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Comprehensive Exam in Macroeconomic Theory–Procedural Instructions

(1) Write your answers only on the paper we provide.

(2) We are distributing a numbered sign–in sheet in a moment. The number next to your signature will be your **student number**.

(3) Every sheet of paper you turn in to us **must** have your **student number** written at the **top–center** of the sheet and **circled**.

(4) Every sheet of paper you turn in to us **must** have a **page number** written at the **top–right corner** of the sheet.

(5) You have 4 hours to complete the exam. When you have finished, or when it is 4:00 pm (whichever comes first), prepare a cover sheet for your exam. This cover sheet should not have a page number, but must have the following things on it:

(a) Your **student number** at the top–center, circled.

(b) The phrase “Macroeconomics Comprehensive Exam”.

(c) The sentence “My last page is page number X”, where X is your total number of pages of answers.

1. (15%)

Consider the OLG model (as discussed in class) in which each cohort lives for two periods. L_t is the population of each cohort. The growth rate of cohorts over time is given by n, i.e., $L_{t+1} = (1+n)L_t$. Each agent born in period t works in period t only, and consumes in period t and $t+1$. Preferences are logarithmic with discount factor β. The agent supplies one unit of labor inelastically and earns a wage rate w_t.

1. (1%) Give the expression for the total population of the economy in period t.
2. (2%) Write the budget constraint in period t for the agent born in t. (Use $c_{1,t}$ to denote consumption in the first period of life in period t.)
3. (4%) For the agent born in t, assume there is a bond that pays a gross rate of interest $1+r_{t+1}$ in period $t + 1$ on savings that were made in period t (i.e., the usual assumption from our class). For this agent, derive the Euler equation linking consumption from t to $t + 1$.
4. (8%) From the above Euler equation, as well as this agent's budget constraint in each period, solve for this agent's consumption in period t as a function of the wage rate w_t.

2. (18%)

Consider the following 2-period model (0,1) with two goods (a, b). The economy has one representative household that supplies one unit of labor inelastically, and maximizes:

$$\theta\ln(c_{a,0}) + (1-\theta)\ln(c_{b,0}) + \beta\,[\theta\ln(c_{a,1}) + (1-\theta)\ln(c_{b,1})] \quad (1)$$

subject to the following budget constraints for period 0 and 1, respectively:

$$c_{a,0} + k_1 + p_{b,0}c_{b,0} = w_0 + R_0k_0 \quad (2)$$

$$c_{a,1} + p_{b,1}c_{b,1} = w_1 + R_1k_1 \quad (3)$$

where good a is the numeraire good. (Hence, $p_{b,i}$ is the relative price of good b in period i.) K_0 is given, and capital fully depreciates. Hence, aggregate capital in period 1 equals aggregate investment in period 0:

$$K1 = I0 \tag{4}$$

The Cobb-Douglas production functions for each good in period 0 are:

$$Y_{a,0} = K_{a,0}^{\alpha}(A_{a,0}L_{a,0})^{1-\alpha} \tag{5}$$

$$Y_{b,0} = K_{b,0}^{\alpha}(A_{b,0}L_{b,0})^{1-\alpha} \tag{6}$$

where $K_{a,0}$ is the use of capital to produce good a in period 0. Output of good a is used for consumption and capital accumulation. However, output of good b is used only for consumption. The production functions for period 1 are similar. All markets are perfectly competitive, and labor and capital are perfectly mobile across the two sectors (a and b).

1. (2%) For period 1, write down the goods market equilibrium condition for good a.
2. (6%) Derive the intertemporal Euler equation linking consumption of good b across the two periods. How does the relative price of good b affect the intertemporal allocation of consumption of this good? (Hint: R_t is measured in units of good a.)
3. (10%) Consider period 1. K_1 was chosen endogenously in the previous period, so is now a state variable. Given K_1, describe in as much detail as you can, how you would solve for the equilibrium amount of labor and capital that is in sector a. Use equations and words.

3. (17%)

Consider the real business cycle (RBC) model as discussed in class. There are a large number (measure N) of identical households. The households have access to a complete set of contingent Arrow-Debreu securities. Each household has one unit of time, which it allocates between labor l_t, and leisure, $1 - l_t$. The household also accumulates capital, and rents the capital to firms. Capital depreciates at rate δ. A representative firm hires labor (at wage W_t) and rent capital (at rate R_t) to produce output and maximize profits. The production function is given by:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{7}$$

where K_t and L_t are economy-wide aggregate capital and labor, respectively. All markets are perfectly competitive. There are two sources of stochastic shocks: an aggregate economy-wide productivity shock and an aggregate government purchase shock (G_t) financed by lump sum taxes

(T_t):

$$ln(A_t) = \rho_a ln(A_{t-1}) + \epsilon_{a,t} \tag{8}$$

$$\ln(G_t) = (1 - \rho_g)\ln(\omega Y) + \rho_g \ln(G_{t-1}) + \epsilon_{g,t} \tag{9}$$

where $\omega < 1$, $\rho_a < 1$, $\rho_g < 1$, Y is the mean of Y_t, $\epsilon_{a,t}$ has a normal distribution with mean 0 and variance $\sigma_a{}^2$ and $\epsilon_{g,t}$ has a normal distribution with mean 0 and variance $\sigma_g{}^2$. These two shocks are uncorrelated.

Importantly, households and firms make decisions in period t *after* they observe the productivity shock and the G shock in period t. Preferences are given by:

$$E_t\left[\sum_{t=0}^{\infty}\beta^t u(c_t, l_t)\right] \tag{10}$$

where c_t is household consumption, and l_t is household labor, in period t. $u(.)$ is given by:

$$u(c_t, l_t) = ln(c_t) + \theta ln(1 - l_t) \tag{11}$$

Denote k_t as capital per household or per capita. The capital/labor ratio is given by $\frac{k_t}{l_t}$.

1. (1%) From the production and utility functions presented above, does G have any consumption or production value? (Only write: Yes or no)

2. (8%) Suppose the G_t persistence parameter ρ_g is close to, but less than 1, e.g., 0.95. Describe qualitatively the impact effect of a positive G shock, i.e., $\epsilon_{g,t} > 0$ on household decisions (given existing prices), i.e., the effects on consumption demand and labor supply in the current period (t), and on the equilibrium levels of output, and the real interest rate. No credit without explanation.

3. (8%) Now, suppose the G_t persistence parameter $\rho_g = 0$, i.e., the shock is i.i.d. **Compare** qualitatively the effects of a positive G shock, i.e., $\epsilon_{g,t} > 0$ (and is the same size as in part 2 above) on household decisions (given existing prices), i.e., the effects on consumption demand and labor supply in the current period (t), and on the equilibrium levels of output and the real interest rate, to the effects in part 1 above. No credit without explanation.

4. (20%)

Consider a simple exchange economy with two consumers, indexed by i = 1; 2; who live forever, and with one perishable consumption good. Time is discrete and indexed by $t = 0, 1, \ldots$. Each consumer values sequences of consumption goods $c^i = \{c^i_t\}_{t=1}^{\infty}$ according to

$$U(c^i) = \sum_{t=0}^{\infty} \beta^t \ln(c^i_t),$$

with $\beta \in (0, 1)$. The endowment processes are given by

$$w_t^1 = 2, \quad and \quad w_t^2 = 0 \quad if \quad t \quad even$$

$$w_t^1 = 0, \quad and \quad w_t^2 = 1 \quad if \quad t \quad odd.$$

(a) Define an *Arrow-Debreu* competitive equilibrium for this economy.

(b) Solve for the equilibrium quantities and prices. Compare the allocations of each type of consumers and provide economic intuition for your findings.

(c) Write down the social planner problem and solve for the allocations.

5. (20%)

Consider an unemployed worker who each period can draw two independently and identically distributed wage offers (w_1 and w_2) from the cumulative probability distribution function $F(w)$. The worker will work forever at the same wage after having once accepted an offer. In the event of unemployment during a period, the worker receives unemployment compensation c. The worker derives a decision rule to maximize $E \sum_{t=0}^{\infty} \beta^t y_t$ where $y_t = w$ or $y_t = c$, depending on whether she is employed or unemployed. Let $v(w)$ be the value of $E \sum_{t=0}^{\infty} \beta^t y_t$ for a currently unemployed worker who has best offer $w = max\{w_1, w_2\}$ in hand.

(a) Formulate Bellman's equation for the worker's problem.

(b) Prove that the worker's reservation wage is *higher* than it would be had the worker faced the same c and been drawing only one offer from the same distribution $F(w)$ each period. Please provide economic intuition.

6. (10%)

The economy is populated by a continuum of workers who provide labor services to finance the purchase of a consumption good. Individuals live for two periods and may choose to work in either of two occupations, labeled occupations S (safe) and R (risky). Wages in each occupation are exogenous, w_S and w_R with $w_R > w_S$.

In period 1 they are exogenously assigned to one of the occupations. Upon entering to the occupation in period 1 they have human capital $h_1^S = h_1^R = 1$. In period 2 human capital is given by

$$h_2^j = h_1^j + e \tag{12}$$

for $j = S,R$ and where e is effort level that can take two values: e_L or e_H with $e_H > e_L$.

After the initial period they have an option to switch. Productivity in the risky occupation is uncertain: it takes the value 1 with probability p and $\gamma_L < 1$ with probability $(1 - p)$.

Workers are risk averse and they order amounts of consumption c according to $u(c) = log(c)$.

(a) Write down the Bellman equation(s).

(b) Write down the occupational decision of the agent in the second period.