract for a non-integrated enterprise (a, b) solves the following maximization problem:

$$\begin{aligned} \max_s \quad & V_A \equiv \pi(e_A, e_B)(1-s)R(a, b) - ce_A^2, & (\mathcal{P}_N) \\ \text{subject to} \quad & U_B \equiv \pi(e_A, e_B)sR(a, b) - c(1-e_B)^2 = u, & (\text{PC}_B) \\ & e_A = \operatorname{argmax}_e \{ \pi(e, e_B)(1-s)R(a, b) - ce^2 \} = \frac{(1-s)R(a, b)}{2c}, & (\text{IC}_A) \\ & e_B = \operatorname{argmax}_e \{ \pi(e_A, e)sR(a, b) - c(1-e)^2 \} = 1 - \frac{sR(a, b)}{2c}, & (\text{IC}_B) \end{aligned}$$

where u is the outside option of the B -supplier. We assume that $u \geq u_0$, where $u_0 \geq 0$ is the reservation utility of all B -suppliers, i.e., the utility any B -supplier obtains if he does not join any firm. Constraint (PC_B) is the *participation constraint* of the B -supplier, whereas constraints (IC_A) and (IC_B) are the *incentive compatibility constraints* of the A -supplier and the B -supplier, respectively. Note that (IC_A) and (IC_B) together imply that $\pi(e_A, e_B) = R(a, b)/2c$, and hence, $\pi(e_A, e_B) \in [0, 1]$ under the assumption that $R(a, b) \in [0, 2c]$. The (maximum) value function of the above maximization problem, denoted by $\phi^N(a, b, u; P)$, is the maximum payoff that accrues to the A -supplier given that the B -supplier consumes u at a given market price P . The following lemma characterizes the optimal revenue share and the value function under non-integration.

Lemma 1 *When the supplier units in an arbitrary enterprise (a, b) stay separate, for a given product market price P , the optimal revenue share (accruing to the B -supplier) and the associated value function are respectively given by:*

$$s(a, b, u; P) = 1 - \frac{\sqrt{R(a, b)^2 - 4cu}}{R(a, b)}, \quad (1)$$

$$\phi^N(a, b, u; P) = \frac{1}{4c} \left[2R(a, b)\sqrt{R(a, b)^2 - 4cu} - (R(a, b)^2 - 4cu) \right], \quad (2)$$

for $0 \leq u \leq \frac{R(a, b)^2}{4c}$.

The participation constraint of the B -supplier determines the optimal revenue share $s = s(a, b, u; P)$ of each type b B -supplier. Note that u must lie between 0, which corresponds to $s = 0$, and $R(a, b)^2/4c$, the level corresponding to $s = 1$. The value function is symmetric with respect to the 45° line (on which $\phi^N(a, b, u; P) = u(b)$ and $s = 0.5$). This implies that total surplus is maximized when the shares across the two non-integrated units are equal (see the non-linear frontier in Figure 1). Equal, or more broadly ‘balanced’, shares yields strong incentives to the managers to better coordinate their decisions, i.e., e_A and e_B move closer to each other. Finally, higher revenue R , holding the shares fixed, also induces better coordination.

4.1.2 Integration

When the two supplier units integrate, the enterprise is acquired by HQ who is conferred with the decision making power. Motivated entirely by incomes, HQ will choose e_A and e_B to maximize the expected revenue $\pi(e_A, e_B)R(a, b)$ as long as $s_{HQ} > 0$. This induces $e_A = e_B$. The HQ breaks even as the market for headquarters is perfectly competitive. The private costs of managerial actions are still borne by the individual supplier units. The aggregate managerial cost, $ce_A^2 + c(1 - e_B)^2$ is minimized when $e_A = e_B = \frac{1}{2}$. The maximum payoff accruing to the A -supplier, given that the B -supplier consumes u , in each enterprise (a, b) is given by:

$$\phi^I(a, b, u; P) = R(a, b) - \frac{c}{2} - u \quad \text{for } 0 \leq u \leq R(a, b) - \frac{c}{2}. \quad (3)$$

The above function is linear in u , i.e., surplus is fully transferable between the two managers since neither the action taken by HQ nor the costs borne by the managers depends on the revenue share. The function $\phi^I(a, b, u; P)$ is strictly increasing in a and b , strictly decreasing in u (with slope -1) and symmetric with respect to the 45° line (see the linear frontier in Figure 1).

Although surplus is fully transferable between the A - and B -suppliers, this form of organization may not be the efficient one as HQ , having a stake in the firm’s revenue, puts too little weight on the managers’ private costs while maximizing the expected revenue.⁸

⁸Note that the first-best surplus, $\frac{R^2}{2c}$, is strictly higher than $R - \frac{c}{2}$, the surplus accrued to an integrated firm as well as $\frac{3R^2}{8c}$, the maximum surplus in a non-integrated firm, which corresponds to $s = \frac{1}{2}$. The diminished output under non-integration reflects the distortionary effect of incomplete contracting. It is similar to the double-marginalization that creates incentives for vertical integration in a world with perfect contracts (see Perry, 1989).

4.1.3 Choice of organization and the bargaining frontier

Having analyzed the optimal contract of an arbitrary enterprise under each ownership structure separately, it is now convenient to analyze the optimal organization and the bargaining frontier associated with a given enterprise. At any given price P and utility u accruing to the B -supplier, each enterprise (a, b) would choose N over I if and only if $\phi^N(a, b, u; P) \geq \phi^I(a, b, u; P)$. We assume that an enterprise would choose to stay separate whenever it is indifferent between N and I . Thus, at any given product market price P , the bargaining frontier of each enterprise (a, b) is given by:

$$\phi(a, b, u; P) = \max \{ \phi^N(a, b, u; P), \phi^I(a, b, u; P) \} \quad \text{for } 0 \leq u \leq u_{\max}(a, b),$$

where

$$u_{\max}(a, b) \equiv \max \left\{ \frac{R(a, b)^2}{4c}, R(a, b) - \frac{c}{2} \right\}.$$

The equality between $\phi^N(a, b, u; P)$ and $\phi^I(a, b, u; P)$ gives two cut-off levels $u_L(a, b)$ and $u_H(a, b)$ of the utility of the B -supplier with $0 < u_L(a, b) \leq u_H(a, b) < u_{\max}(a, b)$. The corresponding revenue shares (of the B -supplier) are $s_L(a, b)$ and $s_H(a, b)$, respectively, which are given by (1).

In an arbitrary enterprise (a, b) , the optimal choice of ownership structure depends on the revenue of the enterprise, $R(a, b)$. One can find two threshold values $R^- \equiv (2 - \sqrt{2})c$ and $R^+ \equiv \frac{2c}{3}$ of $R(a, b)$ with $0 < R^- < R^+ < 2c$ such that when $R(a, b) < R^-$ the managers of the supplier units prefer to stay separate because in this case $\phi^N(a, b, u; P) > \phi^I(a, b, u; P)$ for all $(a, b, u; P)$. On the other hand, the suppliers prefer to integrate when $R(a, b) > R^+$. Interestingly, when $R^- \leq R(a, b) \leq R^+$ there is no clear dominance of one mode of organization over the other. This case is depicted in Figure 1 where $\phi^N(a, b, u; P)$ is the strictly concave frontier and $\phi^I(a, b, u; P)$ is the linear frontier. Clearly, they intersect twice at $u_L(a, b)$ and $u_H(a, b)$. Therefore, for $R^- \leq R(a, b) \leq R^+$, non-integration is chosen by each enterprise (a, b) if and only if $u \in [u_L(a, b), u_H(a, b)]$. The resultant bargaining frontier is given by the upper envelope of $\phi^N(a, b, u; P)$ and $\phi^I(a, b, u; P)$, which is non-concave. Note that when $R(a, b) = R^+$, we have $u_L(a, b) = u_H(a, b)$. Thus to summarize,

Proposition 1 (Bargaining frontier of a given enterprise) *In a given enterprise (a, b) , there exist two threshold values R^- and R^+ of the enterprise revenue $R(a, b)$ with $0 < R^- < R^+ < 2c$ such that*

- (a) *When $R(a, b) < R^-$, the enterprise chooses non-integration over integration. The bargaining frontier is given by $\phi(a, b, u; P) = \phi^N(a, b, u; P)$;*
- (b) *When $R(a, b) \in [R^-, R^+]$, non-integration is preferred if and only if $u \in [u_L(a, b), u_H(a, b)]$. The bargaining frontier is given by*

$$\phi(a, b, u; P) = \begin{cases} \phi^N(a, b, u; P) & \text{if } u \in [u_L(a, b), u_H(a, b)], \\ \phi^I(a, b, u; P) & \text{if } u \in [0, u_{\max}(a, b)] \setminus [u_L(a, b), u_H(a, b)]; \end{cases}$$

- (c) *When $R(a, b) > R^+$, the enterprise chooses integration over non-integration. The bargaining frontier is given by $\phi(a, b, u; P) = \phi^I(a, b, u; P)$.*

Low revenue, i.e., $R < R^-$ implies that an organization puts more emphasis on private benefits relative to the benefits accruing from coordination and chooses non-integration over integration for all levels of u .

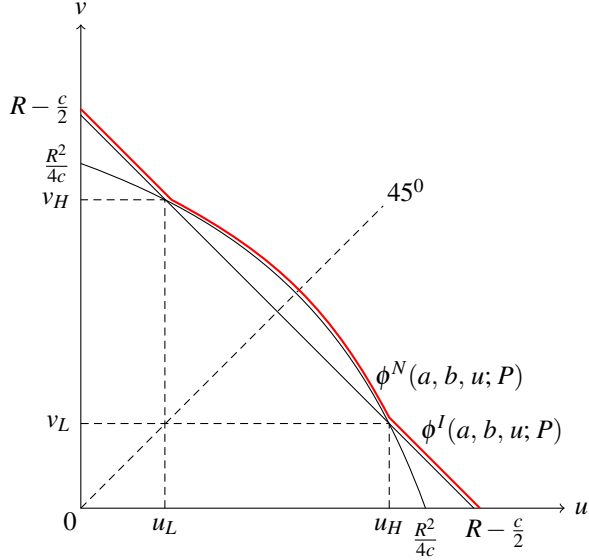


Figure 1: The two bargaining frontiers, $\phi^N(a, b, u)$, the concave function, and $\phi^I(a, b, u)$, the linear function, when $R(a, b) \in [R^-, R^+]$. For intermediate values of the B supplier utility u , N is preferred to I , while for high or low values of u , I is chosen over N . The combined frontier $\phi(a, b, u)$ of a given enterprise (a, b) is the upper envelope of $\phi^N(a, b, u)$ and $\phi^I(a, b, u)$.

On the other hand, for the high-revenue (or high-productivity) enterprises with $R > R^+$, more weight is placed on coordination and revenue maximization, and hence, integration is the optimal choice for all u . For intermediate productivity enterprises, $R \in [R^-, R^+]$, either organizational structure may be optimal, depending on the levels of u , or the revenue share s . For intermediate values of u , an enterprise prefers to stay separate because the corresponding shares s and $1 - s$ are more balanced and so coordination among the two units can be achieved without being integrated. But for the extreme values of u , either high or low, integration is preferred because the shares are tilted in favor of one of the two units and the incentives for revenue maximization are weak.

Proposition 1 allows us to reduce the set $[0, 1] \times \{N, I\}$ of feasible contracts (s, d) to the set of feasible shares $[0, 1]$ only, or equivalently to the set of feasible transfers (to the B -supplier) $[0, u_{max}(a, b)]$. For example, if $R^- \leq R(a, b) \leq R^+$, then a revenue share in the interval $[s_L(a, b), s_H(a, b)]$ or a transfer in $[u_L(a, b), u_H(a, b)]$ is equivalent to the fact that an enterprise (a, b) would never choose to integrate because integration is the strictly dominated mode of organization. On the other hand, a revenue share in $[0, s_L(a, b)) \cup (s_H(a, b), 1]$ or a transfer in $[0, u_L(a, b)) \cup (u_H(a, b), u_{max}(a, b)]$ implies that the enterprise strictly prefers to integrate. Thus, in the equilibrium of the supplier market where the supplier units match with each other optimally, the only thing that would matter is the set of feasible revenue shares, $[0, 1]$, or equivalently the set of feasible transfers, $[0, u_{max}(a, b)]$ for each enterprise (a, b) .⁹

⁹In the proofs of many subsequent results, we will use revenue share s and utility transfer u interchangeably.

4.2 Equilibrium matching

In this section, we analyze the equilibrium matching function $b = \beta(a)$ and show that the matching is positive assortative (PAM), i.e., $\beta(a)$ is non-decreasing in a . PAM in an equilibrium allocation relies on the properties of the bargaining frontier $\phi(a, b, u(b); P)$. Until Section 5, the product market price will be taken as constant, and hence, we will disregard the dependence of ϕ on P for the time being. Recall that in the enterprise formation stage at date 0, each type a A -supplier solves the program (\mathcal{M}_a) to choose a B -supplier, which is given by:

$$\beta(a) = \operatorname{argmax}_b \{\phi(a, b, u(b))\} \equiv \operatorname{argmax}_b \{\max\{\phi^N(a, b, u(b)), \phi^I(a, b, u(b))\}\}.$$

Legros and Newman (2007) propose that the *generalized increasing difference* (GID) is a necessary and sufficient condition for PAM, which is equivalent to the following *single-crossing condition*:¹⁰

$$\phi(a', b'', u'') = \phi(a', b', u') \implies \phi(a'', b'', u'') \geq \phi(a'', b', u') \quad (\text{GID})$$

for any $a'' > a'$, $b'' > b'$ and $u'' > u'$. The left panel of Figure 2 depicts single-crossing. Given that the Pareto frontier under $d = N, I$ is strictly decreasing in u , the implicit function theorem implies that the indifference curve $\phi^d(a, b, u) = \bar{v}$ of each type a under $d = N, I$ can be expressed as $u^d \equiv \psi^d(a, b)$ with $\psi_b^d(a, b) = -\frac{\phi_2^d}{\phi_3^d}$ representing the slope of the indifference curve corresponding to d (we suppress the dependence of ψ^d on \bar{v}). Therefore, the combined indifference curve of each type a is given by:

$$\psi(a, b) = \max\{\psi^N(a, b), \psi^I(a, b)\}.$$

Condition (GID) asserts that the indifference curves of two distinct types, a' and a'' can cross only once. In other words, given $b'' > b'$ and $u'' > u'$, if a' , the lower type, is indifferent between (b', u') and (b'', u'') , then a higher type a'' (weakly) prefers to pay more to be matched with b'' . Thus, proving that the equilibrium matching is PAM in our context boils down to verifying condition (GID) for $\phi(a, b, u) \equiv \max\{\phi^N(a, b, u), \phi^I(a, b, u)\}$. We proceed as follows (proofs are in the Appendix):

1. We first show that $\phi^d(a, b, u)$ satisfies (GID) for $d = N, I$, i.e., $\psi^d(a', b)$ and $\psi^d(a'', b)$ for $a' \neq a''$ cross only once. This property is easily established because $z(a, b)$ is strictly supermodular in (a, b) , and the indifference curve of any type a A -supplier is differentiable everywhere. Therefore, the equilibrium matching is PAM for all (a, b) such that $R(a, b) < R^-$ and $R(a, b) > R^+$.
2. Recall that, for $R(a, b) \in [R^-, R^+]$, there is no clear dominance of one ownership structure over the other, and hence, establishing (GID) is not trivial. Thus, for each a the combined indifference curve is non-differentiable at a unique point \hat{b} which is given by $\psi^N(a, \hat{b}) = \psi^I(a, \hat{b})$. We later show that the indifference curve under one ownership structure is steeper than that under the other for all b , so \hat{b} is unique (See proof of Proposition 3 in the Appendix). Although $\phi^d(a, b, u)$ satisfies (GID) for each $d = N, I$, the indifference curve of each a , being the maximum of two indifference curves, each under a distinct ownership structure, does not necessarily satisfy single-crossing (see the right panel of Figure 2). We show that a sufficient condition for $\phi(a, b, u)$ to satisfy (GID) is that the kink in the indifference curves of the A -suppliers moves to the left as a increases, i.e., $d\hat{b}/da \leq 0$, which always holds in our model.

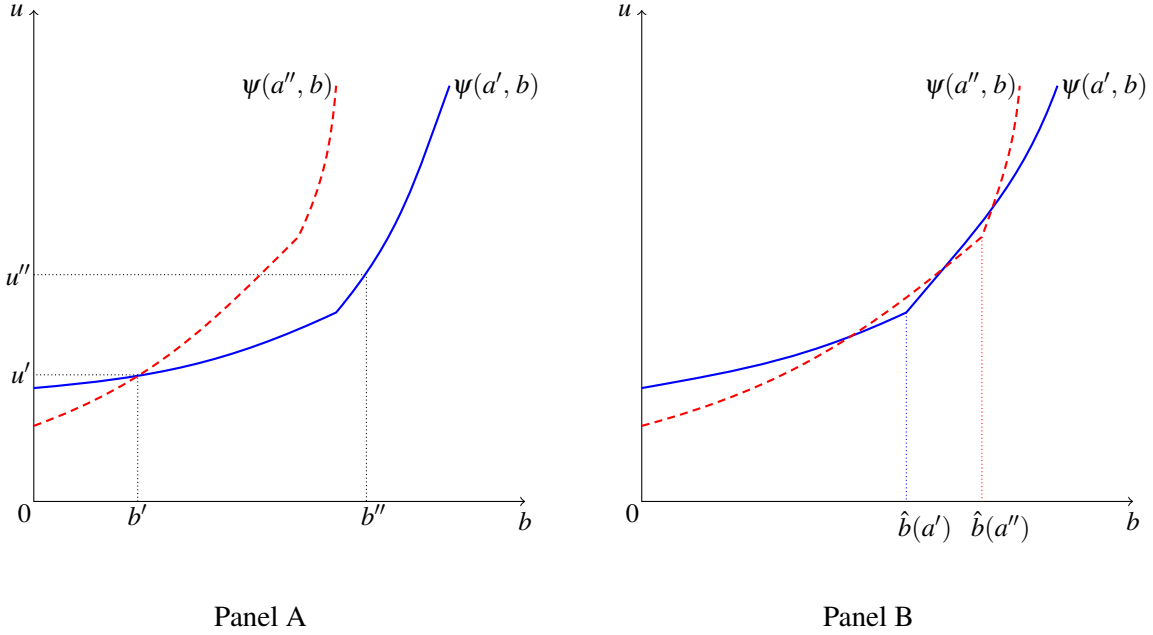


Figure 2: Panel A depicts (GID). The indifference curves $\psi(a', b)$ and $\psi(a'', b)$ cross only once. In other words, if a lower type a' is indifferent between (b', u') and (b'', u'') , then a higher type a'' prefers to pay more than u'' to b'' . Panel B depicts a possible violation of (GID), i.e., the indifference curves of two distinct types a' and a'' cross each other multiple times. To avoid the situation depicted in Panel B it suffices to show that \hat{b} moves to the left following an increase in a .

It turns out that

$$\frac{d\hat{b}}{da} = - \frac{\psi_a^I(a, \hat{b}) - \psi_a^N(a, \hat{b})}{\psi_b^I(a, \hat{b}) - \psi_b^N(a, \hat{b})}.$$

The numerator and the denominator of the above fraction are of the same sign, and hence, \hat{b} is decreasing in a . The intuition is simple. The term $\psi_a^I(a, \hat{b}) - \psi_a^N(a, \hat{b})$ is the contribution of a to a marginal change in the match surplus [in a neighborhood of \hat{b}] when the enterprise (a, b) switches from N to I , whereas the term $\psi_b^I(a, \hat{b}) - \psi_b^N(a, \hat{b})$ is the contribution of b to such marginal changes. Therefore, both a and b gains at the margin by switching from N to I . When these marginals point in opposite directions, PAM may fail to hold around the kink, as Panel B of Figure 2 depicts. Note that the proof of PAM does not hinge on the particular signs of the numerator and the denominator in the sense that $\psi_\theta^I(a, \hat{b}) - \psi_\theta^N(a, \hat{b})$ for $\theta = a, b$ is required only to be of the same sign, i.e., the indifference curve of each a under integration can be steeper or flatter than that under non-integration, yet any equilibrium allocation exhibits PAM.

Two famously used functions are $\psi(a, b) = \max\{ab - \bar{v}, 2a^2b^2 - \bar{v}\}$ (in Cole, Mailath, and Postlewaite, 2001), and $\psi(a, b) = \max\{a^2b - \bar{v}, ab^2 - \bar{v}\}$ (in Kremer and Maskin, 1996). In the first case, (GID) always holds because \hat{b} moves to the left following an increase in a , whereas in the second case, (GID) may fail to hold for all (a, b) as the kink moves to the right.¹¹ The reason is that a and b are treated

¹⁰See Chade, Eeckhout, and Smith (2017, pp. 503-504) for the proof of this assertion.

¹¹It is worth noting that the two aforementioned papers model the matching games when utility is perfectly transferable (TU), and hence, proving (GID) boils down to showing the supermodularity of the exogenously given surplus function $z(a, b) = \max\{z^1(a, b), z^2(a, b)\}$ where z^1 and z^2 describe two distinct production technologies. Our model, on the other hand, analyzes

asymmetrically under two distinct technologies in [Kremer and Maskin \(1996\)](#), whereas in the other case they are treated symmetrically. Ours is similar to [Cole et al. \(2001\)](#) because our production function $z(a, b)$ remains unaltered across the two ownership structures. We state the main result of this section in the following Proposition.

Proposition 2 *The bargaining frontier $\phi(a, b, u)$ satisfies (GID), and hence, the equilibrium matching is positive assortative, i.e., $\beta'(a) \geq 0$.*

4.3 Equilibrium ownership structures

We now analyze the equilibrium ownership structures in the supplier market. In Section 4.1, we have analyzed the optimal contract of each enterprise (a, b) for given levels of enterprise revenue $R(a, b)$ and the utility of each B -supplier. In the market equilibrium, both revenue and utility are endogenized through the equilibrium matching function $a = \alpha(b)$. First, note that condition (MC) implies that $G(\alpha(b)) = F(b)$, and hence, $\alpha(b) = G^{-1}(F(b))$ with $\alpha'(b) = f(b)/g(\alpha(b)) > 0$ for all $b \in B$.

In order to determine the equilibrium organizational structure of the supplier market, we first derive the [equilibrium] *indifference locus* on which any equilibrium enterprise $(\alpha(b), b)$ is indifferent between the two organizational modes, which is depicted in Figure 3. Consider an enterprise $(\alpha(b), b)$ along the equilibrium path whose productivity is given by $\tilde{z}(b) \equiv z(\alpha(b), b)$. Since $z_a, z_b > 0$ and the equilibrium exhibits PAM, we have $\tilde{z}'(\cdot) > 0$ and hence, an inverse function \tilde{z}^{-1} exists. Therefore, for a given level of revenue R of enterprise $(\alpha(b), b)$, we write $b = \tilde{z}^{-1}(R/P) \equiv Z(R)$. Let $b^- = Z(R^-)$ and $b^+ = Z(R^+)$. Clearly, each equilibrium enterprise $(\alpha(b), b)$ can be uniquely identified by b , the productivity of the B -supplier. In Figure 3, a point (b, u) in the ‘type-utility’ space represents the type of the B -supplier and the utility consumed by this type, respectively of an enterprise $(\alpha(b), b)$ formed in an equilibrium allocation.

- If for any equilibrium enterprise $(\alpha(b), b)$ we have $b \in [b_{min}, b^-)$, then this enterprise would strictly prefer non-integration over integration irrespective of the utility allocations. For values of $b \in (b^+, b_{max}]$, on the other hand, an enterprise would strictly prefer to integrate.
- Now let $b \in [b^-, b^+]$. Recall that, for these values of b , there is no clear dominance of one ownership structure over the other. Thus, corresponding to each $b \in [b^-, b^+]$ there are exactly two utility values, $u_L(\alpha(b), b)$ and $u_H(\alpha(b), b)$, which leave enterprise $(\alpha(b), b)$ indifferent between the two ownership structures.¹² Given that $\alpha'(b) > 0$, it is easy to show that $u_L(\alpha(b), b)$ is strictly increasing in b . This is the upward-sloping portion of the *parabola-shaped curve* in Figure 3 that is labelled U_L . On the other hand, $u_H(\alpha(b), b)$ is strictly increasing (decreasing) for low (high) values of b . In Figure 3, the increasing portion of $u_H(\alpha(b), b)$ is labelled U_H^+ and its decreasing portion is labelled U_H^- .

Therefore, U_L, U_H^+, U_H^- and the portion of the vertical line at $b = b^-$ above the point $(b, u) = (b^-, 0.086c)$ together define the indifference locus for each enterprise $(\alpha(b), b)$, which we will denote by $\tilde{u}(b)$. The

a situation where utility is not perfectly transferable (ITU). See [Amir \(2005\)](#) for a survey on applications of supermodularity, complementarity and single-crossing in economics.

¹²Note that, by construction, $u_L(\alpha(b^-), b^-) = 0 < (3/2 - \sqrt{2})c = 0.086c = u_H(\alpha(b^-), b^-)$ and $u_L(\alpha(b^+), b^+) = u_H(\alpha(b^+), b^+) = c/12 = 0.083c$.

indifference locus partitions the ‘type-utility space’ into two disjoint regions in which one ownership structure is preferred to the other by all enterprises $(\alpha(b), b)$ formed in equilibrium.

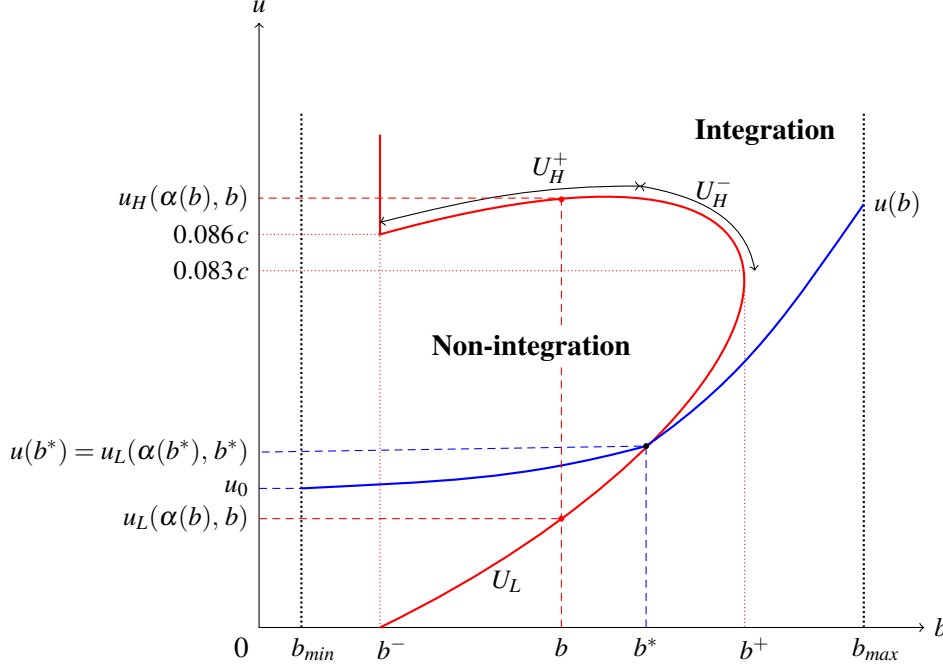


Figure 3: Monotonic ownership structure in the market equilibrium. Enterprises with $b \leq b^*$ stay separate, whereas those with $b > b^*$ integrate.

Next, we derive the utility allocations for the B -suppliers in the market equilibrium. The first-order condition of the optimization problem (\mathcal{M}_a) is given by the following *ordinary differential equation* (ODE):

$$u'(b) = \begin{cases} -\frac{\phi_2^N(\alpha(b), b, u(b))}{\phi_3^N(\alpha(b), b, u(b))} & \text{if } u(b) \in [u_L(\alpha(b), b), u_H(\alpha(b), b)]; \\ -\frac{\phi_2^I(\alpha(b), b, u(b))}{\phi_3^I(\alpha(b), b, u(b))} & \text{otherwise.} \end{cases} \quad (4)$$

Given that $-\phi_2^d(a, b, u)/\phi_3^d(a, b, u) \equiv \psi_b^d(a, b) > 0$ for $d = N, I$, the equilibrium utilities of the B -suppliers, which is implicitly given by:

$$u(b) = u(b_{min}) + \int_{b_{min}}^b u'(x)dx = u_0 + \int_{b_{min}}^b u'(x)dx, \quad (5)$$

is strictly increasing in b . Since the reservation utility u_0 of all the B -suppliers is the outside option of a B -supplier with type b_{min} , in equilibrium we must have $u(b_{min}) = u_0$. The equilibrium utility [of the B -suppliers] function $u(b)$ is depicted in Figure 3. According to the Picard-Lindelöf Theorem (see [Birkhoff and Rota, 1989](#)) a unique solution to the ODE (4) exists (at least in the neighborhood of the initial condition) provided that $u'(b)$ is bounded, Lipschitz continuous in u and continuous in b .¹³ In the

¹³Whenever $u(b) < u_L(\alpha(b), b)$ and $u(b) > u_H(\alpha(b), b)$, the ODE assumes a simple form; all the aforementioned properties are satisfied and an analytical solution can be easily obtained. However, for values of u in the interval

Appendix we show that when u_0 is sufficiently high, the indifference curve of each a is steeper under I than N (see Figure 2).¹⁴ This leads to the following condition:

$$-\frac{\phi_2^N(\alpha(b), b, u(b))}{\phi_3^N(\alpha(b), b, u(b))} < -\frac{\phi_2^I(\alpha(b), b, u(b))}{\phi_3^I(\alpha(b), b, u(b))} \text{ for all } b. \quad (6)$$

Therefore, $u(b)$ is strictly increasing in b , and flatter under non-integration.

The enterprise that is indifferent between the two organizational modes is determined by the intersection of the indifference locus, $\tilde{u}(b)$ and the equilibrium utility function $u(b)$, which is denoted by $(b^*, u(b^*))$. We further assume that $R_{min} \equiv Pz(b_{min}) \in (0, R^-)$. This is equivalent to saying that the minimum productivity is low enough, i.e., $b_{min} < b^- \equiv Z(R^-)$, which ensures that the low-productivity enterprises (with productivity close to b_{min}) choose to stay separate in the market equilibrium. Given the intersection point(s) between the two curves in Figure 3, enterprise $(\alpha(b^*), b^*)$ is an indifferent enterprise in the equilibrium allocation. Since condition (6) holds for all enterprises in equilibrium, and the equilibrium matching is positive assortative, b^* is unique.

Proposition 3 *There is a unique threshold productivity $b^* \in [b^-, b^+]$ of the B -suppliers such that each equilibrium enterprise $(\alpha(b), b)$ chooses non-integration if and only if $b \leq b^*$.*

Note that condition (6) is a single-crossing condition between $(\alpha(b), b)$ and the ownership structure. If I is the more preferred choice, then such single-crossing property implies a positive sorting between enterprise productivity and integration. As a result, high-revenue enterprises choose to integrate, while the low-revenue ones choose to stay separate.

5 Effect of price changes on the equilibrium

5.1 Incidence of integration

We examine how a change in the product market price P affects the fraction of integrated firms in the market equilibrium. First note that, in any equilibrium enterprise $(\alpha(b), b)$, the total revenue, $Pz(\alpha(b), b)$, and hence the match surplus, go up as a result of an increase in the product market price. We will argue that how the measure of integrated firms changes following an exogenous increase in the market price P will depend crucially on how this additional surplus gets divided between the A - and B -suppliers in the marginal enterprise $(a^*(P), b^*(P))$ where $a^*(P) = \alpha(b^*(P))$, i.e., the enterprise which is indifferent between the two ownership structures at the initial price P . Recall that $b^* \in (b^-, b^+)$, otherwise one of the two ownership structures is always dominant. We first state the following useful result.

Lemma 2 *Let the product market price increase exogenously from P to P' . Then,*

$[u_L(\alpha(b), b), u_H(\alpha(b), b)]$, the ODE is much more complicated and an analytical solution to it does not exist. In this region, we require to establish the existence and uniqueness of a solution. Our assumptions ensure that it is continuous in b , because b enters u' through $z(\alpha(b), b)$ which is a continuous function. The term u enters through the share. If a differentiable function has a derivative that is bounded everywhere by a real number, then the function is Lipschitz continuous. If $u_0 = 0$, then $s = 0$ and u' becomes unbounded. Hence, we require u_0 to be strictly positive.

¹⁴When u is low it is easier for A to transfer surplus to B than when u is high, under N . Therefore, when u is high an A -supplier is willing to pay less (than when u is low) for a higher b type under N , whereas the slope under I is independent of u . This implies that the indifference curve of an A -supplier under N becomes flatter as u increases.

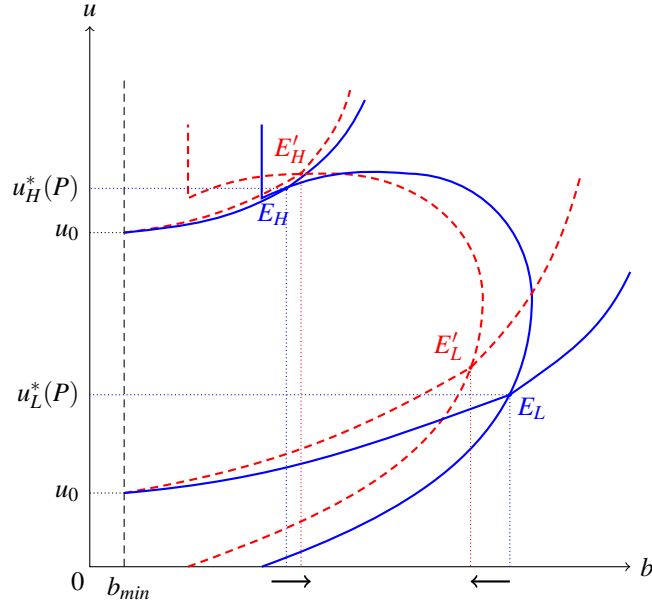


Figure 4: Two possible changes in equilibria. In one case, around E_L , b^* goes down following an increase in price implying more integration. In the other case, around E_H , higher price induces b^* to go up, and hence, less integration.

- (a) *the indifference locus shifts in the way as depicted in Figure 4;*
- (b) *The equilibrium utility of the B-suppliers is higher under P' , i.e., $u(b | P') > u(b | P)$ for all $b > b_{min}$ where $u(b | P)$ is the equilibrium utility of the B-suppliers under the product market price P .*

When the product market price P changes exogenously, the market equilibrium is affected by both (a) a shift in the indifference locus, and (b) a shift in the equilibrium utility $u(b | P)$. Therefore, the effect of a price change on the marginal enterprise $(a^*(P), b^*(P))$ is ambiguous. Figure 4 depicts two such situations. In the first case, point E_L denotes the equilibrium under the initial price P . At E_L , the B-supplier in the indifferent enterprise $(a^*(P), b^*(P))$ consumes $u_L^*(P) \equiv u_L(a^*(P), b^*(P))$. Point E'_L denotes the equilibrium under the increased price P' at which the initially indifferent enterprise $(a^*(P), b^*(P))$ prefers to integrate, and hence, the threshold productivity level b^* decreases implying that the fraction of integrated enterprises increases under the increased product market price. In the other case, a movement from E_H to E'_H implies that the threshold productivity level b^* increases, and hence, more enterprises stay separate in the equilibrium under price P' . Therefore,

Proposition 4 *The effect of an exogenous increase in the product market price on the fraction of integrated enterprises is in general ambiguous.*

The intuition behind Proposition 4 is best understood in terms of Figure 5 which depicts the effect of an increase in the product market price on the marginal enterprise (a^*, b^*) . The bargaining frontier at the initial price P is denoted by $\phi(a^*, b^*, u; P)$. When price increases to P' , the bargaining frontier moves to a higher level, $\phi(a^*, b^*, u; P')$. The shift of the linear part is parallel, whereas the non-linear part shifts

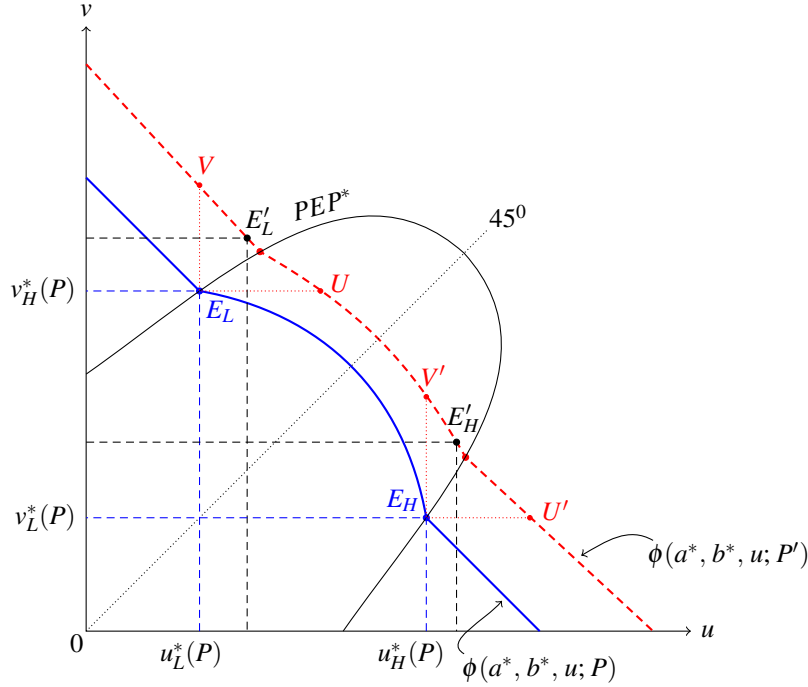


Figure 5: The bargaining frontiers of the marginal enterprise before and after the price increase. If the initial utility allocation is at E_L , then this enterprise prefers to integrate at the new utility allocation E'_L . If, on the other hand, the initial utility allocation is at E_H , then this enterprise prefers to integrate at the new utility allocation E'_H .

out but becomes flatter as the product market price increases.¹⁵ Recall from Figure 1 that, corresponding to each bargaining frontier associated with each arbitrary enterprise (a, b) there are two kinks or indifferent points – namely, $(u_L(a, b), v_H(a, b))$ and $(u_H(a, b), v_L(a, b))$. Joining these kinks associated with different price levels yields the ‘price expansion path’ [for the enterprise (a^*, b^*)], labelled PEP^* . In other words, inside the region enclosed by PEP^* , enterprise (a^*, b^*) prefers to stay separate as the market price changes, whereas outside this region this enterprise prefers to integrate.¹⁶

First, we describe the movement from E_L to E'_L . At E_L , the enterprise (a^*, b^*) is indifferent between the two organizational modes under the product market price P . Therefore, in the initial equilibrium, $u(b^* | P) = u_L^*(P)$ and $v(a^* | P) = v_H^*(P)$. As the product market price increases to P' , the match surplus

¹⁵The absolute value of the slope of the non-linear frontier for a given (a, b) is given by:

$$\left| \phi_3^N(a, b, u; P) \right| = \frac{s(a, b, u; P)}{1 - s(a, b, u; P)}.$$

The above expression is increasing in s and $s(a, b, u; P)$ is decreasing in P , and hence, $\phi_3^N(a, b, u; P)$ becomes flatter as P increases. The imperfect transferability problem under non-integration is mitigated when price, and hence revenue, increases.

¹⁶The price expansion path for any given enterprise is qualitatively the same as the one derived by (Legros and Newman, 2013, Figure I), who assume a quadratic success probability. The only difference is that, under a linear probability of success function, the two paths meet at the 45⁰-line. It is worth noting that the price expansion path is unique when the supplier units are homogeneous, whereas in our case there is a distinct price expansion path associated with each pair of suppliers.

of $(a^*(P), b^*(P))$ increases, and the bargaining frontier shifts out to $\phi(a^*, b^*, u; P')$. Suppose we keep the utility of b^* fixed at its initial level $u_L^*(P)$, and give the entire additional surplus to a^* so that her utility goes up to point V . At this interim utility allocations this enterprise should choose to integrate because point V lies above PEP^* . On the other hand, if the utility of a^* were to be fixed at $v_H^*(P)$, then the entire additional surplus would be given to b^* so that we would end up with a utility allocation at point U . Because U lies below PEP^* , the enterprise would choose to stay separate. But eventually, to preserve stability in the input market, the additional surplus must be divided between the two supplier units. In other words, the utility allocation for $(a^*(P), b^*(P))$ must lie on the segment of the frontier $\phi(a^*, b^*, u; P)$ between points U and V , e.g. at point E'_L which is shown to be above the price expansion path. Therefore, the marginal enterprise chooses to integrate, and hence, the fraction of integrated firms increases following an increase in price (a movement from E_L to E'_L in Figure 4). The logic behind a movement from E_H to E'_H for the marginal enterprise is similar. In particular, in this case the new utility allocation for the marginal enterprise lies inside the region enclosed by PEP^* , and hence, $(a^*(P), b^*(P))$ chooses to stay separate following a price rise, and the measure of integrated enterprises decreases.

How the increased surplus following an increase in price should be divided in the marginal firm $(a^*(P), b^*(P))$, i.e., the exact location of E'_L or E'_H will depend on how the additional surplus is divided in each of the infra-marginal firms. It is typical with endogenous sorting under two-sided heterogeneity that the equilibrium surplus division in any firm has spillovers in the firms higher in ranking because the utility of the B -suppliers must follow the allocation rule given by (5). For example, if the reservation utility u_0 , i.e., the utility of the B -suppliers in the least productive enterprises is very high, then the equilibrium at E_H is more likely to occur,¹⁷ and hence, an exogenous increase in price may induce less integration. Given the utility allocation in the indifferent enterprise [at the initial market price] $(u_H^*(P), v_L^*(P))$, because the bargaining frontier around E_H is flatter under the increased price it is now marginally less costly for the B -supplier to transfer surplus to his partner. Consequently, the share of surplus is more balanced (in favor of the A -supplier), and this enterprise may end up staying separate in the equilibrium under the increased price.

Given the nature of the ODE (4), it is not possible to solve for $u(b)$ analytically, and hence, we cannot have a clear sufficient condition to describe which of the two opposing forces described above (and when) dominates. In the following numerical example we show that higher product market price can lead to less integration.

Example 1 (higher prices leading to less integration) Let $z(a, b) = a^{0.3}b^{0.5}$. We set $c = 0.25$ and $u_0 = 0.012$. Furthermore, we assume that both a and b are uniformly distributed on $[0.25, 0.4]$ and $[0.3, 0.5]$, respectively, so that $\alpha(b) = 0.025 + 0.75b$. Using the above data we solve the ODE (4) numerically using Matlab.¹⁸ It turns out that, for $P = 0.59$ and $P' = 0.6$, we have $b^*(P) = 0.45 < 0.5 = b^*(P')$ implying that a greater measure of enterprises stay separate under the higher product market price P' .

In the above example, $u(b | P)$ intersects the indifference locus at U_H^+ , and hence, a movement from E_H to E'_H in Figure 4. If we consider a movement from E_L to E'_L , and force the utility of $b^*(P)$ to stay at its initial level $u_L^*(P)$, then this enterprise would prefer to integrate at E'_L . This is because the already unbalanced distribution of surplus in favor of the A -supplier in the marginal enterprise, since we started with a low utility for the B -supplier, becomes even more unbalanced following a price rise. This in turn

¹⁷This is only one possible explanation for the emergence of E_H as equilibrium. It may be the case that u_0 is low, but $u(b)$ is steep enough so that $u(b)$ intersects the indifference locus at any point on U_H^+ .

¹⁸The Matlab codes are available upon request.

harms coordination under non-integration, and hence, integration is more likely to dominate. This is the mechanism in Legros and Newman (2013), where all suppliers are homogeneous and $u(b) = 0$. In this case the indifferent enterprise at P prefers N at P' even when $u(b)$ is fixed.¹⁹

5.2 Organizationally augmented industry supply

We derive now the industry supply curve (OAS) as a function of the product market price. Consider an arbitrary enterprise (a, b) . If this enterprise stays separate, then its expected output is given by:

$$q^N(a, b, P) = \pi(e_A(a, b), e_B(a, b))z(a, b) = \frac{R(a, b)}{2c} \cdot z(a, b) = \frac{Pz(a, b)^2}{2c}.$$

The output of enterprise (a, b) is strictly increasing in the product market price P . On the other hand, if this enterprise integrates, its expected output is given by:

$$q^I(a, b, P) = \pi(e_A(a, b), e_B(a, b))z(a, b) = z(a, b).$$

The output of an integrated enterprise does not depend on the product market price. Clearly, an integrated enterprise produces greater expected output because $R(a, b) \leq 2c$ for all (a, b) . The organizationally augmented industry supply is the expected output aggregated across all the enterprises in equilibrium, which is given by:

$$Q(P) = \int_{b_{min}}^{b^*(P)} q^N(\alpha(b), b, P) dF(b) + \int_{b^*(P)}^{b_{max}} q^I(\alpha(b), b, P) dF(b). \quad (\text{OAS})$$

A change in the product market price P affects $Q(P)$ via two channels – a rise in P (i) augments the output q^N of each non-integrated enterprise, but leaves the integrated output q^I unaltered and (ii) changes the fraction of integrated enterprises by changing the threshold productivity $b^*(P)$ of the indifferent enterprise. Moreover, these two effects may be countervailing. Whether the augmented industry supply curve $Q(P)$ is increasing or decreasing in P depends on the sign of $db^*(P)/dP$. Clearly, if the values of P are close to zero, then non-integration is the preferred ownership structure for all the enterprises because $b^-(P)$ is arbitrarily high. Therefore, $Q(P)$ must be increasing for low product market prices. But for high values of P if the threshold productivity $b^*(P)$ increases, the reduction in the number of integrated enterprises, and hence, the aggregate expected integration output may outweigh the increase in the aggregate expected non-integration output. Consequently, $Q(P)$ may decrease.

Proposition 5 *For low product market prices the organizationally augmented industry supply $Q(P)$ is increasing in P . However, $Q(P)$ may have a backward-bending segment. A necessary condition for the backward-bending supply curve is a negative correlation between the product market price and integration, i.e., $\frac{db^*(P)}{dP} > 0$.*

In the following example we show that the industry supply curve can be backward-bending.

¹⁹However, (Legros and Newman, 2013, footnote 16) do recognize the possibility of a non-monotonic association between price and integration when u is fixed at a high level, but no further analysis is provided. In our model, this is analogous to the indifferent enterprise offering a fixed utility $u_H^*(P)$ to the B -supplier in Figure 5.

Example 2 (Backward-bending OAS) We maintain the same parameter specifications as in Example 1. Figure 6 depicts a backward-bending organizationally augmented supply curve which is S-shaped. For low and high levels of the product market prices the quantity supplied is increasing in P , whereas for intermediate price levels the industry supply is decreasing in the market price. For example, consider a price increase from $P = 0.59$ to $P' = 0.6$. In this case, the output decreases from $Q(P) = 0.32$ to $Q(P') = 0.28$. The arc-elasticity of supply with respect to P and P' is given by -7.93 , i.e., a 1% increase in the product market price implies an approximate decrease in the industry supply $Q(P)$ by about 7.9% on average.

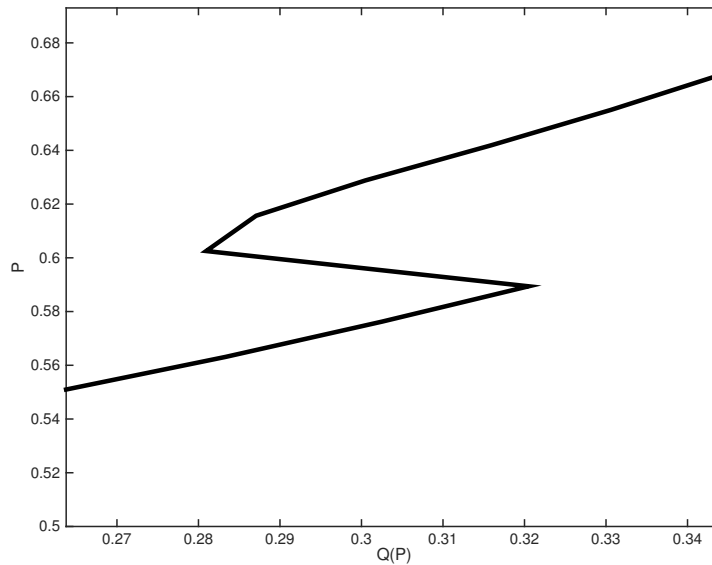


Figure 6: A backward-bending industry supply. The OAS is increasing in P for low and high levels of market price, whereas it is decreasing for the intermediate price levels.

For intermediate price levels, when an increase in P implies a decrease in the measure of integrated enterprises in equilibrium, we have a downward-sloping industry supply curve. On the other hand, there is more integration and increasing supply following an increase in P for extreme values of the product market price. To close our model, the equilibrium product market price P^* is determined by the intersection of the OAS, $Q(P)$ and the demand curve, $D(P)$.

6 Empirical implications

The model derives four predictions: i) positive assortative matching among input suppliers with heterogeneous productivities, ii) integration is more likely for high productivity organizations, while low productivity ones choose to remain separate, iii) price can be positively or negatively associated with the incidence of integration and iv) the industry supply curve (OAS) can be increasing or it may have a backward-bending segment.

The first two predictions are generally supported by the recent literature, e.g., [Hortaçsu and Syverson \(2007\)](#) and [Atalay et al. \(2014\)](#). The empirical evidence on the third prediction has been mixed – [Hastings \(2004\)](#), [Alfaro et al. \(2016\)](#) and [McGowan \(2017\)](#) find a positive relation, whereas the findings of [Hortaçsu and Syverson \(2007\)](#) show a negative association between price and integration.

Recall that [Proposition 5](#) suggests three possibilities as price increases: (a) a backward-bending OAS accompanied by less integration, (b) an upward-sloping OAS and less integration and (c) an upward-sloping OAS along with more integration. With this taxonomy in mind our model is amenable to analyze implications of exogenous changes in the product market price. If we are in case (a), then we would observe (i) a lower product price, (ii) a higher aggregate output and (iii) a greater number of integrated firms. This prediction is consistent with the findings of [Hortaçsu and Syverson \(2007\)](#). [Legros and Newman \(2013\)](#) consider two distinct levels (high and low) of exogenously given enterprise productivities and introduce a technology shock that increases the measure of high productivity firms. This shifts the (upward-sloping) OAS out and, given a fixed downward-sloping demand, results in a lower price, higher output and more integration (because high productivity firms integrate). Thus, our model offers an explanation of the same phenomenon which is complementary to that of [Legros and Newman \(2013\)](#), in the sense that we rely on a demand rather than a technology shock.

A lower price can be associated with lower aggregate output but more integration even if the OAS is not backward-bending, which would be implied by case (b) in the above taxonomy. This case is more likely to occur when the initial product market price is not too low. On the other hand, when the initial market price is low, case (c) is more likely to occur, which implies that lower market price and output would be associated with less integration. This prediction conforms to the finding of [Alfaro et al. \(2016\)](#) who establish a positive correlation between market price and integration, by restricting their analysis to highly competitive sectors, which presumably are characterized by low market prices.

7 Conclusion

We analyze the determinants of firm boundaries when firms of different productivities interact in a perfectly competitive product market. The choice of ownership structure in a given enterprise depends on the trade-off between coordination and private benefits: non-integration, a mode based on contingent revenue shares, puts too much weight on the private costs of managerial actions and hinders coordination; integration, which is based on delegation of decision rights to an outsider, facilitates coordination but ignores private benefits. Neither mode of organization thus achieves efficiency. Unbalanced revenue shares between the two units induce the managers to opt for integration because coordination if they remain separate is poor. Balanced revenue shares, on the other hand, harmonize incentives and make non-integration more likely to dominate.

When supplier units are vertically differentiated with respect to productivity, competition for high-quality units arises naturally in the supplier market. We model such competition as a two-sided matching game, which endogenizes revenue share or utility allocation in each enterprise. In an equilibrium allocation, the matching is positive assortative, i.e., more productive suppliers match together to form enterprises. Thus, ex-ante differences in input productivity imply ex-post differences in firm revenue. Moreover, high-revenue firms opt to integrate because incentive to coordinate is high in such firms, whereas low-revenue enterprises stay separate.

Our paper contributes to the extant literature pertaining to OIO by introducing two-sided heterogeneity and sorting. Within each enterprise, a price change alters the endogenously determined distribution of surplus between the two suppliers. It can induce more or less coordination under non-integration, and consequently, the product market price may be positively or negatively associated with the decision to integrate. Moreover, the possible negative correlation between price and integration may give rise to a backward-bending industry supply curve. These findings generate interesting testable implications.

The present model yields interesting normative implications with respect to *managerial firms*. When managers are partial revenue claimants, they tend to underweight enterprise revenues in favor of private benefits because the perceived price is lower than the actual market price. Essentially, the impact of partial revenue claims by the managers on ownership structures is similar to the impact market price has when firms are non-managerial. The presence of managerial firms thus yields an equilibrium that is organizationally inefficient (e.g. [Leibenstein, 1966](#)) because the true market price is unchanged. Thus, the presence of managerial firms may imply either ‘too little’ or ‘too much’ integration in equilibrium relative to the social optimum with non-managerial firms. This can have interesting policy implications related to corporate governance. Furthermore, taxes that affect the market price can also have similar implications on the efficiency of organizational choice.

Appendix: Proofs

Proof of Lemma 1. Substituting for e_A and e_B from the incentive compatibility constraints (IC_A) and (IC_B), the optimal contracting problem in an arbitrary enterprise (a, b) reduces to:

$$\begin{aligned} \max_{s \in [0, 1]} V_A(s; R) &\equiv \frac{R^2}{4c}(1 - s^2), & (\mathcal{P}'_N) \\ \text{subject to } U_B(s; R) &\equiv \frac{R^2}{4c}s(2 - s) = u. & (PC'_B) \end{aligned}$$

From (PC'_B) it follows that

$$s(a, b, u; P) = 1 - \frac{\sqrt{R(a, b)^2 - 4cu}}{R(a, b)}.$$

We ignore the other root since it is strictly larger than 1. The bargaining frontier under N is given by:

$$\phi^N(a, b, u; P) = \frac{R(a, b)^2(1 - s^2)}{4c} = \frac{1}{4c} \left[2R(a, b)\sqrt{R(a, b)^2 - 4cu} - \{R(a, b)^2 - 4cu\} \right]. \quad (A1)$$

This completes the proof of the lemma.

Proof of Proposition 1. First, consider the case when non-integration completely dominates integration in a given enterprise (a, b) . In this case the minimum non-integration surplus must be strictly higher than the maximum surplus under integration, i.e.,

$$\frac{R^2}{4c} > R - \frac{c}{2} \iff \left[R - c(2 - \sqrt{2}) \right] \left[R - c(2 + \sqrt{2}) \right] > 0$$

Since $R \leq 2c$, the above holds for $R < (2 - \sqrt{2})c = 0.586c \equiv R^-$. Next, consider the case when integration completely dominates non-integration in a given enterprise (a, b) . In this case the minimum

non-integration surplus must be strictly higher than the maximum surplus under non-integration, i.e.,

$$R - \frac{c}{2} > \frac{3R^2}{8c} \iff \left(R - \frac{2c}{3}\right)(R - 2c) < 0$$

Since $R \leq 2c$, the above holds for $R > \frac{2c}{3} = 0.666c \equiv R^+$. This completes the proofs of Parts (a) and (c). To show Part (b), note first that $\phi^N(a, b, u; P)$ intersects the linear function $\phi^I(a, b, u; P)$ exactly twice since both are symmetric with respect to the 45° -line and the non-linear frontier is strictly concave. The two intersection points are given by:

$$u_L(a, b) = \frac{1}{8} \left[4R(a, b) - 2c - \frac{R(a, b)}{c} \cdot \sqrt{(2c - R(a, b))(2c - 3R(a, b))} \right], \quad (\text{A2})$$

$$u_H(a, b) = \frac{1}{8} \left[4R(a, b) - 2c + \frac{R(a, b)}{c} \cdot \sqrt{(2c - R(a, b))(2c - 3R(a, b))} \right]. \quad (\text{A3})$$

Note that $u_H(a, b) = R(a, b) - \frac{c}{2} - u_L(a, b)$. Thus, for the existence of the two intersection points it suffices to show that $u_L(a, b) \geq 0$, which holds if $R(a, b) \leq -\frac{(2+\sqrt{10})c}{3}$ or $R^- \leq R(a, b) \leq R^+$ or $2c \leq R(a, b) \leq (2 + \sqrt{2})c$. Given that $0 \leq R(a, b) \leq 2c$, this case occurs when $R^- \leq R(a, b) \leq R^+$. This completes the proof of the proposition.

Proof of Proposition 2. We prove the result in the following steps:

1. We prove that, under each ownership structure $d = N, I$, the bargaining frontier for a each (a, b, u) satisfies **GID**, i.e.,

$$\phi^d(a', b'', u'') = \phi^d(a', b', u') \implies \phi^d(a'', b'', u'') \geq \phi^d(a'', b', u') \quad (\text{GID}^d)$$

for any $a'' > a', b'' > b'$ and $u'' > u'$.

2. We establish **GID** for the combined bargaining frontier

$$\phi(a, b, u) = \max\{\phi^N(a, b, u), \phi^I(a, b, u)\};$$

STEP 1: Because the bargaining frontier under each ownership structure $d = N, I$ is differentiable everywhere, **(GID)^d** is equivalent to the *Spence-Mirrlees condition* which is given by:

$$\psi_{ab}^d(a, b) = \frac{\partial}{\partial a} \left[-\frac{\phi_2^d(a, b, u)}{\phi_3^d(a, b, u)} \right] > 0 \quad \text{for } d = N, I, \quad (\text{SM})$$

where $\psi^d(a, b)$ is the indifference curve of each a under ownership structure $d = N, I$.

Let a given enterprise (a, b) with transfer u to the B -supplier chooses $d = N$. Since the contract must be optimal under this given ownership structure, the bargaining frontier is given by equation **(A1)**. We differentiate **(PC_B)** with respect to $\theta = a, b$ and u , respectively to obtain the following:

$$\frac{\partial s}{\partial \theta} = -\frac{s(2-s)R_\theta}{(1-s)R} \quad \text{for } \theta = a, b, \quad (\text{A4})$$

$$\frac{\partial s}{\partial u} = \frac{2c}{(1-s)R^2}. \quad (\text{A5})$$

Denote by ϕ_i^d the partial derivative of ϕ^d with respect to the i -th argument, and by ϕ_{ij}^d the cross-partial with respect to the i -th and the j -th arguments. Differentiating (A1) with respect to a , b and u , and using (A4) and (A5) we obtain

$$\phi_1^N(a, b, u) = \frac{1-s+s^2}{1-s} \cdot \frac{RR_a}{2c} > 0, \quad (\text{A6})$$

$$\phi_2^N(a, b, u) = \frac{1-s+s^2}{1-s} \cdot \frac{RR_b}{2c} > 0, \quad (\text{A7})$$

$$\phi_3^N(a, b, u) = -\frac{s}{1-s} < 0. \quad (\text{A8})$$

Note that condition (SM) is equivalent to

$$\frac{\phi_{21}^d(a, b, u)}{\phi_2^d(a, b, u)} - \frac{\phi_{31}^d(a, b, u)}{\phi_3^d(a, b, u)} > 0.$$

Differentiating (A7) with respect to a we get

$$\frac{\phi_{21}^N}{\phi_2^N} = \frac{R_{ab}}{R_b} + \frac{R_a}{R} + \left(\frac{2s-1}{1-s+s^2} + \frac{1}{1-s} \right) s_a = \frac{R_{ab}}{R_b} + \frac{1-3s+s^3}{(1-s)^2(1-s+s^2)} \cdot \frac{R_a}{R}. \quad (\text{A9})$$

On the other hand, Differentiating (A8) with respect to a we get

$$\frac{\phi_{31}^N}{\phi_3^N} = -\frac{2-s}{(1-s)^2} \cdot \frac{R_a}{R}. \quad (\text{A10})$$

Therefore, equations (A9) and (A10) together imply

$$\frac{\phi_{21}^N}{\phi_2^N} - \frac{\phi_{31}^N}{\phi_3^N} = \frac{R_{ab}}{R_b} + \left[\frac{1-3s+s^3}{(1-s)^2(1-s+s^2)} + \frac{2-s}{(1-s)^2} \right] \frac{R_a}{R} = \frac{R_{ab}}{R_b} + \frac{3}{1-s+s^2} \cdot \frac{R_a}{R} > 0. \quad (\text{A11})$$

The above is true because $R_\theta = Pz_\theta > 0$ for $\theta = a, b$, $R_{ab} = Pz_{ab} > 0$ and $s \in [0, 1]$. Therefore, (GID^d), or equivalently, (SM) holds for $d = N$.

Next, consider that enterprise (a, b) chooses $d = I$, and hence the associated bargaining frontier is given by (3). Note that $\phi_2^I = R_b$, $\phi_{21}^I = R_{ab}$, $\phi_3^I = -1$ and $\phi_{31}^I = 0$. Therefore,

$$\frac{\phi_{21}^I(a, b, u)}{\phi_2^I(a, b, u)} - \frac{\phi_{31}^I(a, b, u)}{\phi_3^I(a, b, u)} = \frac{R_{ab}}{R_b} > 0, \quad (\text{A12})$$

which proves that (GID^d) holds for $d = I$. This establishes single-crossing under each ownership structure $d = N, I$.

STEP 2: To establish single-crossing for the combined frontier, note first that $\phi(a, b, u) = \phi^N(a, b, u)$ if $R(a, b) < R^-$, and $\phi(a, b, u) = \phi^I(a, b, u)$ if $R(a, b) > R^+$. Therefore, GID holds for each enterprise (a, b) with $R(a, b) < R^-$ and $R(a, b) > R^+$. Next, we prove the single-crossing condition for $R(a, b) \in [R^-, R^+]$. Recall that the indifference curve of each a under ownership structure $d = N, I$ is given by $\phi^d(a, b, \psi^d(a, b)) = \bar{v}$, and hence,

$$\psi_\theta^d(a, b) = -\frac{\phi_2^d(a, b, u^d)}{\phi_3^d(a, b, u^d)} \quad \text{for } \theta = a, b. \quad (\text{A13})$$

Define by

$$H(s) \equiv \frac{1-s+s^2}{s} \cdot \frac{R}{2c}.$$

Then, from (A6)-(A8) it follows that

$$\begin{aligned}\psi_a^I(a, b) - \psi_a^N(a, b) &= R_a(a, b)[1 - H(s)], \\ \psi_b^I(a, b) - \psi_b^N(a, b) &= R_b(a, b)[1 - H(s)].\end{aligned}$$

For each a , \hat{b} is defined by $\psi^I(a, \hat{b}) = \psi^N(a, \hat{b})$. Thus,

$$\frac{d\hat{b}}{da} = -\frac{\psi_a^I(a, \hat{b}) - \psi_a^N(a, \hat{b})}{\psi_b^I(a, \hat{b}) - \psi_b^N(a, \hat{b})} = -\frac{R_a(a, \hat{b})[1 - H(s)]}{R_b(a, \hat{b})[1 - H(s)]} = -\frac{z_a(a, \hat{b})}{z_b(a, \hat{b})} < 0.$$

The above inequality proves the single-crossing under the combined ownership structures, and hence, the bargaining frontier $\phi(a, b, u)$ satisfies (GID) for all (a, b, u) . This situation is depicted in Panel A of Figure 2. Therefore, it follows from Legros and Newman (2007) that the equilibrium matching is PAM.

Proof of Proposition 3. We first show that, under the assumption that $u_0 > \tilde{u}$ for some $\tilde{u} > 0$, inequality (6) holds. From (A7), (A8), $\phi_2^I = R_b$ and $\phi_3^I = -1$, it follows that

$$-\frac{\phi_2^N(a, b, u)}{\phi_3^N(a, b, u)} < -\frac{\phi_2^I(a, b, u)}{\phi_3^I(a, b, u)} \iff H(s)R_b < R_b \iff H(s) < 1. \quad (\text{A14})$$

The above inequality implies that the indifference curve of each a is steeper under I than that under N for all b , and hence, they cross each other only once. This situation is depicted in the left panel of Figure 2 where the steeper portion of each indifference curve corresponds to I . Since $\lim_{s \rightarrow 0} H(s) = \infty$, $H(1) = R/2c \leq 1$ and $H'(s) = -\frac{1-s^2}{s^2} \cdot \frac{R}{2c} < 0$ for all $s \in [0, 1]$, there is a unique $\tilde{s} \in (0, 1)$ such that $s > \tilde{s}$ implies inequality (A14). Clearly, when $s \leq \tilde{s}$, the inequality in (A14) is reversed implying that the indifference curve of each a is flatter under I than that under N for all b . Because under non-integration, $R^2s(2-s) = 4cu$, $s > \tilde{s}$ if and only if $u > \tilde{u}$ where \tilde{u} is given by $R^2\tilde{s}(2-\tilde{s}) = 4c\tilde{u}$. Therefore, a sufficient condition for (A14) to hold for all (a, b, u) is that $u_0 > \tilde{u}$. Condition (6) is nothing but condition (A14) along the equilibrium path. Thus, (A14) implies (6). Clearly, (6) implies a single-crossing condition between $u_N(b)$ and $u_I(b)$ where $u_N(b)$ denotes the portion of $u(b)$ to the left of the equilibrium indifference locus and $u_I(b)$ is the portion of $u(b)$ to the right of it. Thus, it trivially follows that no $b < b^*$ will choose I and no $b > b^*$ will choose N . Therefore, b^* is unique.

Proof of Lemma 2. We first prove part (a). Differentiating (A2) with respect to P we obtain

$$\frac{du_L}{dP} = \frac{z[2c(c-R) + 2c\sqrt{(2c-R)(2c-3R)} - (2c-R)(2c-3R)]}{4c\sqrt{(2c-R)(2c-3R)}}.$$

It is easy to show that $2c\sqrt{(2c-R)(2c-3R)} - (2c-R)(2c-3R) \geq 0$ for $R \leq 2c/3$. Therefore, the numerator of the above expression is always strictly positive for $R \in [R^-, R^+]$, and hence, $du_L(a, b)/dP > 0$ for all (a, b) with $R(a, b) \in [R^-, R^+]$. Next, differentiating (A3) with respect to P we get

$$\frac{du_H}{dP} = \frac{z[2c^2 + 3R^2 - 6cR + 2c\sqrt{(2c-R)(2c-3R)}]}{4c\sqrt{(2c-R)(2c-3R)}}.$$

The numerator of the above expression is strictly decreasing in R on $[R^-, R^+]$, positive at $R = R^-$ and negative at $R = R^+$. This proves part (a) of the Lemma.

Finally, we prove part (b). Let $h(s) \equiv \frac{1-s+s^2}{s}$. Note that

$$-\frac{\phi_2^N(a, b, u(b))}{\phi_3^N(a, b, u(b))} = \frac{R(a, b)R_b(a, b)}{2c} \cdot h(s(a, b, u(b))).$$

To prove the Lemma, it suffices to show that $du(b)/dP > 0$ for $b \leq b^*$ because for any integrated enterprise (a, b) , we have $u'(b) = -\frac{\phi_2^I}{\phi_3^I} = R_b = Pz_b$, and hence, $du'(b)/dP = z_b > 0$. We differentiate (5) with respect to P to obtain

$$\frac{du(b)}{dP} = \int_{b_{min}}^b \frac{Pz_b}{2c} \left[PH'(s) \left\{ \frac{ds}{dP} + \frac{ds}{du} \frac{du(x)}{dP} \right\} + 2h(s) \right] dx, \quad (\text{A15})$$

Note that

$$h'(s) = -\frac{1-s^2}{s^2} < 0.$$

On the other hand, differentiating (PC'_B) we obtain

$$\frac{ds}{dP} = -\frac{s(2-s)}{P(1-s)} < 0, \quad \text{and} \quad \frac{ds}{du} = \frac{2c}{R^2(1-s)} > 0.$$

Clearly, $du_0/dP = 0$. Second, for values of b arbitrarily close to b_{min} , $du(b)/dP > 0$. Since in this case $du(x)/dP$ is arbitrarily close to zero, and the other terms in the integrand of (A15) are positive, $du(b)/dP$ cannot be negative for any b in the neighborhood of b_{min} . Now, suppose by way of contradiction that $du(b)/dP$ becomes negative for some b (farther from b_{min}). Consider the b , denoted by \tilde{b} , at which $du(\tilde{b})/dP = 0$. Given, as we showed above, that the integrand of (A15) is positive initially, then at \tilde{b} it must be negative (so that the positive and negative areas cancel each other out in the integration). But the integrand of (A15) evaluated at \tilde{b} is strictly positive, a contradiction. Given that the integrand of (A15) is always positive, part (b) of the lemma follows.

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