Microeconomics Comprehensive Exam, **Part One**

August, 2025

Department of Economics, University of Houston

1 ***60 pts.*** Senstivie periods for investments

HISD investigates "sensitive periods" during which childhood skill investments are more effective. In an experiment, half of the children in each of a, b, and c ($a < b < c$) age groups are randomly assigned to z hours of a weekly skills training program at a cost of \$1 per child-hour for one year. Skills, $q_t \in R_+$, at ages t and $t+1$ are measured for each child.

1. *12 pts.* Considering the relationship between q and z, develop a definition of "sensitive periods" based on dynamic production function considerations.
2. *12 pts.* Based on your definition, under what assumptions does the experiment identify "sensitive periods"?
3. *12 pts.* Given unlimited resources, how would you set up an ideal experiment to identify "sensitive periods" under weaker sets of production function assumptions?
4. *12 pts.* The experiment finds positive effects at all ages, with higher effects for younger children. Based on your prior responses, is age a the most "sensitive"?
5. *12 pts.* Set up and solve to the extent possible a planning problem to determine the optimal allocation of training times across grades.

2 ***60 pts.*** Production and degradation

Robinson Crusoe (RC) spends v^y units of time generating output $Y = v^y$ for consumption $C = Y$. RC spends v^e units of time reducing environmental degrading D. Utility is quasiconcave in C and island environment/scenery B. Given baseline $B_{t=0}$, $B_t = B_{t-1} - D_t$. Degradation is increasing in v^y and decreasing in v^e.

1. *12 pts.* Propose functional forms for utility and degradation functions and discuss properties.
2. *12 pts.* At t, set up RC's static utility maximization problem and solve to the extent possible.
3. *12 pts.* As RC's overall time resource available for v^y and v^e increases, given your assumptions, under optimal choices, would Y_t and B_t have a positive or negative relationship, or something else? (Higher GDP cleaner environment? or higher GDP and dirtier environment?)
4. *12 pts.* Respond to (3) again, now consider alternative returns to scale assumptions for the degradation production function.
5. *12 pts.* Set up the dynamic optimization problem over infinite horizon. Discuss optimality conditions and optimal time-use paths.

Microeconomics Comprehensive Exam, **Part Two**

August, 2025

Department of Economics, University of Houston

Read all questions carefully before you begin. All twelve parts are worth an equal amount, but some parts are more difficult than others. Be strategic and judicious with your time. Think slowly and calmly. Good luck!

1. Consider the case of a seller S trying to sell an object to a single buyer B. The object has value $v \in \{H, L\}$, $H > L > 0$. S knows the value, but B does not. The interaction between the two parties is as follows:

   - **Stage 1.** S offers a price p
   - **Stage 2.** B can spend money to learn about the value (e.g., hire somebody to appraise it). More specifically, an appraisal generates a signal of $x \in \{H, N\}$ (High type or No-Info). For an appraisal of accuracy q:

     $$P(x = H|v = H) = q \tag{1}$$
     $$P(x = H|v = L) = 0 \tag{2}$$

     Put another way, L-type objects always yield no signal, but H-type objects may yield a conclusive H signal with probability q, but they may also yield an inconclusive N signal with probability $1 - q$. The cost of a quality q appraisal is $c \cdot q$.
   - **Stage 3.** B can either buy or pass. Let's assume that if B indifferent between buying and passing, she buys.

   (a) As a warmup, let's pretend that there were no appraisers (no stage 2). What price should S offer?

   (b) OK, now we are going to prove that as information gets cheaper, in the Perfect Bayesian Equilibrium, B extracts all of the surplus, and S receives the lowest possible payoff (i.e., L). This is a fairly complicated proof, so I'll guide you in steps. I also will not require you to make any formal mathematical arguments (i.e., no algebra necessary). But you need to provide cogent, intuitive arguments in clear English. If it helps you to use algebra instead (or to complement your arguments), then you should feel free.

      i. Argue that type L sellers can't charge a price that results in information acquisition.
      ii. Following your argument in part (b), argue that B will never acquire any information in equilibrium.
      iii. Now consider an equilibrium in which S randomizes over prices and B doesn't acquire information. Argue that this is Pareto dominated by a different equilibrium offer and describe that offer. That is, argue that either B or S can get a strictly higher payoff under an alternative pure strategy offer, and the other one is at least as well off.
      iv. Argue that as information gets cheaper ($c \to 0$), the single equilibrium price should converge to L.

   (c) Give a 1-3 paragraph explanation for this surprising result! You should discuss this in the context of your answer to part (a). Can you think of several settings in which these informational forces come into play? Can you think of some in which they don't? Do the logic and implications of the model apply in these settings or not? Why or why not?

2. My two kids want to watch TV, but the house is a mess. I tell them that for every item of theirs that they pick up, they get to watch a little bit of TV. Formally, each of them chooses effort e_i, and the total number of items that are put away is given by $Y = e_1 + e_2 + \epsilon$ where $\epsilon \sim N\left(0, \sigma^2\right)$. (For simplicity, you can assume that they can pick up a fraction of an item.) I contract to let them watch $m_1(Y)$ and $m_2(Y)$ minutes of TV respectively. Their private cost of effort is given by $c(e) = \frac{1}{2}e^2$, the value of an item that is put away is normalized to 1, and a minute of TV is worth 1.

   (a) As a warmup, consider the scenario where effort is observable and contractible. What effort level does the first best contract induce?

   (b) Now suppose that effort is not observable, and I commit to an equal sharing rule: $m_i(Y) = \frac{Y}{2}$. Now how much effort do my kids put in?

   (c) Compare your answers in parts (a) and (b) and explain why they. What do you think would happen if I committed to an unequal sharing rule, e.g. $m_1(Y) = \frac{2}{3}Y$ and $m_2(Y) = \frac{1}{3}Y$. Discuss in 3-5 sentences.

   (d) What if I offered linear contracts, i.e., $m_i(Y) = \alpha + \beta Y$. (Assume that the kids have an outside option that gives them 0). In terms of the contract parameters, how much effort do my kids put in now? Compare it with your answers to parts (b) and (c). What condition is necessary for this contract to be implementable?

   (e) Now suppose my kids are risk averse. To save you some time, you can assume that their utility is given by $U(m_i, e_i) = E(m_i) - \frac{\rho}{2}\text{Var}(m_i) - \frac{1}{2}ce_i^2$ where $\rho > 0$ captures risk aversion. Derive the IC constraint for the linear contract from part (d). How much effort to my kids put in now? (Hint: You should first derive $\alpha^\star$ as a function of β, then $\beta^\star$ as a function of the parameters. $e^\star$ should follow easily from that.)

   (f) Now in the craziest counterfactual of all, suppose I had n kids. Under an equal sharing contract and risk neutrality, how much effort would they put in? What happens to total effort as $n \to \infty$? Is my house cleaner or messier? Discuss this result in 2-3 sentences.