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Comprehensive Exam in Macroeconomic Theory–Procedural Instructions

(1) Write your answers only on the paper we provide.

(2) We are distributing a numbered sign–in sheet in a moment. The number next to your signature will be your **student number**.

(3) Every sheet of paper you turn in to us **must** have your **student number** written at the **top–center** of the sheet and **circled**.

(4) Every sheet of paper you turn in to us **must** have a **page number** written at the **top–right corner** of the sheet.

(5) You have 4 hours to complete the exam. When you have finished, or when it is 2:00 pm (whichever comes first), prepare a cover sheet for your exam. This cover sheet should not have a page number, but must have the following things on it:

(a) Your **student number** at the top–center, circled.

(b) The phrase "Macroeconomics Comprehensive Exam".

(c) The sentence "My last page is page number X", where X is your total number of pages of answers.

1. (15%) Neoclassical growth model (NGM)

Suppose there are $L_0 = 1$ households in the economy. Population growth $= n > 0$, i.e., $L_{t+1} = (1 + n)L_t$. Hence, there are $\frac{L_t}{L_0}$ members per household. TFP growth = 0. The preferences of each **household** are given by:

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{L_t}{L_0}\right) U(c_t) \tag{1}$$

where c_t is consumption per capita (equivalently, per household **member**). Each household member supplies one unit of labor inelastically, and earns a wage W_t (per unit of labor). $U(c_t) = \ln(c_t)$. The household also owns the capital stock, which it rents to firms at rate R_t (per unit of capital). The household can accumulate capital in the usual way. Capital depreciates at a rate δ.

Firms rent capital and hire labor at wage rate W_t in order to maximize profits. The production function is our usual Cobb-Douglas function (with TFP growth =0):

$$Y_t = K_t^{\alpha}(AL_t)^{1-\alpha} \tag{2}$$

(a) (7) Solve for the steady-state capital stock, output, and consumption per capita. (Hint: Set up the social planner's problem for the household.)

(b) (3) Now suppose population growth decreases from n to n' with $n' < n$. Solve for the steady-state capital, output, and consumption per capita.

(c) Compare the three variables in the two steady-states. (5) What is the intuition for any differences (or non-differences) betweeen them?

2. (17%) Lucas Tree Asset-Pricing Model

There is a representative consumer who maximizes expected utility over consumption:

$$E_t\left[\sum_{t=0}^{\infty} \beta^t U(c_t)\right] \tag{3}$$

The consumption good is produced by one "Lucas tree". Output, y_t, is exogenous and follows a Markov process:

$$y_{t+1} = G(y_t, \epsilon_{t+1}) \tag{4}$$

where ϵ_t is an iid shock with distribution $\phi(.)$.

There is one share in the Lucas tree. A share entitles the owner as of the beginning of t to all of the tree's output in period t. Right after the owner gets the output, the shares are traded. The household sells its shares at a price p_t per share, and then it can buy new shares at that price, too. Paying the price p_t entitles the owner to the output of the tree next period, y_{t+1}, as well as the right to sell the share of (claim to) the tree at price p_{t+1}. Denote π_t to be the amount of the share the consumer holds at the beginning of period t (and purchased in period $t-1$).

(a) (3) Write down the household's budget constraint

(b) (5) Now, let us jump ahead to the Euler equation:

$$u'(c_t)p_t = \beta E_t\left[u'(c_{t+1})(y_{t+1} + p_{t+1})\right] \tag{5}$$

Show how the Euler equation equation can be manipulated to yield the following:

$$E(R_{t+1}) = \frac{1 - Cov(m_{t+1}, R_{t+1})}{E_t(m_{t+1})} \tag{6}$$

Clearly, a big part of your solution will be to define R_{t+1} and m_{t+1}.

(c) (9) (2) With the above equation, derive the expected return for a risk-free asset. (2) Write down the expression for the risk premium, which is the difference between the expected return of a risky asset and that of a risk-free asset. (5) How does the risk premium depend on the covariance? Give as much intuition as possible.

3. **(18%) Real Business Cycle (RBC) model** Consider the RBC model taught in class. There are a large number (measure N) of identical households. The households have access to a complete set of contingent Arrow-Debreu securities. Each household has one unit of time, which it allocates between labor and leisure. The household also accumulates capital, and rents the capital to firms. A representative firm hires labor (at wage W_t) and rent capital (at rate R_t) to produce output and maximize profits. All markets are perfectly competitive. Capital depreciates at rate δ. The source of uncertainty is an aggregate economy-wide productivity shock:

$$ln(A_t) = \rho ln(A_{t-1}) + \epsilon_t \tag{7}$$

where ϵ_t has a normal distribution with mean 0 and variance σ^2. Importantly, households and firms make decisions in period t *after* they observe the productivity shock in period t. Preferences are given by:

$$E_t\left[\sum_{t=0}^{\infty}\beta^t u(c_t, l_t)\right] \tag{8}$$

where c_t is household consumption, and l_t is household labor, in period t. $u(.)$ is given by:

$$u(c_t, l_t) = ln(c_t) + \theta ln(1 - l_t) \tag{9}$$

The household has one unit of time to allocate between labor, l_t, and leisure, $1-l_t$. Denote k_t as capital per household or per capita. The capital/labor ratio is given by $\frac{k_t}{l_t}$. The production function is given by:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{10}$$

where K_t and L_t are economy-wide aggregate capital and labor, respectively.

(a) (2) Write the household budget constraint

(b) (4) Derive the Euler equation for the household's intertemporal consumption choice

(c) (12) Here are four stylized facts about business cycles:

i. Employment is positively correlated with output

ii. TFP is positively correlated with output

iii. The real interest rate has almost no correlation with output iv. Consumption and investment are positively correlated with output

(5) Which of these facts can be explained by the RBC model? Why? (3) Which one is not? Why not? (4) How could you modify the model (different shock, different parameters of existing shocks), to help make the model explain this fact?

4. (15%) Consider the two-sided search model we studied in class, but change the model so the number of matches is given by

$$m_t = \mu m(u_t; v_t)$$

where μ is a constant that denotes matching efficiency. Matching efficiency could stand in for information technologies that make it easier for workers to find firms posting vacancies and for firms with vacancies to find workers, for example.

(a) Set up the value functions of the problem.

(b) Construct a steady state equilibrium.

(c) Determine the effects on total surplus, unemployment, vacancies, output, and wages, of an *increase* in matching efficiency in the steady state, and explain your results giving economic intuition.

5. (15%) An economy consists of two infinitely lived consumers named i = 1;2. There is one nonstorable consumption good. Consumer i consumes c_t^i at time t. Consumer i ranks consumption streams by

$$\sum_{t=0}^{\infty} \beta^t u(c_t^i),$$

where $\beta \in (0,1)$ and $u(c)$ is increasing, strictly concave and twice continuously differentiable. Consumer 1 is endowed with a stream of the consumption good $y_t^1 = 1,0,0,1,0,0,1,\ldots$. Consumer 2, is endowed with a stream of the consumption good $y_t^2 = 0,1,1,0,1,1,0,\ldots$. Assume there are complete markets with time-0 trading.

(a) Define a competitive equilibrium.

(b) Compute a competitive equilibrium.

6. (20%) Consider the following version of the overlapping generations model with capital. Time is discrete ($t = 0,1,2,\ldots$). Individuals live for two periods, and only work when young, and there is no population growth. An individual born at time t chooses consumption, savings and hours worked when young to maximize:

$$U(cy,t,co,t+1) = log(cy,t) + \beta log(co,t+1)$$

with $0 < \beta < 1$.

There are two types of individuals in this economy that are differentiated by the efficiency units they are endowed (born) with. Type-H individuals are endowed with z_H, while type-L individuals are endowed with z_L, with $z_H > z_L$. N_i; $i = H;L$; young individuals are born at each date so the total size of each new cohort is $N = N_H + N_L$.

Competitive firms have access to a Cobb-Douglas technology that uses capital (K) and labor services (L); via a Cobb-Douglas technology. In turn, labor types are imperfect substitutes in production (Cobb-Douglas form as well). Formally, output Y is given by:

$$Y = K\alpha L1{-}\alpha$$

with $L = L_H^{\phi} L_L^{1-\phi}$.

(a) Formulate the problem of young individuals born at t.

(b) Derive the law of motion for the capital-labor ratio.

(c) Suppose now at time $t = t_0 - 1$ the economy is in the steady state. At the start of t_0, newborn migrants are allowed to enter the economy, once and never again. These migrants have the same preferences and endowments of natives, and never leave. We assume that this changes the composition of the labor pool on a permanent basis. We now have $N_H' > N_H$ and $N_L' > N_L$; respectively, for all t. Describe the dynamic adjustment of the rate of return of capital.

(d) What are the long-run effects of the entry of migrants on labor earnings? Economist A argues that labor earnings of both types decline. Is she right? Explain providing economic intuition.