Fan Wang, Vikram Maheshri
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# Microeconomics Comprehensive Exam Questions

Procedural Instructions:

(1) Write your answers only on the paper we will provide.

(2) We will be distributing a numbered sign-in sheet in a moment. The number next to your signature will be your student number.

(3) Every sheet of paper you turn in to us must have your student number written at the top center of the sheet and circled.

(4) Every sheet of paper you turn in to us must have a page number written at the top-right corner of the sheet.

(5) When you have finished, or when time is up (whichever comes first), prepare a cover sheet for your exam. This cover sheet should not have a page number, but must have the following things on it:

(a) Your student number at the top-center, circled.

(b) The phrase "Microeconomics Comprehensive Exam August 2022".

(c) The sentence "My last page is page number X," where X is your total number of pages of answers.

Read all questions carefully before you begin. You should plan to spend approximately one minute per point. Good luck!

1. *70 pts. Frankly my deer, I don't give a horn.* Usually in class or when you solve a problem set or exam, you (1) are given the assumptions and structure of a model, (2) derive certain mathematical implications and (3) give these implications economic interpretations. This question is going to be very different, as I am going to ask you to do step (1) yourself. That is, we will start from an interesting phenomenon, and then you will come up with a way to mathematically model it. Moreso than any other question, this does not have a single correct answer. If you use your imagination and explain yourself clearly, you will do very well on this problem.

   Take a look at the picture of a buck (male deer), standing next to a doe (female deer). The most obvious difference between them is that the buck has big antlers (i.e., horns on his head), but the doe does not. Assume – reasonably – that the horns do not have any intrinsic value to the buck's survival. Moreover, they are actually costly to the buck: they require resources to grow. For this part of the problem, also assume – unreasonably – that the buck can perfectly control the size of his antlers.

   (a) *5 pts.* Why would a buck grow antlers if they are costly and don't help it survive? What model from class does this remind you of? Explain in 1-2 sentences.

   (b) *5 pts.* Building on the intuition from the previous question, who is the principal and who is the agent?

   (c) *20 pts.* How would you model the buck's decision (remember, we're assuming that he can choose his antler size)? The more details you can provide, the better. More concretely:
      i. What are the important variables here? Define them with words.
      ii. What are the objectives of the principal and agent?
      iii. What are the important parameters here? (These could be specific Greek letters or they could be functions like $u(\cdot)$ and $v(\cdot)$.) Define them with words.

   (d) *10 pts.* What restriction(s) on the parameters do you need to make in order for your model to capture your intuition from part (a)? Explain what these restriction(s) mean in words in the context of the deer example.

   (e) *10 pts.* Solve for the equilibrium antler size in your model. Interpret the equilibrium in 1-2 sentences. Describe how the changes in certain parameters affect the equilibrium in clear English.

Figure 1: Buck (L) and Doe (R)

(f) *20 pts.* OK, now let's be a bit more realistic. Clearly a buck can't just choose how big his antlers are going to be. How are these "choices" made in nature? (Hint: no specific person makes these choices, nevertheless they are clearly "made"). How might you enrich your model to account for this realism. You don't have to actually do any of this formally, but you need to describe how you would approach the problem. For instance, would you add more actors to the model besides the doe/buck? Would you add dynamics? Would you add some kind of strategic interaction? This is an open-ended question for you to really stretch your brain.

2. *50 pts. Teamwork makes the dream work?* OK, now on to a more traditional problem where we explore moral hazard in a collaborative setting. Suppose that there is a team of two risk neutral workers, 1 and 2, who take actions $a_1 \geq 0$ and $a_2 \geq 0$ to produce output $Q = Q(a_1,a_2)$. We assume that $\frac{\partial Q}{\partial a_i} > 0$, $\frac{\partial^2 Q}{\partial a_i^2} < 0$ and $\frac{\partial^2 Q}{\partial a_i \partial a_j} \geq 0$. That is, there is diminishing marginal productivity in own actions, but increasing marginal productivity in your teammate's action.

Worker $i$'s utility is given by

$$u_i = w_i - g_i(a_i) \qquad (1)$$

where $w_i$ is how much of the output that they enjoy (i.e., their wage) and $g_i(\cdot)$, a strictly increasing and convex function, captures the worker $i$ specific disutility of effort in taking an action.

Finally, individuals' actions are unobservable, only the total output is observable. Hence wages can only be a function of observable output, i.e. $w_i(Q)$. (Assume for simplicity that $w_i$ is differentiable for all $i$.) Finally, the team shares the total surplus (output), so $w_1+w_2 = Q$. For now, the team has no manager.

(a) *5 pts.* Explain in 2-3 sentences with clear logic why there is an incentive for workers to free-ride off of their teammate's effort. To get credit, you must indicate what component of the model implies this.

(b) *5 pts.* Do you think that free riding will lead to an inefficient outcome or not? If so, will workers exert too little or too much effort? Don't derive this mathematically, just make a clear argument in words.

(c) *10 pts.* Let's solve for the second-best outcome. Write out worker *i*'s problem taking worker *j*'s action as given and show that the first order condition of this problem can be expressed as

$$g'(a_i) = \frac{\partial w_i(Q(a_i, a_j))}{\partial a_i} \cdot \frac{\partial Q(a_i, a_j)}{\partial a_i} \tag{2}$$

Interpret this FOC in one sentence.

(d) *10 pts.* Now let's solve for the first-best outcome. Consider a social planner who is able to force workers to take specific actions. Write down this planner's problem and show that the first order conditions of this problem can be expressed as

$$g_i'(a_i) = \frac{\partial w_i(Q(a_i, a_j))}{\partial Q} \frac{\partial Q(a_i, a_j)}{\partial a_i} + \frac{\partial w_j(Q(a_i, a_j))}{\partial Q} \frac{\partial Q(a_i, a_j)}{\partial a_j} \tag{3}$$

Interpret this FOC in one sentence. (Hint: you may want to use words like "marginal benefit" and "marginal cost".) Then give a 1-2 sentence explanation of how your answers in parts (c) and (d) differ.

(e) *10 pts.* Now we are going to figure out how to implement the first-best outcome in the second-best world! Suppose that the team asks an external manager to set wages. (Note, the manager still cannot observe actions, only total output.) Let $Q_{FB}$ be the first best level of output that the social planner could implement and $a^{FB}_i$ be the associated actions taken.

Consider the following wage contract:

$$w_i(Q) = \bar{w}_i \text{ if } Q \geq Q_{FB} \tag{4}$$
$$= 0 \text{ otherwise} \tag{5}$$

where $\bar{w}_1 + \bar{w}_2 = Q_{FB}$ and $\bar{w}_i \geq g_i\left(a_i^{FB}\right)$ for all *i*. Explain why this is a Nash Equilibrium, i.e., neither worker has an incentive to deviate from their first best actions.

(f) *10 pts.* Do you find your answer to part (f) surprising? Why or why not? Can you think of an example where there is moral hazard in a team setting, but a manager/coach is not hired to implement the optimal wages? In your example, why do you think the teammates don't solve the moral hazard problem with a third party?