Tariffs versus Anti-dumping Duties*

Emin M. Dinlersoz† Can Dogan‡
University of Houston University of Houston

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Abstract

Tariffs and anti-dumping duties are two important tools used to protect industries from foreign competition and to generate revenue. While both tools have been studied in separate contexts, little is known about their effects on domestic and import prices, exporting firms’ profits, domestic revenue, domestic industry protection, and domestic and foreign welfare. These effects are investigated in a two-country framework where a firm dominant in its home market exports to a foreign market served by an oligopoly, a setup that represents many important anti-dumping duty filings in the U.S. against foreign firms. Conditions under which one tool dominates the other for a given criterion are provided.

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†Department of Economics, 204 McElhinney Hall, Houston, TX 77204-5019. E-mail: edinlers@mail.uh.edu
‡Department of Economics, 204 McElhinney Hall, Houston, TX 77204-5019. E-mail: cdogan@mail.uh.edu
1 Introduction

Tariffs and anti-dumping duties are two important policy tools that protect industries and generate revenue to the countries that initiate them. While the two have been extensively studied in separate contexts, there is little work that compares them explicitly.\footnote{See, e.g., Bahgwati and Kemp (1969), Leith (1971), Brander and Krugman (1983), Reitzes (1993), Blonigen (2002), Blonigen and Prusa (2003), and Bown and Crowley (2007).} This paper does so with respect to their effects on domestic and import prices, industry protection, exporting firm’s profits, and domestic and foreign welfare. The goal is to be able to say under what conditions one tool is superior to the other in terms of a well-defined criterion. If the superiority of one tool can be established under a given criterion, the intervention in an otherwise free trade environment should be performed using the superior tool, depending on the importance of that criterion. For instance, when industry protection is a prominent goal in an intervention and an anti-dumping duty is found to be uniformly more effective in promoting that goal compared to a tariff, then the prescribed policy action would be to hold back tariffs and use anti-dumping duties instead.

Tariffs and anti-dumping duties, while sharing the common goal of industry protection, differ in the ways they are treated in theory. A tariff is usually set to maximize either domestic revenue or welfare, whereas an anti-dumping duty is frequently designed to make up for the gap between a foreign firm’s price at home and the price it charges for its exports, provided that the gap is positive. An anti-dumping duty thus mainly serves the purpose of industry protection, and as a by product, produces revenue but is not designed to maximize protection, revenue, or welfare. Consequently, an anti-dumping duty in its simplest form, unlike a tariff, is not a solution to an optimization problem.

In practice, too, there are important differences between the implementation of the two tools. Tariffs are imposed on all foreign firms exporting a given good to a domestic market. Anti-dumping duties, on the other hand, target a specific foreign firm or a few firms claimed to be "dumping", i.e. selling at prices that are deemed "unfair". The goal is to restore "fair pricing" by the foreign firm, where the definition of what is "fair" is an important practical question. Tariffs are usually determined by the U.S. International Trade Commission (ITC) and are subject to the approval of the Congress. On the other hand, anti-dumping duties are decided solely by the ITC and the Department of Commerce, with little or no involvement on the part of the Congress.\footnote{Obviously, it is impossible to claim that the government and ITC move entirely independently on the implementation of the two tools. Governments can implement policies that can affect the initiation and the outcome of anti-dumping duties. For example, "Continued Dumping and Subsidy Offset Act of 2000", also referred to as the "Byrd Amendment", which was repealed in 2005, proposed the redistribution of the anti-dumping duty revenues to the injured domestic firms that initiated the duty. This redistribution of the duty certainly increased incentives for filing anti-dumping duties.} It is also not uncommon to find anti-dumping duty filings in cases where there is already a tariff in effect. This article abstracts from any politico-economic issues and ignores possible coexistence of the two tools in an industry. The aim is to compare the tools mutually exclusively.

To carry out the comparison in a tractable yet realistic framework, a model of trade between two countries is developed, where a dominant firm in one of the countries exports to a market inhabited by a group of oligopolistically competitive firms. The specific structure of the model draws upon a few key
observations on the nature of many major anti-dumping cases filed in the U.S.

First, an anti-dumping petition requires some coordination among domestic firms against a foreign firm that is claimed to be dumping. Just as in the case of cartels, this type of coordination is more easily attained in concentrated markets. A select set of cases are reviewed below as examples of U.S. anti-dumping cases which were filed by a single domestic firm or group of firms in highly concentrated markets. To reflect this observation, the domestic industry is modeled as an oligopoly where the number of firms are allowed to vary, so that the role of the competitiveness of the domestic market can be assessed, including the limiting case of a competitive market.

Second, the foreign industry in the model consists of a single firm, representing a foreign market where a dominant firm has the ability to set prices. While such a setup is not applicable to all anti-dumping cases, an important subset of the cases involve a single dominant foreign firm, as reviewed below. A dominant firm is more likely to be the subject of dumping scrutiny. Because the dominant firm can set a price above the competitive level at home, the gap between its home and export prices is larger as long as the foreign market is more competitive than its home market. This gap is more likely to induce anti-dumping filings, as the widely-used definition of "dumping" compares a foreign firm’s home price with its export price.

Third, tariffs are usually targeted at an entire foreign industry, not necessarily at a single firm, whereas anti-dumping duties mostly aim a single foreign firm and a narrowly defined set of products. Therefore, the model should be seen as focusing on a case where tariffs are directed to a foreign industry with a dominant firm, where the exports from all other foreign firms can be ignored to a first approximation.

A number of compromises are made to obtain a tractable framework that can convey a set of basic results. First, trade is unilateral. This simplification is reasonable in all the dumping cases reviewed below, which involve a foreign firm exporting to the U.S. but no or negligible trade in the other direction. Second, there are no transportation costs, although they can be introduced to the foreign firm’s cost structure without altering the core analysis. Third, there is no explicit dynamics. Consequently, dumping that can arise from penetration or predatory pricing is ruled out. Any transitory dynamics in pricing is also ignored. Fourth, and relatedly, there is no uncertainty as to whether a duty or a tariff will be imposed by a domestic entity. The analysis compares three static equilibria pertaining to a free trade regime, an anti-dumping duty regime, and a tariff regime. It is assumed that a trade barrier in the form of tariff or a duty is imposed at some point in time during a free trade regime and the foreign firm alters its behavior thereafter in response to that barrier. All the mutual comparisons between different regimes pertain to "long term", that is, after all adjustments to a barrier have taken place, i.e. no temporary dynamics.

All results in the paper revolve around the following key parameters: i) the efficiency of domestic versus foreign production as reflected in the marginal costs of the firms, ii) the own price-elasticities of demand in the two countries, iii) the cross-price elasticity between the domestic and the foreign good, and iv) the number of firms in the domestic market, which determines the competitiveness of

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3See, Brander and Krugman (1983) for a model of reciprocal dumping, which involves only two firms, one in each country.
the domestic market. Dumping arises in the model solely from the differences in the two countries’ demand elasticities, which allow the foreign firm to engage in international price discrimination. The comparison of a tariff regime and the anti-dumping duty regime depends on how large the dumping margin is compared to the tariff rate. The tariff rate is related only to domestic demand elasticity, whereas the dumping margin is a function of both the domestic and the foreign demand elasticity. This key difference allows the comparison of the two tools based on how the dumping margin respond to changes in the foreign demand elasticity. In general, one tool does not dominate the other uniformly under all criteria. However, the analysis reveals that, under certain restrictions, a revenue-maximizing tariff can be shown to be superior to an anti-dumping duty under many criteria as long as the foreign demand elasticity is large enough, keeping all else constant. The predictions of the model on the direction of change in prices when a tariff or an anti-dumping duty is imposed are all also broadly in line with the existing empirical evidence.

The rest of the paper is organized as follows. In the next section, some important examples from U.S. anti-dumping cases are provided to motivate the modeling approach. Section 3 lays out the model. Section 4 analyzes the model under three regimes: free trade, anti-dumping duty and tariff. Section 5 compares the three regimes on prices, profits, revenue, industry protection and welfare. Section 6 considers some important extensions. Section 7 concludes. Proofs of results that are not obvious are deferred to Appendix A for brevity.

2 Empirical motivation

The structure of the model is inspired by a number of major U.S. anti-dumping cases such as Cemex, a major global cement producer based in Mexico. The Mexican cement industry has been highly concentrated for a long time. Cemex is the dominant producer in Mexico and it accounted for 71% of the domestic production in 1989, shortly before the filing of the dumping case against it. Cemex has grown substantially since then, and through a series of strategic acquisitions, it has secured a dominant position in the world market. Strict restrictions placed on the import of cement by the Mexican government were instrumental in Cemex’s path to dominance at home and its expansion worldwide.\(^4\)

The U.S. cement industry, on the other hand, has an oligopolistic structure, composed of a handful of mostly foreign-owned firms with several plants across the U.S. In 1990, cement producers in southern states successfully lobbied for an anti-dumping duty against Cemex, which accounted for a large amount of the cement imported to southern U.S. In contrast, the U.S. has been exporting a substantially smaller amount of cement to Mexico, so trade is virtually unilateral. The unusually low price of cement imported by Cemex has allegedly allowed Cemex to obtain a disproportionate share of the U.S. market. Producers in the southern U.S. states have accused Cemex of selling its portland cement below fair market value and they have petitioned the U.S. government for antidumping relief.

The case of Cemex is ideal in terms of the main elements motivating the model considered here: a dominant foreign firm competing with a group of domestic firms in an environment where the trade is

\(^4\)See, e.g., Chapter 8 in Spulber (2007).
almost unilateral. However, this case is by no means special, as further examples suggest.

Another recent case in point comes from the U.S. Steel industry, filed against Východné Slovenské Železiarne (VSŽ), a major steel producer in Kosice, Slovakia. Like Cemex, VSŽ is a dominant firm in its home market. In 1997, shortly before the case was filed, as the largest company in Slovakia it alone accounted for as much as 8% of the gross national product of Slovakia. It was also one of the largest Central European companies, and the second largest employer and the biggest Slovak exporter.

The preliminary determination by the Department of Commerce in the investigation of VSŽ was issued on December 28, 1999, which found that certain steel products from VSŽ were being sold in the United States at less than fair value. Subsequently, an anti-dumping duty was placed on exports from VSŽ. As in the case of Cemex, the investigation against VSŽ was petitioned by a group of U.S. steel firms. The U.S. steel industry is composed of a number of large integrated companies, some foreign-based, and a large number of smaller firms called minimills. The seven largest integrated companies accounted for 48% of the entire output in 2001. Two of the petitioners, U.S. Steel and Ispat, were ranked among the top 20 producers worldwide. As in the case of Cemex, the structure of the domestic market in this case can also be represented by competition among a small number of firms with some market power. At the time of the duty, U.S. steel exports to Slovakia was negligible compared to the U.S. imports. The U.S. exports increased sharply, however, following the acquisition of VSŽ by U.S. Steel in 2001.

A third example is the imports of color television sets from Korea. The Korean consumer electronics industry includes more than 150 small firms, but is dominated by three: Gold Star Co., Ltd., Samsung Electronics Co., Ltd., and Daewoo Electronics Co., Ltd., whose market shares in color television set production in 1988, 5 years after the initiation of the dumping duty, were 33%, 31.7%, and 17.5% respectively. At the time of the initiation of the antidumping action, trade between the U.S. and Korea was almost unilateral due to Korean restrictions on imports of color televisions. No imports were allowed before 1982, from 1982 to 1985 a recommendation of the Korean Producers Association was required for imports, which was lifted after 1986. Imports were still restricted by customs tariffs ranging from 35% to 40% after 1986, which kept the ratio of imports to domestic production less than 1% through the 1980’s and allowed the big three companies to charge near-monopoly prices in Korea. On the other hand, there was a rapid expansion in exports to the U.S. by Korean companies. Exports rose 13.9% the year before the initiation of the antidumping action and 207% during the year of initiation. Import-competing firms in the U.S. demanded anti-dumping restrictions against Korean exports as they saw their domestic market shares erode. Therefore the United States imposed the first anti-dumping duty on imports of Korean color television sets from the big three producers in 1983. At the first annual review in 1984, the duty levied on Samsung Electronics Co., Ltd. was 52.5%.

The model in the next section is based on the common features of the cases reviewed in this section.

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5The petitioning firms consisted of Bethlehem Steel Corporation, Gulf States Steel, Inc., Ispat Inland Inc., LTV Steel Company Inc., National Steel Company, Steel Dynamics, Inc., U.S. Steel Group (a unit of USX Corporation), and Weirton Steel Corporation. Also petitioning were parties with similar interests: United Steelworkers of America and Independent Steelworkers Union. For more on this case, see the Department of Commerce web page: http://www.ita.doc.gov/media/FactSheet/FactSheet1229.htm

6See Bark (1993) for the details of this case.
The environment of these cases provides a plausible framework for a tractable analysis of the issues considered.

3 The model

Consider two markets: "foreign" and "domestic". An asterisk identifies the variables and functions for the foreign market. A single firm in the foreign market exports to the domestic market served by \( N \) identical oligopolists, none of which exports to the foreign market. The foreign firm is a monopoly in the foreign market, representing the case of a dominant firm in its extreme form.

Two imperfectly substitutable goods are sold in the domestic market: the import good and the domestic good. The foreign demand for the import good is \( D(p^*) \). The domestic demand for the import good is \( D^i(p^i, p) \) and the domestic demand for the domestic good is \( D(p, p^i) \), where \( p \) and \( p^i \) are the prices of the domestic and import goods, respectively. All demand functions are twice continuously differentiable\(^7\) and satisfy

\[
\begin{align*}
D^{*'} &< 0, \\
D^i(p^i, p) &\equiv D(p^i, p), \\
D(p, p^i) &\equiv D^i(p^i, p), \\
D_1 &< 0, \\
D_2 &> 0.
\end{align*}
\]

Property (1) and first part of property (3) are standard. Property (2) imposes symmetry, which simplifies notation and analysis, but otherwise is not essential. The second part of property (3) implies that the domestic good and the import good are substitutes.

Let \( Q^i \) and \( Q \) be the total quantity of imports and the total quantity of the domestic good, respectively. The domestic oligopolists compete in a Cournot fashion by choosing outputs. To facilitate the exposure of an oligopolist’s output choice problem, a twice-differentiable inverse demand function \( P(Q, p^i) \) is assumed to exist for the domestic demand such that

\[
Q_1 < 0, \\
Q_2 > 0.
\]

The first part of (4) follows from the first part of (3). The second part implies that the domestic price strictly increases as the price of the import good increases.

The foreign firm’s marginal cost is \( c^* > 0 \) for both exports and home output. Each domestic firm also has a marginal cost \( c > 0 \).\(^8\) The analysis proceeds with general demand functions satisfying properties (1)-(4). Let \( \varepsilon^*(p) = -\frac{D^{*'}(p)p}{D^*(p)} \) and \( \varepsilon(p_1, p_2) = -\frac{D_1(p_1, p_2)p_1}{D(p_1, p_2)} \) be the own-price elasticities associated with the demand functions \( D^*(p) \) and \( D(p_1, p_2) \). Also let \( \gamma(p_1, p_2) = \frac{D_2(p_1, p_2)p_2}{D(p_1, p_2)} \) be the cross-price elasticity of \( D(p_1, p_2) \). It will be useful at times to restrict attention to the class of demand functions that exhibit the following additional properties

\[
\begin{align*}
\varepsilon^{*'} &\geq 0, \\
\varepsilon_1 &\geq 0, \\
\varepsilon_2 &\leq 0,
\end{align*}
\]

\(^7\)Except at a countable set of prices.

\(^8\)At this point, no special relationship between \( c \) and \( c^* \) is imposed, although \( c^* < c \) may be plausible for the case where the foreign technology is superior.
\[ \gamma_1 \leq 0, \gamma_2 \geq 0. \]  

(7)

These restrictions imply that the own-price elasticity is non-decreasing in own-price and the own-price elasticity is non-increasing in the price of the substitute. In addition, two main classes of demand functions will be of particular interest as they are frequently used in the trade literature. These are constant-elasticity (or iso-elastic) demand functions

\[ D^*(p) = Ap^{-\varepsilon^*}, \]

\[ D(p_1, p_2) = a p_1^{-\varepsilon p_2^\gamma}, \]  

(8)

where \( \varepsilon^*, \varepsilon, \gamma, A, a > 0 \), and linear demand functions

\[ D^*(p) = A - Bp, \]

\[ D(p_1, p_2) = a - bp_1 + dp_2, \]  

(9)

where \( A, B, a, b, d > 0 \). Note that these two classes of demand functions satisfy (1)-(7).

4 Analysis

The solution to the model is characterized under three separate regimes: free trade, anti-dumping duty, and tariff. Three distinct equilibria will be considered, each corresponding to a different regime. The first one is the equilibrium that emerges under free trade where a tariff or an anti-dumping duty is absent. Under free trade even the threat of an anti-dumping duty or a tariff is assumed away. The second and third equilibria are those that result when a trade barrier – a tariff or a duty – has actually been imposed, and all firms have responded to that barrier. The latter equilibria can emerge either when no firm anticipates a trade barrier or when all firms anticipate a trade barrier. Anticipation of a barrier may affect the prices even before the barrier is actually imposed. However, the only equilibria studied here are ex-post equilibria: those that result when a trade barrier has actually been imposed and all firms have responded to that barrier. Any temporary ex-ante equilibria that may prevail under the threat of a barrier before that barrier is imposed is not considered. The focus here is on comparing the ex-post equilibrium after a barrier is imposed with that under the complete absence of that barrier.

4.1 Free trade

Under free trade, the foreign firm maximizes its total profit

\[ \max_{p^*, p^i} \pi^*(p^*, p^i) \equiv D^*(p^*)p^* - c^* + D(p^i, p)p^i - c^*, \]

where the first term is the home profit and the second term is the export profit. A domestic firm’s output is the solution to

\[ \max_q \pi(q) \equiv q(P(Q, p^i) - c), \]
where $q$ is the quantity supplied by the firm.\footnote{A remark on the choice of prices by the foreign firm versus the choice of quantities by the domestic firms is in order. If all domestic firms compete by choosing prices, standard Bertrand result applies in the domestic market under the assumption of no product differentiation in the domestic good. Introducing a model of horizontally differentiated products, such as Hotelling’s linear city model, in the domestic sector avoids the Bertrand outcome, but does not necessarily generate a simpler analytical framework. In particular, a model with many differentiated products in the domestic sector also complicates the specification of substitutability between the import good and the domestic goods.}

To ensure unique global interior solutions that can be characterized by first order conditions, $\pi^*(p^*, p')$ is assumed to be strictly concave individually in $p^*$ and $p'$, and $\pi(q)$ is assumed to be strictly concave in $q$.\footnote{The conditions for strict concavity are standard. It can be verified that strict concavity holds for constant-elastic and linear demand functions, and for log-concave demand functions in general.} The first order conditions for the foreign firm’s problem are

$$ D^*(p^*) (p^* - c^*) + D^*(p^*) = 0, \quad (10) 
$$

$$ D_1(p^*, p') (p^* - c^*) + D(p^*, p') = 0. \quad (11) 
$$

For the domestic oligopoly, a symmetric equilibrium is considered. For a domestic firm, the first order condition for profit maximization is

$$ (P (Q, p') - c) + q P_1 (Q, p') = 0, \quad (12) 
$$

where $Q = Nq$. Let $\alpha = -\frac{QP_1}{P_1^*} = \frac{DP_1}{D^*_1}$ denote the "relative curvature" of the inverse domestic demand. A sufficient condition for the stability and uniqueness of symmetric equilibrium in the domestic market is

$$ N + 1 - \alpha > 0, \quad (13) 
$$

which is assumed to hold hereafter.\footnote{See Seade (1980a,b). Schlee (1993) shows that this condition implies both uniqueness and symmetry of the Cournot equilibrium.} The stability condition holds for all $N \geq 1$ for constant-elastic demand in (8) with $\varepsilon > 1$, and for linear demand in (9). It holds in general for all log-concave demand functions.

Let $\varepsilon^*(p^*)$ and $\varepsilon(p^*, p')$ be the own-price elasticities associated with $D^*(\cdot)$ and $D(\cdot, \cdot)$, respectively. Letting the subscript “$f$” denote free trade variables in equilibrium, the free trade equilibrium prices are

$$ p_f^* = c^* \left( \frac{\varepsilon^*(p_f^*)}{\varepsilon^*(p_f^*) - 1} \right), \quad p_f = c^* \left( \frac{\varepsilon(p_f^*, p_f^*)}{\varepsilon^*(p_f^*, p_f^*) - 1} \right), \quad p_f = c \left( \frac{N \varepsilon(p_f^*, p_f^*)}{N \varepsilon(p_f^*, p_f^*) - 1} \right). 
$$

For the prices to be positive, elasticities must satisfy $\varepsilon^*(p_f^*) > 1$, $\varepsilon(p_f^*, p_f^*) > 1$, and $\varepsilon(p_f^*, p_f^*) > 1/N$. Because $p_f^*$ is unique and does not depend on other prices, uniqueness of equilibrium under free trade requires only that the pair $(p_f, P_f')$ be unique.\footnote{It can be verified that uniqueness holds under constant-elastic and linear demand functions for the equilibria under free trade, anti-dumping duty, and tariff. See Appendix B for a formal treatment of uniqueness in these cases.}

To be able to talk about the imposition of an anti-dumping duty, dumping must prevail under free trade. According to the WTO, an agreement on the presence of dumping requires evidence on
the existence of two effects: dumping itself, and material injury to the domestic industry. In its most widely used definition, dumping occurs when the foreign firm’s price at home is greater than its export price, i.e. \( p_f^i > p_f^j \). Dumping can occur, for instance, when the foreign firm’s marginal cost for exports is sufficiently lower than its marginal cost at home, everything else constant, although such a cost difference is not expected. The empirical trade literature has generally found that exports tend to cost more, both in terms of fixed and marginal costs involved, and therefore only relatively more efficient firms are able to export.\(^{13}\) Here, the marginal cost is assumed to be the same for a good sold at home and a good exported. Furthermore, the model abstracts from any dynamics that may involve predatory or penetration pricing. In the absence of such dynamics, the only way dumping can be obtained here is when the price elasticity at home is lower than the export price elasticity, i.e. \( \varepsilon^*(p_f^i) < \varepsilon(p_f^j, p_f) \). The last inequality is assumed to hold hereafter. The free trade prices are thus those that would prevail when the demands in the two countries have different elasticities and the foreign firm engages in international price discrimination.

The second requirement, material injury, takes place, according to the amended Tariff Act of 1930, when the foreign firm exports a large enough quantity and charges an import price low enough to induce actual or potential decline in domestic price, profits, output, sales or market share.\(^{14}\) In theory, unless one defines precisely what constitutes a "material injury", any amount of foreign competition causes loss of profits for domestic firms under very general conditions. In the model, the effect of a decline in \( p_i^j \) on a domestic firm’s profit, \( \frac{d\pi(q)}{dp_i} \), is always negative.\(^{15}\) Therefore, there is an unambiguous association between industry protection as measured by domestic profits and import price. Consequently, the domestic industry is injured in terms of either output or price or both whenever there is a fall in the import price. Under certain conditions, the sales and market share of domestic firms also decrease.\(^{16}\) It will be sufficient here to assume that the free trade import price is low enough to induce material injury in terms of domestic profits. Observe also that the foreign firm’s profit always decreases as the domestic price decreases.\(^{17}\)

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\(^{13}\) See, for instance, the recent survey by Bernard, Jensen, Redding, and Schott (2007).

\(^{14}\) ITC may also consider the negative effect of imports on productivity, return on investments, capacity utilization, cash flow, inventories, employment, wages, growth, investment and the ability to raise capital. These dimensions are not considered here.

\(^{15}\) This result follows from the envelope theorem:

\[
\frac{d\pi(q)}{dp} = \frac{\partial \pi(q)}{\partial q} \frac{\partial q}{\partial p} + \frac{\partial \pi(q)}{\partial p} = \frac{\partial \pi(q)}{\partial p} = qP_2 > 0.
\]

\(^{16}\) It can be shown that for constant-elastic demand domestic output, sales, and market share decline and domestic price does not change as import price falls. For linear demand, domestic output, sales, price and market share all decline.

\(^{17}\) Again, by the envelope theorem: \( \frac{\partial \pi(p^*, p)}{\partial p} = (p^i - c^*)D_2 > 0 \).
4.2 Anti-dumping duty

When an anti-dumping duty is imposed, the foreign firm must pay a duty per unit exported, equal to \( p^* - p^i \) as long as \( p^* > p^i \). The foreign firm’s problem under the duty is then

\[
\max_{p^*, p^i} \pi^*(p^*, p^i) \equiv D^*(p^*)(p^* - c^*) + D(p^i, p)(p^i - c^*) - I(p^* > p^i)D(p^i, p)(p^* - p^i),
\]

where \( I(\cdot) \) is the indicator function. A domestic firm’s problem is the same as in free trade. To ensure unique interior solutions for the case where dumping prevails under the duty, i.e. when \( I(p^* > p^i) = 1 \), \( \pi^*(p^*, p^i) \) is assumed to be jointly strictly concave in \((p^*, p^i)\). Individual strict concavity in \( p^* \) and \( p^i \) is also assumed for this case to facilitate the analysis.\(^{18}\)

The system of first order conditions, assuming that dumping prevails under the duty, is\(^{19}\)

\[
\begin{align*}
D''(p^*)(p^* - c^*) + D^*(p^*) - D(p^i, p) &= 0, \quad (15) \\
D_1(p^i, p)(2p^i - c^* - p^*) + 2D(p^i, p) &= 0, \quad (16) \\
(P(Q, p^i) - c) + qP_1(Q, p^i) &= 0. \quad (17)
\end{align*}
\]

As opposed to the case of free trade, all three prices are now interrelated because the foreign firm is now penalized for the gap between its home and export prices.

From (15), it can be verified that if the foreign firm dumps under the duty, it must hold that \( D^*(p^*) > D(p^i, p) \), i.e. the foreign firm’s home demand must exceed the demand for its exports. An increase in \( p^* \) has three effects on the foreign firm’s profits: a decrease in profits due to decreasing demand at home represented by the first term in (15), an increase in profits at home due to an increase in markup represented by the second term in (15), and a decrease in export profits due to increasing dumping margin represented by the last term in (15). Because the first and third effects are negative and the three effects must balance each other out, the magnitude of the second effect must exceed that of the third. As a result, when the duty is in place, the domestic firm lowers its price to a level at which the foreign firm’s domestic output exceeds exports.\(^{20}\)

Let \( \lambda(p) = \frac{D(p^i, p)}{D^*(p^i)} \) be the foreign firm’s exports-to-home-output ratio, where \( p = \{p^*, p^i, p\} \). The preceding discussion implies \( \lambda(p) < 1 \) for (15) to hold. Using the subscript “ad” to denote the equilibrium

\(^{18}\)Strict concavity in \( p^* \) is guaranteed under the concavity of home profits under free trade, which was assumed earlier. Strict concavity in \( p^i \) requires for all \( p^* \geq p^i > c \)

\[
D_{11}(2p^i - p^* - c) + 4D_1 < 0,
\]

which is ensured by the strict concavity in \( p^i \) under free trade. Joint concavity requires the Hessian of \( \pi^*(p^*, p^i) \) to be negative-definite, which is equivalent to

\[
(D''''(p^* - c^*) + 2D'''(2p^i - c^* - p^*) + 4D_1) - D_1^2 > 0,
\]

for all \( p^* \geq p^i > c \) and for any given \( p \). Concavity conditions are satisfied by constant-elasticity demand functions.

\(^{19}\)If there is no dumping when the duty is in place, the system of first order conditions is identical to that under free trade.

\(^{20}\)However, there need not be any definitive relationship between the foreign output and the exports when there is dumping but no duty is in place.
variables under the anti-dumping duty, the foreign firm’s prices can be written as

\[ p^{*\text{ad}} = c^*(\frac{\varepsilon^*(p^{*\text{ad}})}{(\varepsilon^*(p^{*\text{ad}}) - (1 - \lambda(p^{*\text{ad}})))}, \tag{18} \]

\[ p^{i\text{ad}} = c^*(\frac{\varepsilon(p^{i\text{ad}}, p^{ad})}{\varepsilon(p^{i\text{ad}}, p^{ad}) - 1}) \left(\frac{2\varepsilon^*(p^{*\text{ad}}) - (1 - \lambda(p^{*\text{ad}}))}{2\varepsilon^*(p^{*\text{ad}}) - 2(1 - \lambda(p^{*\text{ad}}))}\right), \tag{19} \]

and the domestic price is

\[ p_{\text{ad}} = \frac{N\varepsilon(p_{\text{ad}}, p^{i\text{ad}})}{N\varepsilon(p_{\text{ad}}, p^{i\text{ad}}) - 1}. \tag{20} \]

The dumping margin is

\[ \frac{p^{*\text{ad}} - p_{\text{ad}}^i}{p^{i\text{ad}} - p_{\text{ad}}} = \frac{\varepsilon(p^{i\text{ad}}, p^{ad})(1 - \lambda(p^{ad})) - 2\varepsilon^*(p^{*\text{ad}})}{\varepsilon(p^{i\text{ad}}, p^{ad}) (2\varepsilon^*(p^{*\text{ad}}) - (1 - \lambda(p^{*\text{ad}})))}. \tag{21} \]

Because the dumping margin must be non-negative, it must hold that \( \frac{\varepsilon(p^{i\text{ad}}, p^{ad})(1 - \lambda(p^{ad})) - 2\varepsilon^*(p^{*\text{ad}})}{\varepsilon(p^{i\text{ad}}, p^{ad}) (2\varepsilon^*(p^{*\text{ad}}) - (1 - \lambda(p^{*\text{ad}})))} \geq \frac{2}{1 - \lambda(p^{*\text{ad}})} \), i.e. when the duty is in place the export demand must be more than twice as elastic as the home demand. Under constant-elastic demand, this condition implies that even under free trade the import demand must be sufficiently more elastic than the home demand.

An important question is whether the foreign firm continues to dump after the duty is imposed.

**Proposition 1** The foreign firm continues to dump (weakly) under the anti-dumping duty, i.e. \( p^{*\text{ad}} \geq p^{i\text{ad}} \), where \( p^{*\text{ad}} = p^{i\text{ad}} \) if and only if \( \frac{\varepsilon(p^{i\text{ad}}, p^{ad})}{\varepsilon^*(p^{*\text{ad}})} = \frac{2}{1 - \lambda(p^{*\text{ad}})}. \)

The result in Proposition 1 follows because if the firm does not dump when the duty is present, it would not have dumped under free trade to start with. In other words, no dumping under the duty implies that the profit maximizing choices of prices are in a region where the firm chooses not to dump. But these choices must then be precisely those that must maximize profits under free trade, a contradiction with the fact that the firm is assumed to dump under free trade.

It can also be shown that in response to the duty the foreign firm lowers its home price and raises its export price, thereby lowering its dumping margin.

**Proposition 2** The foreign price is lower and the import price is higher under anti-dumping duty compared to free trade, i.e. \( p_f^* > p_{\text{ad}}^i \) and \( p_{\text{ad}}^i < p_f^i \). The dumping margin is lower compared to free trade.

It is important to note that the foreign firm responds to the anti-dumping duty by adjusting both prices, not just one. Depending on the relative elasticities of the foreign and import demands and the sizes of the two demands, the firm can end up changing the two prices only a little or by a large amount. When the import demand in the domestic country is very small, \( \lambda(p) \) is close to zero and the firm lowers its home price only slightly, as can be verified from a comparison of the expressions for \( p_f^* \) and \( p_{\text{ad}}^i \). On the other hand, if the import demand is very high, i.e. \( \lambda(p) \) is large, then the firm lowers its home price more, whereas its import price rises little.
Proposition 2 lines up with the evidence cited in Blonigen and Park (2004), which suggests that a large percentage of dumping margins decline in response to a anti-dumping dumping duty. There is also evidence that the home price of the foreign firm declines in response to an anti-dumping duty. A good example is the case of color TV sets from Korea discussed earlier. Bark (1993) reports that there was no trend for domestic TV prices in Korea before the U.S. antidumping case between 1980 and 1983. But when the Korean companies began to adjust to reduce the bite of the antidumping order, domestic prices began to fall.

Foreign output unambiguously falls when a duty is imposed. The effect of the duty on the quantity of imports and domestic output depends on the elasticities. The direct effect of an increase in import price due to a duty is a decline in imports. However, if the domestic price rises sufficiently in response, imports can increase. Thus, depending on the relative magnitudes of own and cross-price elasticities, imports and domestic output can increase or decrease.

4.3 Tariffs

Consider now the introduction of a per unit ad-valorem tariff, \( \tau \in [0, 1) \). The foreign firm’s problem becomes

\[
\max_{p^*, p^j} \pi^*(p^*, p^j) \equiv D^*(p^*) (p^* - c^*) + D(p^j, p)(p^j - c^* - \tau p^i).
\]

Free trade corresponds to the case \( \tau = 0 \). A domestic firm’s problem is the same as in free trade. As before, for a unique interior solution to the foreign firm’s problem, \( \pi^*(p^*, p^j) \) is assumed to be strictly concave in \( p^j \).

The first order conditions now become

\[
D^*(p^*) (p^* - c^*) + D^*(p^*) = 0,
\]

\[
D_1(p^j, p)(p^j - c^* - \tau p^i) + D(p^j, p)(1 - \tau) = 0,
\]

\[
(P(Q, p^j) - c) + qP_1(Q, p^j) = 0,
\]

and the equilibrium prices are

\[
p^*(\tau) = c^* \left( \frac{\varepsilon^*(p^*(\tau))}{\varepsilon^*(p^*(\tau)) - 1} \right),
\]

\[
p^j(\tau) = c^* \left( \frac{\varepsilon(p^j(\tau), p(\tau))}{\varepsilon(p^j(\tau), p(\tau)) - 1} \right) \frac{1}{1 - \tau},
\]

\[
p(\tau) = c \left( \frac{N \varepsilon(p(\tau), p^j(\tau))}{N \varepsilon(p(\tau), p^j(\tau)) - 1} \right).
\]

\(21\) Strict concavity requires for \( \tau > 0 \)

\[
D_{11}((1 - \tau)p^j - c^*) + 2D_1(1 - \tau) < 0,
\]

for all \( p^j \) and for any given \( p \). The derivative of the left hand side with respect to \( \tau \) is \(-D_{11}p^j - 2D_1\). The strict concavity under free trade (at \( \tau = 0 \)) assumed earlier implies \( D_{11}(p^j - c^*) + 2D_1 < 0 \). Therefore, a sufficient condition for strict concavity under tariff to hold is \(-D_{11} - 2D_1 < 0 \). This last condition holds for both constant-elastic and linear demand functions.
Note that \( p^*(\tau) = p_f \), regardless of the magnitude of tariff. Consequently, foreign output does not change in response to a tariff. Whether the import price and domestic price rise in response to a tariff depends on the elasticity of the import and domestic demand. As in the case of a duty, a change in import price due to an increase in tariff has the direct effect of reducing imports, but if the induced change in domestic price is positive and large enough, imports can increase. It is possible to characterize the responses of the prices and the outputs to changes in tariff. Total differentiation of (25) and (26) with respect to \( \tau \), and the derivatives of the import and domestic market clearing conditions with respect to \( \tau \) yield the following result.

**Proposition 3** Under (6) and (7),

i) Import price is strictly increasing and domestic price is non-decreasing in tariff,

ii) An increase in tariff leads to a decline in imports if and only if

\[
\frac{d \ln p(\tau)}{d \ln p^i(\tau)} < \frac{\varepsilon(p^i(\tau), p(\tau))}{\gamma(p(\tau), p^i(\tau))}.
\]

iii) An increase in tariff leads to a rise in domestic output if and only if

\[
\frac{d \ln p(\tau)}{d \ln p^i(\tau)} < \frac{\gamma(p(\tau), p^i(\tau))}{\varepsilon(p(\tau), p^i(\tau))}.
\]

The responses of prices and outputs to a change in tariff can be characterized explicitly for constant-elastic and linear demand in (8) and (9), respectively.

**Proposition 4** i) If the demand functions are constant-elastic, an increase in tariff leads to an increase in import price, no change in domestic price, an increase in domestic output and a decline in exports, regardless of the number of domestic firms, \( N \).

ii) If the demand functions are linear, an increase in tariff leads to an increase in domestic and import prices, and an increase in domestic output. Exports fall as tariff increases as long as \( N \geq 3 \), but may increase or decrease if \( N \leq 2 \). If \( d > \sqrt{3b} \), exports increase when \( N \leq 2 \).

iii) When the demand functions are linear, the magnitudes of the responses of domestic and import prices and domestic output characterized in (ii) become smaller as \( N \) increases, whereas the magnitude of the response of exports becomes larger. When the number of domestic firms is very large, i.e. \( N \to \infty \), domestic price does not change, import price increases, domestic output increases, and imports fall in response to an increase in tariff.

The difference in the responses across the two classes of demand functions arise from the fact that when the demand has constant elasticity, a change in import price has no effect on the elasticity of the domestic demand, whereas in the case of linear demand the elasticity of the domestic demand changes. The sufficient condition in part (ii) of Proposition 4 is intuitive. The quantity of imports can increase only when the domestic demand is sufficiently responsive to changes in import price, i.e. when \( d \) is large enough. When there is a small number of firms in the domestic market, the responsiveness of the domestic demand to changes in import price is higher: a given amount of change in tariff induces a
larger change in domestic demand. Coupled with a high cross-price elasticity, imports can thus increase even as the import price goes up.

Part (iii) of Proposition 4 pertains to the case of an interaction between a dominant foreign firm and an (approximately) competitive domestic industry. When there is a large number of domestic firms, the domestic industry’s behavior approximates that of a competitive industry in the sense that price tends to marginal cost. In that case, a shift in domestic demand due to an increase in tariff has no effect on domestic price. Therefore, imports must fall as a result of the increase in import price.

4.3.1 The revenue- and welfare-maximizing tariffs

Two important tariffs are the one that maximizes the government revenue and the "optimal" tariff that maximizes domestic welfare, both of which are frequently used in the literature. After the investigation of the analytically simpler revenue-maximizing tariff first, the welfare maximizing-tariff will be discussed.

The sequence of moves in the tariff-setting game is assumed to be of the Stackelberg form. The government leads by setting a tariff. The foreign firm and the domestic firms then follow simultaneously by choosing their prices in response to the tariff. The government sets the tariff anticipating that the firms will respond optimally to the tariff. The revenue-maximizing tariff is the solution to

\[
\max_{\tau \in (0,1)} R(\tau) \equiv \tau p^i(\tau) D(p^i(\tau), p(\tau)).
\]

For tractability, \( R(\tau) \) is assumed to have a unique interior maximizer \( \tau^* \in (0,1) \), strictly increasing over \( (0, \tau^*) \), and strictly decreasing over \( (\tau^*, 1) \). These properties hold for constant-elastic demand and for linear demand, except for one special case.\(^{22}\) Note also that \( R(0) = 0 \), and it can be shown that \( \lim_{\tau \to 1} R(\tau) = 0 \) when import demand is constant-elastic with \( \varepsilon > 1 \) or when it is linear.

The revenue-maximizing tariff satisfies the first order condition

\[
p^i(\tau) D(p^i(\tau), p(\tau)) + \tau |p^i(\tau) D(p^i(\tau), p(\tau))| + p^i(\tau) p^i(\tau) D(p^i(\tau), p(\tau)) + p^i(\tau) D_2(p^i(\tau), p(\tau)) p'(\tau) = 0,
\]

which yields the implicit solution for \( \tau^* \) as

\[
\tau^* = \frac{1}{(\varepsilon p^i(\tau^*), p(\tau^*)) - 1} \left( \frac{d \ln p^i(\tau)}{d \tau} \right)_{\tau=\tau^*} - \gamma(p^i(\tau^*), p(\tau^*)) \left( \frac{d \ln p(\tau)}{d \tau} \right)_{\tau=\tau^*}.
\]

Under the case where demand is constant-elastic, the revenue-maximizing tariff simplifies to

\[
\tau^* = \frac{1}{\varepsilon}.
\]

In the case of linear demand, however, the expression for \( \tau^* \) is complicated.\(^{24}\)

\(^{22}\)The only case that does not satisfy these properties is linear demand with \( N \leq 2 \) and \( d > \sqrt{3b} \). In this case both \( p'(\tau) \) and \( D(p^i(\tau), p(\tau)) \) are strictly increasing in \( \tau \) and the government can increase revenues by setting \( \tau \) arbitrarily close to 1.

\(^{24}\)In the case of linear demand, the revenue-maximizing is one of the roots of a 3rd degree polynomial in \( \tau \).
The optimal tariff, $\tau^o$, is defined as the tariff that solves the following problem

$$\max_{\tau \in (0,1)} W(\tau) \equiv N \pi(q(\tau)) + \int_{p(\tau)}^{\infty} D(p, p'(\tau))dp + \int_{p'(\tau)}^{\infty} D(p, p(\tau))dp + R(\tau),$$

(29)

where the first term is the profit in the domestic sector, the second and third terms are consumer surpluses from domestic and foreign consumption, and the fourth term is the government revenue. For tractability, it is assumed that the objective function in (29) has a unique interior maximizer $\tau^o$, strictly increasing over $(0, \tau^o)$, and strictly decreasing over $(\tau^o, 1)$.

While an explicit derivation of $\tau^o$ is difficult, an indirect approach can be used to compare it with $\tau^r$. The derivative of $W(\tau)$ evaluated at $\tau = \tau^r$ can be written as

$$Q'(\tau^r)(p(\tau^r) - c) - p''(\tau^r)Q'(\tau^r) + \left( \int_{p(\tau^r)}^{\infty} D_2(p, p'(\tau^r))dp \right) p''(\tau^r) + \left( \int_{p'(\tau^r)}^{\infty} D_2(p, p(\tau^r))dp \right) p'(\tau^r).$$

Because the last two terms are non-negative, a sufficient condition for $W'(\tau^r) > 0$ or, equivalently, $\tau^o > \tau^r$, is that the first two terms add up to a positive number. This requires

$$Q'(\tau^r)(p(\tau^r) - c) > p''(\tau^r)Q'(\tau^r),$$

(30)

which essentially states that the marginal increase in total profit from the domestic good due to demand expansion must overcome the marginal decline in the sales of the foreign good due to a higher import price. As discussed earlier, in certain cases the domestic demand can decline when a tariff is imposed. In such cases, the first two terms are negative and a more general condition that ensures $W'(\tau^r) > 0$ is that the last two terms are larger than the first two in absolute value.

### 4.3.2 Some special tariffs

Consider now some special tariffs that facilitate the comparison of tariff and anti-dumping duty regimes. The first one is the tariff $\tau^{\text{pro}}$ that gives the same protection as the anti-dumping duty. Industry protection is measured by the profit of a domestic firm, which is strictly increasing in import price. In other words, the domestic industry is better protected as import price increases. Because a domestic firm’s profit is strictly decreasing in tariff, $\tau^{\text{pro}}$ is the unique solution to $p'(\tau^{\text{pro}}) = p'_{ad}$, which yields, using (19) and (25),

$$\tau^{\text{pro}} = \frac{1 - \lambda(p_{ad})}{2\varepsilon(p_{ad}) - (1 - \lambda(p_{ad}))}.$$  

(31)

The second special tariff, $\tau^{\text{rev}}$, is the one that gives the same revenue to the government as the anti-dumping duty

$$R(\tau^{\text{rev}}) = R_{ad} = D(p_{ad}^*, p_{ad})(p_{ad}^* - p_{ad}^i).$$

In general $\tau^{\text{rev}}$ may not exist, and if it exists, it may not be unique. Because $R(\tau)$ is continuous, has a unique interior maximizer, and $R(0) = 0$, the existence of $\tau^{\text{rev}}$ requires $R(\tau^r) \geq R_{ad}$. If, in addition,
\[ \lim_{\tau \to 1} R(\tau) < R_{ad}, \] then there must exist two distinct values \( \tau_1^{rev} \) and \( \tau_2^{rev} \) such that \( 0 < \tau_1^{rev} < \tau < \tau_2^{rev} \).

Finally, consider the tariff, \( \tau^{ad} \), that provides the same profit to the foreign firm as under the duty. Let \( \pi_f, \pi_{ad}, \pi(\tau) \) be the foreign firm’s equilibrium profits under free trade, anti-dumping duty, and tariff \( \tau \), respectively. Because the firm continues to dump under the duty, it must hold that

\[ \pi_{ad} > \lim_{\tau \to 1} \pi(\tau) = D^*(p_f^*)(p_f - c^*), \]

for otherwise the firm would have chosen not to export under the duty. Moreover, since \( \pi(\tau) \) is continuous and strictly decreasing and \( \pi(0) = \pi_f \geq \pi_{ad} \), there exists a unique tariff rate, \( \tau^{ad} \in (0, 1) \) such that \( \pi(\tau^{ad}) = \pi_{ad} \).

Next, some of the special tariff rates defined so far will be compared. This comparison obviously depends on the relative magnitudes of the domestic and foreign demand elasticities. The following analysis is based on the observation that the revenue-maximizing and optimal tariff do not depend on the elasticity of the foreign demand at all, whereas the anti-dumping duty depends on that elasticity. Keeping the own-price and cross-price elasticities for the domestic and import demand constant, a change in the foreign demand elasticity affects all tariff rates \( \tau^{pro}, \tau^{rev} \) and \( \tau^{ad} \), but not \( \tau^r \) or \( \tau^o \). Comparative statics with respect to the foreign demand elasticity is thus simpler than considering changes in the domestic and import elasticities, which affect all tariff rates.

To formalize, consider the parameterized version of the foreign demand elasticity, \( \varepsilon^* (\cdot; \Theta^*) \), where \( \Theta^* \) is a parameter vector.\(^{26}\) Suppose that for some \( \theta^* \in \Theta^* \), it holds that

\[ \frac{\partial \varepsilon^* (\cdot; \Theta^*)}{\partial \theta^*} > 0. \tag{32} \]

Assumption (32) also implies that the total derivative is also positive, i.e. \( \frac{d \varepsilon^* (p_{ad}^*; \Theta^*)}{d \theta^*} > 0. \tag{27} \) The following can be established using (32).

**Proposition 5** Assume (5)-(7). Given any set of values for parameters other than \( \theta^* \),

i) If

\[ \frac{d \varepsilon^* (p_{ad}^*; \theta^*)}{d \theta^*} < \frac{d \varepsilon^* (p_{ad}^*)}{d \theta^*}, \tag{33} \]

then there exists a unique \( \theta^*_R \) such that \( R(\tau^r) > R_{ad} \) and \( \tau_1^{rev} < \tau^r < \tau_2^{rev} \) for \( \theta^* > \theta^*_R \).

ii) If the reverse of (33) holds, there exists a unique \( \theta^{pro}_* \) such that \( \tau^r > \tau^{pro} \) and \( \tau_2^{rev} > \tau^{pro} \) for \( \theta^* > \theta^{pro}_* \).

In other words, when the foreign demand is sufficiently elastic, a revenue-maximizing tariff can dominate an anti-dumping duty in revenue and protection. The condition (33) requires that the elasticity

\(^{26}\)For instance, for the constant-elasticity demand functions \( \Theta^* = \{\varepsilon^*\} \), and for linear demand functions \( \Theta^* = \{A, B\} \).

\(^{27}\)That is, when the elasticity is higher at all price levels, the firm must choose a price where the elasticity is higher than the elasticity at the optimal price that was chosen under the case with lower elasticity. Otherwise the firm could have achieved the same level of elasticity when \( \theta^* \) was lower by choosing a higher price and that would have increased the profits. For constant-elastic demand, setting \( \theta^* = \varepsilon^* \), and for linear demand functions setting \( \theta^* = B \) achieves the desired monotonicity for the foreign demand.
of the foreign demand increase faster than the elasticity of the import demand as $\theta^*$ increases. This condition ensures that the gap between foreign and import prices decreases as $\theta^*$ increases. Part (i) may still hold without this sufficient condition. The reverse of condition (33), on the other hand, implies that the import price continues to fall faster than the foreign price and the gap between the foreign and import prices increases as $\theta^*$ increases. In this case, the revenue-maximizing tariff provides more protection when the foreign demand is sufficiently elastic. A revenue-equivalent tariff, $\tau^{rev}_{2}$, is also superior to an anti-dumping duty in terms of protection under these conditions. Once again, the reverse of condition (33) is only sufficient for Part (ii), and the result may still hold when it is not satisfied.

For the case of the constant-elastic demand, the following relationships can be stated.

**Proposition 6** If the demand functions are constant-elastic,

- i) $R(\tau^r) > R_{ad}$ and $\tau^{rev}_{1} < \tau^r < \tau^{rev}_{2}$,
- ii) $\tau^r < \tau^{pro}$,
- iii) If $\frac{\varepsilon^2}{4\varepsilon - 2} < \varepsilon^*$, then $\tau^{pro} > \tau^{rev}_{2}$.

5 Comparison of the three regimes

Now that the solutions to the model under the three regimes are characterized, the effects of the two policy tools on key variables can be analyzed. The propositions stated below apply to revenue-maximizing tariff, $\tau^r$, but they are also valid for the optimal tariff, $\tau^o$, provided that $\tau^o$ exists and $\tau^o < \tau^r$.

5.1 Prices

By Proposition 2, the import price is always strictly higher under the duty than that under free trade, and the foreign price is always strictly lower. By Proposition 3, the import price under tariff is always strictly higher than that under free trade, whereas the foreign price is the same under both regimes. Domestic prices are weakly higher under a duty or a tariff compared to the free trade regime. Overall, a tariff does not affect consumption or production in the foreign market, but an anti-dumping duty does.

The comparison of the import and domestic prices under a tariff versus a duty depends on how much the foreign firm raises its export price over its free trade level in response to a duty or a tariff. It was shown in the proof of Part (i) of Proposition 5 that $p^{i}_{ad}$ strictly increases as $\theta^*$ increases. Therefore, the following result is immediate.

**Proposition 7** Assume (5)-(7). Given any set of values for parameters other than $\theta^*$, $p^{i}_{ad} > p^{i}(\tau^r)$ and $p_{ad} \geq p(\tau^r)$ for $\theta^* < \theta^{pro}_{opt}$ as long as (33) holds.

Under constant-elastic demand, the fact that $\tau^{pro} > \tau^r$ implies $p^{i}_{ad} = p^{i}(\tau^{pro}) > p^{i}(\tau^r)$, i.e. the antidumping duty leads to a higher import price than the optimal tariff. On the other hand, domestic prices are the same: $p_{ad} = p(\tau^r)$. Because neither the duty nor the optimal tariff affects the domestic
price, the net effect is a higher reduction in import demand under the duty compared to the case of the revenue-maximizing tariff.

5.2 The tariff rate and the dumping margin

The dumping margin exceeds the revenue-maximizing tariff rate if

\[ \frac{p_{ad}^*}{p_{ad}^i} - 1 > \tau^r. \tag{34} \]

It is easy to verify that the left hand side of (34) is strictly increasing in \( \theta^* \) when

\[ \frac{d \ln p_{ad}^i}{d \ln p_{ad}^*} > 1, \tag{35} \]

In other words, a 1% decrease in \( p_{ad}^* \) must be associated with more than a 1% decrease in \( p_{ad}^i \) for the dumping margin to exceed the revenue-maximizing tariff. Thus, under (35), when the foreign demand elasticity is sufficiently high, the dumping margin exceeds the revenue-maximizing tariff rate.

**Proposition 8** If (35) holds, then given any set of values for parameters other than \( \theta^* \), there exists some \( \theta_d^* \) such that the dumping margin exceeds the revenue-maximizing tariff rate as long as \( \theta^* > \theta_d^* \).

Note that condition (35) implies the reverse of condition (33), but not vice versa. Thus, condition (35) is more general than condition (33), and also guarantees that the revenue-maximizing tariff provides more protection than the anti-dumping duty as long as \( \theta^* > \theta_{pro}^* \).

For the constant-elastic demand, (34) simplifies to

\[ \frac{2 \varepsilon^*}{2 \varepsilon^* - (1 - \lambda(p_{ad}))} > \frac{1}{\varepsilon - 1}, \]

which holds because the left hand side is greater than one and the right hand side is less than 1, as \( \varepsilon > 1 \). As a result, under constant-elastic demand the dumping margin is always higher than the revenue-maximizing tariff rate. The gap between the dumping margin and the tariff rate increases as \( \varepsilon \) gets larger or as \( \varepsilon^* \) gets smaller. Recent evidence suggests that dumping margins are several times higher than the tariff rates. On average, anti-dumping duties are 10 to 20 times higher than the tariff levels.\(^{28}\) Proposition 8 implies that such margins emerge in the model for general demand functions when the foreign demand elasticity is relatively high compared to the elasticity of import demand.

5.3 Industry protection

The profit of a domestic firm is the main measure of industry protection. Because domestic firms are identical, this measure is also the average profit in the domestic industry. Alternatively, one can use the market share of a domestic firm or all domestic firms as a measure of protection. Under certain

\(^{28}\) See Prusa (2001).
The domestic industry’s market share is higher the higher the profit of a domestic firm, so using the profit of a domestic firm to measure protection is not very restrictive.\(^{29}\)

Clearly, the free trade regime offers the minimum industry protection. While the revenue-maximizing tariff ensures the highest revenue to the government, it does not always provide as much protection as the anti-dumping duty. An immediate implication of Proposition 7 is that the protection is strictly higher under the anti-dumping duty as long as the foreign demand elasticity is sufficiently low. For constant elastic demand in particular, the fact that \(p_{ad} > p^i(\tau^r)\) implies that the domestic industry is less protected under the revenue-maximizing tariff than under the anti-dumping duty.

As discussed earlier, if there exists two tariffs such that \(0 < \tau_1^{rev} < \tau^r < \tau_2^{rev}\), then the government can achieve the same revenue as in the case of anti-dumping duty by choosing either \(\tau_1^{rev}\) or \(\tau_2^{rev}\). However, if the government’s objective is to ensure the highest possible industry protection subject to maintaining the same revenue as in the case of the anti-dumping duty, the tariff \(\tau_2^{rev}\) is superior to \(\tau_1^{rev}\), because \(p^i(\tau_1^{rev}) < p^i(\tau_2^{rev})\). If \(\tau^{rev}\) is unique, implementing it offers the same revenue as the duty, but less protection than the revenue-maximizing tariff and the duty. Finally, in cases where \(\tau^{rev}\) does not exist the revenue-maximizing tariff is inferior to the anti-dumping duty from a revenue standpoint.

Note also that it is possible that \(\tau_2^{rev} > \tau^{pro}\). In that case, \(\tau_2^{rev}\) provides more protection than the anti-dumping duty. Thus, in certain cases one can implement a tariff that not only provides the same revenue as the anti-dumping duty but also one that offers strictly more protection.

### 5.4 Revenue

Part (i) of Proposition 5 identified a condition for the tariff revenue to exceed anti-dumping revenue: the foreign demand elasticity must be sufficiently high. When the demand is constant-elastic, the tariff revenue is always higher, as implied by the first part of Proposition 6. Thus, the government strictly prefers the revenue-maximizing tariff to the anti-dumping duty in terms of revenue under constant-elastic demand.

### 5.5 Profits of the foreign firm

It is straightforward to see that the foreign firm’s profit is highest under free trade. Because a tariff leaves foreign firm’s free-trade profit at home unchanged, the foreign firm’s profit at home is always strictly higher under the optimal tariff than under the anti-dumping duty. On the other hand, profit from exports under the optimal tariff can be higher or lower than that under the duty. Because the export profit under the optimal tariff depends only on the elasticity of the import demand, while the export profit under the duty depends on the elasticities of both home and import demands, which profit

\[\text{profit} = \frac{pq}{Npq + p^iQ^i}.\]

It can be shown that, as \(p^i\) increases, \(pq\) increases and \(p^iQ^i\) declines when demand is constant-elastic. In the case of linear-demand, the same conclusion holds as long as \(N \geq 3\). In both cases, a domestic firm’s profit increases.
is higher depends on the relative magnitudes of the two elasticities. By the envelope theorem, the derivative of the foreign firm’s profit with respect to $\theta^*$ under the anti-dumping duty is

$$\frac{\partial \pi_{ad}}{\partial \theta^*} = \frac{\partial D^s(p_{ad}^*)}{\partial \theta^*}(p_{ad}^* - c^*) + D_2(p_{ad}^*, p_{ad})(2p_{ad}^* - p_{ad} - c^*)y.$$ 

Because $y < 0$, the last term is negative. Thus a sufficient condition for $\frac{\partial \pi_{ad}}{\partial \theta^*} < 0$ to hold is that $\frac{\partial D^s(p_{ad}^*)}{\partial \theta^*} < 0$. The latter condition holds, for instance, when the demand is constant-elastic, or when it is linear and the parameter of interest is $\theta^* = B$. The following can now be stated.

**Proposition 9** Assume (5)-(7) and $\frac{\partial D^s(p_{ad}^*)}{\partial \theta^*} < 0$. Then, given any set of values for parameters other than $\theta^*$, there exists a unique $\theta^*_\pi$ such that $\pi(\tau^r) > \pi_{ad}$ and $\tau^r > \tau^{ad}$ for $\theta^* > \theta^*_\pi$.

Proposition 9 identifies a region where the foreign firm and the domestic government both prefer the revenue-maximizing tariff over a duty. When the foreign demand elasticity is sufficiently high, the two incentives line up. In addition, if the government’s objective is industry protection rather than revenue generation, the government prefers the duty over the tariff, as a consequence of Proposition 6. In that case, the foreign firm and the government have conflicting interests. Also, if the sufficient condition in Proposition 9 does not hold, the government and the foreign firm may have conflicting interests: the government prefers the tariff over a duty, but the firm may not.

In the special case of constant-elastic demand, the fact that $p_{ad}^* > p_{ad}$ implies that the exports are lower under the duty. Because the tariff duty paid by the firm is greater than the anti-dumping duty by Proposition 6, the foreign firm can make higher or lower profit under the tariff regime. The following proposition identifies how elastic foreign demand needs to be for Proposition 9 to hold in the case of constant-elasticity demand.

**Proposition 10** When the demand functions are constant-elastic, $\pi(\tau^r) > \pi_{ad}$ and $\tau^{ad} > \tau^r$ if $\epsilon^* > \frac{\epsilon (\epsilon + 1)}{\delta + 1}$, provided that $\epsilon^* < \frac{\epsilon (1 - \lambda(p_{ad}))}{2}$.

### 5.6 Welfare

For the foreign country, welfare is simply the sum of consumer surplus and producer surpluses. For the domestic country, welfare has three components: consumer surplus, producer surplus, and government revenue. Consider the case of a tariff first. When a tariff is imposed, the foreign country’s welfare unambiguously declines, as the foreign firm’s profit becomes lower and consumer surplus does not change. In the domestic market consumer surplus can increase or decrease. Both the domestic and import prices are now higher, which implies lower consumer surplus, but depending on the cross-elasticity of the import demand, the quantity of imports may also increase, possibly leading to higher surplus. The domestic firms make higher profit, leading to higher producer surplus. The government tariff revenue provides an additional source of welfare.

Consider next the anti-dumping duty. When a duty is imposed under free trade, consumer surplus in the foreign country increases as the foreign price becomes lower. But the foreign firm’s profit is also lower compared to free trade. Overall, welfare in the foreign country can be lower or higher compared
to free trade. In the domestic market, both prices are higher. As in the case of tariffs, whether total consumer surplus falls depends again on the cross-elasticity of the import demand. Domestic firms’ profits are higher and the government makes a positive revenue. Table 1 summarizes the component-wise changes in welfare moving from a free trade regime to a tariff or a duty regime in the foreign and the domestic country.

<table>
<thead>
<tr>
<th>Welfare change</th>
<th>Tariff</th>
<th>Anti-dumping Duty</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Component</strong></td>
<td>Foreign</td>
<td>Domestic</td>
</tr>
<tr>
<td>Consumer</td>
<td>0</td>
<td>–/+</td>
</tr>
<tr>
<td>Producer</td>
<td>–</td>
<td>+</td>
</tr>
<tr>
<td>Government</td>
<td>NA</td>
<td>+</td>
</tr>
<tr>
<td>Total</td>
<td>–</td>
<td>–/+</td>
</tr>
</tbody>
</table>

Table 1. The components of welfare change vis a vis free trade

Table 1 makes it clear that the welfare can go either way in the domestic country under a tariff or a duty. To analyze further the change in welfare when a tariff or duty is imposed, consider the welfare under anti-dumping duty

\[
W_{ad} = N\pi(q_{ad}) + \int_{p_{ad}}^{\infty} D(p, p_{ad}) dp + \int_{p_{ad}}^{\infty} D(p, p_{ad}) dp + R_{ad}.
\]

It was shown in Proposition 5 Part (i) that as the foreign demand becomes more elastic, the tariff revenue \(R(\tau)\) exceeds the anti-dumping duty revenue \(R_{ad}\). Thus, a sufficient condition for \(W_{ad}\) to be lower than \(W(\tau)\) is that the first three components of \(W_{ad}\) be strictly decreasing in \(\theta^*\). This requires

\[
[D_1(p_{ad}, p_{ad})(p_{ad} - c) + x \int_{p_{ad}}^{\infty} D_2(p, p_{ad}) dp - D_i(p_{ad}, p_{ad}) x + y \int_{p_{ad}}^{\infty} D_2(p_{ad}, p_{ad}) dp < 0,
\]

where \(x = \frac{dp_{ad}}{dp}\) and \(y = \frac{dp_{ad}}{dp}\). Equivalently,

\[
\left[D_1(p_{ad}, p_{ad})(p_{ad} - c) + \int_{p_{ad}}^{\infty} D_2(p_{ad}, p_{ad}) dp \right] y > \left[-D_2(p_{ad}, p_{ad})(p_{ad} - c) + D_i(p_{ad}, p_{ad}) - \int_{p_{ad}}^{\infty} D_2(p, p_{ad}) dp \right] x.
\]

In other words, the change in welfare due to a unit change in domestic price must exceed the change in welfare due to a unit change in import price. As long as, condition (36) is satisfied, the welfare from the anti-dumping duty falls below the welfare from the tariff for sufficiently elastic foreign demand. Condition (36) embeds both the technology and demand parameters. For constant-elastic demand, a more specific statement can be made.

**Proposition 11** If all demand functions are constant elastic, the revenue-maximizing tariff provides a higher welfare than the anti-dumping duty if

\[
\frac{c^*}{c} < \left(\frac{(2\varepsilon^2 - 3\varepsilon + 1)N}{\gamma((N + 1)\varepsilon - 1)}\right)^{1\over\varepsilon + \gamma - 1} \left(1 - \frac{N - 1}{N\varepsilon - 1}\right) \left(1 - \frac{(1 - \lambda(p_{ad}))}{(2\varepsilon^*- (1 - \lambda(p_{ad}))}\right).
\]


From proposition 11, one can easily derive the following corollary

**Corollary 1** If demand functions are constant elastic, then the optimal tariff provides a higher welfare than the anti-dumping duty if (37) holds.

### 6 Extensions

In this section, two important extensions are considered briefly. The first one allows free entry in the domestic industry and the second one considers competition in the foreign country.

#### 6.1 Free entry in the domestic market

The analysis so far has focused on a domestic industry with a fixed number of firms. How do the results change if there is free entry to the domestic industry? To accommodate free entry, consider a one-time sunk entry cost $F > 0$ that applies to each domestic firm. A domestic firm’s profit then becomes

$$\pi(q; N) = q(P(Q) - c) - F.$$  

Because $\pi(q; N)$ strictly decreases with $N$, free entry means that the equilibrium number of entrants is the smallest integer such that

$$\pi(q; N) \geq 0 \text{ and } \pi(q, N + 1) < 0.$$  

Under free entry the profit of a domestic firm is no longer an informative measure of industry protection. With the integer requirement in place, as the import price increases, there is a region where profits of a domestic firm increases before the next firm enters. Entry implies lower profits for all firms, thus industry protection is lower. But if the import price increases further profits start to increase again until the next firm enters and industry protection improves until entry occurs. Therefore, for the case with free entry the number of firms in the domestic industry or the total domestic output is a better measure of industry protection.

In general, allowing for free entry changes the analysis in that the changes in prices and profits as import price increases are no longer monotonic and continuous. However, because domestic profits and the import price still decline –even though non-monotonically– as the foreign demand elasticity increases, all propositions in Section 5 that require a high foreign demand elasticity still remain valid.

#### 6.2 Competition in the foreign market

Competition in the foreign market implies a higher own-price elasticity in foreign demand and a lower price in the foreign market under very general conditions. In essence, the effect of an increase in competition in the foreign market is similar to an increase in foreign demand elasticity due to an increase in $\theta^*$. Competition between foreign firms has no effect on tariffs, however, as tariffs depend only on the elasticity of demand for the imported good in the domestic country. Therefore, much of the analysis so far remains valid. Increased competition in the foreign market renders all the results that require a high foreign demand elasticity more likely to hold.
7 Conclusion

This paper has provided a comparison of two important trade barriers, tariffs and anti-dumping duties, in a framework that embeds the key elements of many major dumping cases in the U.S. The model proposed is stylized to focus on a common case where a firm dominant in its home market exports to a market inhabited by an oligopoly. The model also produces a dominant foreign firm–domestic competitive fringe interaction as a special case. Dumping arises due to differences in demand elasticities in the two countries and the foreign firm engages in international price discrimination.

The comparison of the two tools depends critically on the magnitudes of the elasticities in the foreign and domestic markets. A tariff depends only on the elasticity of the import demand in the domestic country, whereas an anti-dumping duty depends, in addition, on the elasticity of the foreign demand. The magnitude of the gap between the two elasticities determines whether one tool dominates the other under a given criterion. While one tool does not dominate the other uniformly under all criteria, it was shown that, under certain restrictions, when the foreign demand elasticity is sufficiently high, a revenue-maximizing tariff can dominate an anti-dumping duty in terms of revenue, protection, and domestic welfare. A high foreign elasticity encourages the use of a tariff rather than an anti-dumping duty. The findings also point to the importance of the knowledge of demand elasticities in the two countries. In particular, the model suggests that the estimates of both the level and derivatives of the elasticities are useful in determining whether the use of tariffs are more attractive in terms of government revenue and protection. The analysis was confined to a static environment. It would be important to see how much dynamic considerations, such as dynamic pricing by the foreign firm can change the results, especially when a trade barrier is anticipated.

References


A Proofs

Proof of Proposition 1. If $p^*_{ad} < p^i_{ad}$, the foreign firm does not pay any anti-dumping duty. Therefore, the prices $p^*_{ad}$ and $p^i_{ad}$ must maximize the sum of the first two terms in (14), i.e. the free trade profit function. It must then hold that $p^*_{ad} = p^f_j$ and $p^i_{ad} = p^j_f$. But since dumping is assumed under free trade, it must be that $p^f_j > p^j_f$, a contradiction with $p^*_{ad} < p^i_{ad}$. Thus, $p^*_{ad} \geq p^i_{ad}$. From (21), it is easy to see that the equality $p^*_{ad} = p^i_{ad}$ holds if and only if

$$\frac{\varepsilon(p^*_{ad}; p_{ad})}{\varepsilon^*(p^*_{ad})} = \frac{2}{1 - \lambda(p_{ad})}.$$  

Proof of Proposition 2. Replacing $p^*_{ad}$ with $p^j_f$ in the first order condition (15) evaluated at equilibrium prices $p^i_{ad}$ and $p_{ad}$ yields

$$D^*(p^*_j)(p^*_j - c^*) + D^*(p^*_f) - D(p^i_{ad}; p_{ad}) < 0,$$

because the first two terms sum to zero by the first order condition (10) that determines the free trade price $p^f_j$. Since $D^*(p^*) (p^*_j - c^*) + D^*(p^*_f)$ is strictly decreasing in $p^*$ by strict concavity, $p^*_{ad}$ must be lower than $p^*_j$ for (15) to hold. Similarly, replacing $p^i_{ad}$ with $p^j_f$ in (16) evaluated at $p^*_{ad}$ and $p_{ad}$ yields

$$D_1(p^j_f, p_{ad})(p^i_{ad} - c^*) + D(p^j_f, p_{ad}) + D_1(p^j_f, p_{ad})(p^i_{ad} - p^*_ad) + D(p^j_f, p_{ad}) > 0. \quad (38)$$
The first two terms add up to zero, by the first order condition (11) that determines $p_f^i$. Since $D_1 < 0$ and $p_{ad}^* ≥ p_f^i$, the last two terms sum to a positive number. Because the left hand side of (??) is strictly decreasing in $p_f^i$ by strict concavity, $p_{ad}^*$ must be strictly higher than $p_f^i$. It follows directly that the dumping margin is also lower compared to that under free trade. ■

Proof of Proposition 3. i) Straightforward differentiation yields

$$p_{ii}^i(\tau) = \frac{c^*}{1 + c^*} \left( \frac{\varepsilon(p_f^i(\tau),\varepsilon_f(\tau))}{\varepsilon(f(\tau),f(\tau)) - 1} \right) \frac{1}{1 - \tau} > 0,$$

(39)

$$p_i^i(\tau) = \frac{-cN\varepsilon_2(p(\tau),p_i^i(\tau))p_{ii}^i(\tau)}{(N\varepsilon(p(\tau),p_i^i(\tau)) - 1)^2 + N\varepsilon_1(p(\tau),p_i^i(\tau))},$$

where the signs follow because $\varepsilon_1(p_i^i(\tau),p_i^i(\tau)) ≥ 0$ and $\varepsilon_2(p(\tau),p_i^i(\tau)) ≤ 0$.

ii, iii) Parts (ii) and (iii) follow from the following derivatives

$$\frac{d\ln Q_i^i(\tau)}{d\tau} = -\varepsilon(p_i^i(\tau),p(\tau))\frac{d\ln p_i^i(\tau)}{d\tau} + \gamma(p_i^i(\tau),p(\tau))\frac{d\ln p(\tau)}{d\tau},$$

(40)

$$\frac{d\ln Q_i^i(\tau)}{d\tau} = -\varepsilon(p(\tau),p_i^i(\tau))\frac{d\ln p_i^i(\tau)}{d\tau} + \gamma(p(\tau),p_i^i(\tau))\frac{d\ln p_i^i(\tau)}{d\tau}.$$

Proof of Proposition 4. i) Using (39) with the constant elastic demand functions one obtains

$$p_{ii}^i(\tau) = \frac{e^* \varepsilon^2}{c^* + (\varepsilon - 1)^2} > 0,$$

because $\varepsilon > 1$ and $p_i^i(\tau) = 0$. From (40), it follows that

$$Q_i^i(\tau) = \gamma \frac{d\ln p_i^i(\tau)}{d\tau} > 0,$$

$$Q_{ii}^i(\tau) = -\varepsilon \frac{d\ln p_i^i(\tau)}{d\tau} < 0.$$

Note that none of the responses depend on $N$.

ii) For linear demand in (9), one obtains

$$p_{ii}^i(\tau) = \frac{(N + 1)b^2c^*}{(1 - \tau)^2(2b^2(N + 1) - d^2)} > 0, \quad p_i^i(\tau) = \frac{dbraco^*}{(1 - \tau)^2(2b^2(N + 1) - d^2)} > 0,$$

$$Q_i^i(\tau) = \frac{Nbd^2c^*}{(1 - \tau)^2(2b^2(N + 1) - d^2)} > 0, \quad Q_{ii}^i(\tau) = \frac{-b(Nb^2(N + 1) - d^2)c^*}{(1 - \tau)^2(2b^2(N + 1) - d^2)} \begin{cases} < 0 & N ≥ 3 \\ ≥ 0 & N ≤ 2 \end{cases}.$$  

The signs follow because for all $N ≥ 1$ it must hold that

$$2b^2(N + 1) - d^2 > 0,$$

for prices to be positive in equilibrium.\textsuperscript{30} Furthermore,

$$b^2(N + 1) - d^2 > 0,$$

\textsuperscript{30}The proof is as follows. For prices to be positive, it must hold that $\varepsilon(p_i^i(\tau)) > 1$ and $\varepsilon(p(\tau)) > 1$. Note that $\varepsilon(p_i^i(\tau)) = \frac{bp^i(\tau)}{a - bp^i(\tau)} > 1$ implies $p_i^i(\tau) > \frac{a + dp^i(\tau)}{2b}$. On the other hand, $\varepsilon(p(\tau)) = \frac{bp(\tau)}{a - bp(\tau)} > \frac{1}{N}$ implies $p(\tau) > \frac{a + dp(\tau)}{N(N + 1)}$. These two restrictions on prices yield $p(\tau) > \frac{a + dp(\tau)}{\beta(N + 1)}$, or equivalently, $[2b^2(N + 1) - d^2]p(\tau) > a(2b + d) > 0$, which implies $2b^2(N + 1) - d^2 > 0$. 

\n
24
for $N \geq 3$, because for $N \geq 3$, $b^2(N + 1) - d^2 \geq 4b^2 - d^2$, and the last expression is greater than zero for $N \geq 3$ since $2b^2(N + 1) - d^2 \geq 4b^2 - d^2 > 0$ for $N \geq 1$. For $N \leq 2$, $b^2(N + 1) - d^2 < 0$ if $3b^2 - d^2 < 0$. Thus, a sufficient condition for $Q''(\tau)$ to be positive for $N \leq 2$ is $3b^2 < d^2$ or $d > \sqrt{3}b$.

iii) Assuming $N$ is a real number for the time being, we can differentiate the responses of prices and quantities to changes in tariff, and obtain their rates of change with $N$ as follows

\[
\begin{align*}
\frac{dp''(\tau)}{dN} &= \frac{-d^2b^2c^*}{(1-\tau)^2 (2b^2(N+1) - d^2)^2} < 0, \\
\frac{dp'(\tau)}{dN} &= \frac{-2b^3dc^*}{(1-\tau)^2 (2b^2(N+1) - d^2)^2} < 0, \\
\frac{dQ'(\tau)}{dN} &= \frac{-d^3b^2c^*}{(1-\tau)^2 (2b^2(N+1) - d^2)^2} < 0, \\
\frac{dQ''(\tau)}{dN} &= \frac{-d^3b^2c^*}{(1-\tau)^2 (2b^2(N+1) - d^2)^2} < 0.
\end{align*}
\]

Finally, using part (ii)

\[
\begin{align*}
\lim_{N \to \infty} p'(\tau) &= 0, \quad \lim_{N \to \infty} p''(\tau) = \frac{c^*}{2(1-\tau)^2} > 0, \\
\lim_{N \to \infty} Q'(\tau) &= \frac{dc^*}{2(1-\tau)^2} > 0, \quad \lim_{N \to \infty} Q''(\tau) = -\frac{bc^*}{2(1-\tau)^2} < 0.
\end{align*}
\]

**Proof of Proposition 5.**  

i) Let $x = \frac{dp'}{d\theta^*}$, $y = \frac{dp_{ad}}{d\theta^*}$, $z = \frac{dp_{ad}'}{d\theta^*}$. An increase in $\theta^*$ implies a change in anti-dumping duty revenue equal to

\[
(D_1(p_{ad}^i, p_{ad}) x + D_2(p_{ad}^i, p_{ad}) y) (p_{ad}^* - p_{ad}') + D(p_{ad}^i, p_{ad}) (z - x)
\]

\[
= -D_1(p_{ad}^i, p_{ad}) c^* x + D(p_{ad}^i, p_{ad}) z + D_2(p_{ad}^i, p_{ad}) (p_{ad}^i - p_{ad}') y,
\]

where the equality follows from the first order condition (16). Thus, if $x, y, z \leq 0$ (with at least one strict inequality), the anti-dumping duty revenue increases as $\theta^*$ decreases.

Total differentiation of (20) with respect to $\theta^*$ yields,

\[
y = \left( -\frac{N c2(p_{ad}, p_{ad}^i)}{(N c(p_{ad}, p_{ad}^i) - 1)^2 + c N c1(p_{ad}, p_{ad}^i)} \right) x.
\]

Because $c2(p_{ad}, p_{ad}^i) \leq 0$, the term inside the parentheses (42) is non-negative. Therefore, $\text{sign}(y) = 0$ or $\text{sign}(y) = \text{sign}(x)$. Furthermore, total differentiation of (16) with respect to $\theta^*$ yields

\[
z = \left( \frac{((N c - 1)^2 + c N c1) (D_11 \Delta + 4D_1) - c N c2 (D_12 \Delta + 2D_1)}{D_1((N c - 1)^2 + c N c1)} \right) x.
\]

But $(D_11 \Delta + 4D_1) < 0$ by the strict concavity of the firm’s profit under anti-dumping duty. In addition, $(D_12 \Delta + 2D_1) < 0$, because $D_12 < 0$.\(^{31}\) Thus, the term in the parentheses in (43) is positive. Therefore, $\text{sign}(z) = \text{sign}(x)$.

\(^{31}\)To see the $D_12 < 0$, note that

\[
\gamma_1 = \frac{D_21 p^i D - D_22 p^i D_1}{D^2} < 0,
\]

implying that $D_21 \leq \frac{D_22 D_1}{D^2} < 0$. By Young’s theorem $D_12 = D_21$ and the result follows.
Now, consider a decrease in \( \theta^* \), all other parameter values fixed. In response, the free trade price \( p_f^* \) increases and \( p_f^\prime \) does not change. The home price under the duty \( p_{ad}^* \) must then increase. Suppose not, i.e. \( p_{ad}^* \) is lower. But the foreign firm would have then chosen to reduce its price at the first place when \( \theta^* \) was higher. Therefore, \( p_{ad}^* \) must be higher. As a result, \( z < 0, x < 0 \) and \( y \leq 0 \). But then (41) is negative. Therefore, an increase in the foreign demand elasticity implies a lower duty revenue. Also note that
\[
\varepsilon^* (p_{ad}^*) \leq \frac{\varepsilon (p_{ad}^\prime, p_{ad}) (1 - \lambda (p_{ad}^*))}{2} < \frac{\varepsilon (p_{ad}^\prime, p_{ad})}{2}.
\]
Therefore, whenever \( \frac{d\varepsilon (p_{ad}^\prime, p_{ad})}{dp_{ad}} < 0 \) or \( \frac{d\varepsilon (p_{ad}^\prime, p_{ad})}{dp_{ad}} = \frac{d\varepsilon^* (p_{ad}^*)}{dp_{ad}} \), as \( \theta^* \) increases \( \varepsilon^* (p_{ad}^*) \) eventually equals \( \frac{\varepsilon (p_{ad}^\prime, p_{ad}) (1 - \lambda (p_{ad}^*))}{2} \) and the anti-dumping revenue becomes zero. Because \( R(\tau^r) \) is independent of \( \theta^* \), there must then exist some \( \theta_R^* \) such that the duty revenue exceeds the tariff revenue.

\[\text{ii) Note that } \tau_{pro} = \frac{1 - \lambda (p_{ad}^*)}{2\varepsilon^* (p_{ad}^*) - 1} < \frac{1}{2\varepsilon^*(p_{ad}^*) - 1}.\]
Using (28), a sufficient condition for \( \tau_{pro} < \tau^r \) is that there exist a \( \theta^* \) such that
\[
(\varepsilon (p_{ad}^\prime (\tau^r), p(\tau^r)) - 1) \frac{d\ln p(\tau)}{d\tau} \bigg|_{\tau=\tau^r} - \gamma (p_{ad}^\prime (\tau^r), p(\tau^r)) \frac{d\ln p(\tau)}{d\tau} \bigg|_{\tau=\tau^r} < 2\varepsilon^*(p_{ad}^*) - 1 \tag{44}
\]
The reverse of (33) implies that, as \( \theta^* \) increases, the right hand side of (44) continues to decline and at the same time the dumping margin does not disappear. Because \( \tau^r \) is independent of \( \theta^* \), the left hand side is independent of \( \theta^* \). Therefore, there must then exist a unique \( \theta_{pro}^* \) such that \( \tau_{pro} < \tau^r \) for \( \theta^* > \theta_{pro}^* \).

**Proof of Proposition 6.** i) The tariff revenue is greater than the duty revenue when
\[
\left( \frac{\varepsilon - 1}{\varepsilon} \right)^{\varepsilon} \frac{\varepsilon}{\varepsilon - 1} > \frac{\varepsilon (1 - \lambda) - 2\varepsilon^*}{2(2\varepsilon^* - 2(1 - \lambda))} \left( \frac{2\varepsilon^* - 2(1 - \lambda)}{2\varepsilon^* - (1 - \lambda)} \right) \epsilon
\]
which can be rewritten, after setting \( k = \frac{2\varepsilon^* - (1 - \lambda)}{1 - \lambda^*} \), as
\[
\left( \frac{\varepsilon - 1}{\varepsilon} \right)^{\varepsilon - 1} \left( \frac{k - 1}{k - 2} \right)^{\varepsilon - 1} \left( \frac{k - 1}{\varepsilon - k} \right) - 1 > 0. \tag{45}
\]
Because \( k > 1 \), it holds that \( \varepsilon - 1 > \varepsilon - k \), and the left hand side of (45) is thus greater than
\[
\left( \frac{\varepsilon - 1}{\varepsilon} \right)^{\varepsilon - 1} \left( \frac{k - 1}{k - 2} \right)^{\varepsilon - 1} \left( \frac{k - 1}{\varepsilon - 1} \right) - 1.
\]
The first term above strictly decreases as \( \varepsilon \) increases and converges to \( \varepsilon^{-1} \) as \( \varepsilon \) diverges to infinity. Therefore, the left hand side of (45) is strictly greater than
\[
v^{-1} \left( \frac{k - 1}{k - 2} \right)^{\varepsilon - 1} \left( \frac{k - 1}{\varepsilon - 1} \right) - 1.
\]
Let \( F(\varepsilon; k) = \left( \frac{k - 1}{k - 2} \right)^{\varepsilon - 1} \left( \frac{k - 1}{\varepsilon - 1} \right) \). Given any \( k \), \( \lim_{\varepsilon \rightarrow 1} F(\varepsilon; k) = \lim_{\varepsilon \rightarrow +\infty} F(\varepsilon; k) = +\infty \). Therefore, the function \( F(\varepsilon; k) \) attains a unique minimum value over \( \varepsilon \in (k, +\infty) \) at
\[
\varepsilon_{\min} = \left( \ln \left( \frac{k - 1}{k - 2} \right) - 1 \right) \left( \ln \left( \frac{k - 1}{k - 2} + 1 \right) \right),
\]
where the function value is

\[ F(\varepsilon; k) = \left( \frac{k - 1}{k - 2} \right)^{\left( \frac{k - 1}{k - 2} - 1 \right)} \left( \frac{k - 1}{\ln \left( \frac{k - 1}{k - 2} \right)} \right) . \]

The derivative of \( F(\varepsilon; k) \) is

\[ -\left( \frac{k - 1}{k - 2} \right)^{\left( \frac{k - 1}{k - 2} - 1 \right)} \frac{2 \ln \left( \frac{k - 1}{k - 2} \right)}{k - 2} \left( 2 \ln \left( \frac{k - 1}{k - 2} \right) - k \ln \frac{k - 1}{k - 2} + 1 \right) < 0, \]

for \( k \in (2, +\infty) \). Application of L’Hopital’s rule yields \( \lim_{k \to \infty} F(\varepsilon; k) = e \). Therefore, \( F(\varepsilon; k) \geq e \), for all \( \varepsilon > k \). Thus, \( e^{-1} F(\varepsilon; k) > 1 \) and, as a result, (45) holds. The facts that \( R(0) = 0, \lim_{\tau \to 1} R(\tau) = 0 \) when \( \varepsilon > 1 \) implies the existence of \( \tau_1^{rev}, \tau_2^{rev} \) and that \( \tau_1^{rev} < \tau < \tau_2^{rev} \).

\( ii) \quad \tau^{pro} = \frac{1}{\varepsilon - 1} > 1 \quad \Rightarrow \quad \varepsilon = \tau^{r} \), where the first inequality follows from the fact that \( \varepsilon > \frac{2 \varepsilon^{*}}{1 - \lambda(p_{ad})} \).

\( iii) \quad \text{Note by definition of } \tau_2^{rev} ,

\[ \tau_2^{rev} (1 - \tau_2^{rev}) = \frac{\varepsilon - 1}{\varepsilon} \left( \frac{2 \varepsilon^{*} - 2(1 - \lambda(p_{ad}))}{2 \varepsilon^{*} - 2(1 - \lambda(p_{ad}))} \right)^{\varepsilon - 1} \frac{\varepsilon}{\varepsilon - 1} (1 - \lambda(p_{ad})) - 2 \varepsilon^{*} \}

\[ < \frac{\varepsilon - 1}{\varepsilon} \left( \frac{2 \varepsilon^{*} - 2(1 - \lambda(p_{ad}))}{2 \varepsilon^{*} - 2(1 - \lambda(p_{ad}))} \right)^{\varepsilon - 1} \frac{\varepsilon}{\varepsilon - 1} (1 - \lambda(p_{ad})) - 2 \varepsilon^{*} \] \[ = \frac{\varepsilon - 1}{\varepsilon} \left( \frac{2 \varepsilon^{*} - 2(1 - \lambda(p_{ad}))}{2 \varepsilon^{*} - 2(1 - \lambda(p_{ad}))} \right)^{\varepsilon - 1} \tau^{pro} (1 - \tau^{pro})^{\varepsilon - 1} ,

where the inequality follows from the fact that \( \tau^{pro} > \frac{1}{\varepsilon - 1} \) and the second equality from \( 1 - \tau^{pro} = \frac{2 \varepsilon^{*} - 2(1 - \lambda(p_{ad}))}{2 \varepsilon^{*} - 2(1 - \lambda(p_{ad}))} \). As a result,

\[ \frac{\tau^{rev} (1 - \tau^{rev})^{\varepsilon - 1}}{\tau^{pro} (1 - \tau^{pro})^{\varepsilon - 1}} < \frac{\varepsilon - 1}{\varepsilon} \frac{(1 - \lambda(p_{ad})) - 2 \varepsilon^{*}}{2 \varepsilon^{*} - 2(\lambda(p_{ad}))} . \]

But the right hand side is less than 1 if

\[ \frac{\varepsilon^{2}}{4 \varepsilon^{*} - 2} < \varepsilon^{*} . \] (46)

Because \( \tau^{pro} \) and \( \tau_2^{rev} \) are both greater than \( \tau^{r} \) and the fact that \( R(\tau) \) is strictly decreasing to the right of \( \tau^{r} \), (46) is sufficient for \( \tau^{pro} > \tau_2^{rev} \) to hold. ■

**Proof of Proposition 7.** The results \( p^{i}(\tau^{r}) > p^{i}_{f} \) and \( p(\tau^{r}) = p_{f} \) follow directly from the definitions of these prices under constant elastic demand functions. Finally, to see \( p^{i}_{ad} > p^{i}(\tau^{r}) \), note that

\[ p^{i}_{ad} = c^{*} \frac{\varepsilon}{\varepsilon - 1} \left( \frac{2 \varepsilon^{*} - 2(1 - \lambda(p_{ad}))}{2 \varepsilon^{*} - 2(1 - \lambda(p_{ad}))} \right)^{\frac{1}{\varepsilon - 1}} \]

\[ = c^{*} \frac{\varepsilon}{\varepsilon - 1} \left( \frac{1 - \lambda(p_{ad})}{1 - \lambda(p_{ad})} \right) - 1 \frac{1}{\varepsilon} \left( \frac{\varepsilon}{\varepsilon - 1} \right)^{\frac{1}{\varepsilon - 1}} > c^{*} \frac{\varepsilon}{\varepsilon - 1} \left( \frac{\varepsilon}{\varepsilon - 1} \right)^{\frac{1}{\varepsilon - 1}} > c^{*} \left( \frac{\varepsilon}{\varepsilon - 1} \right)^{2} = p^{i}(\tau^{r}) ,

\]
where the first inequality follows from the fact that \( \varepsilon > \frac{2\varepsilon^*}{(1-\lambda(p_{ad}))} \) and the second from the fact that \( \frac{\varepsilon - 1}{\varepsilon - 2} > \frac{\varepsilon}{\varepsilon - 1} \).

**Proof of Proposition 10.** The revenue-maximizing tariff provides higher export profit than the duty if

\[
D(p^i(\tau^r), p(\tau^r))((1 - \tau^r)p^i(\tau^r) - c^*) > D(p^i_{ad}, p_{ad})(p^i_{ad} - c^*) - D(p^i_{ad}, p_{ad})(p^*_i - p^i_{ad}).
\]

Under constant elasticity, \( p^i_{ad} > p^i(\tau^r) \). Thus,

\[
D(p^i(\tau^r), p(\tau^r))((1 - \tau^r)p^i(\tau^r) - c^*) > D(p^i_{ad}, p_{ad})(p^i_{ad} - c^*)
\]

(47)

Subtracting the total tariff and the total duty from both sides of (47) and rearranging yields

\[
D(p^i(\tau^r), p(\tau^r))((1 - \tau^r)p^i(\tau^r) - c^*) - (D(p^i_{ad}, p_{ad})(p^i_{ad} - c^*) - D(p^i_{ad}, p_{ad})(p^i_{ad} - c^*))
\geq D(p^i_{ad}, p_{ad})(p^*_i - p^i_{ad}) - D(p^i(\tau^r), p(\tau^r))\tau^r p^i(\tau^r).
\]

Therefore, a sufficient condition for the export profit under the revenue-maximizing tariff to exceed the export profit under the duty is

\[
D(p^i_{ad}, p_{ad})(p^*_i - p^i_{ad}) - D(p^i(\tau^r), p(\tau^r))\tau^r p^i(\tau^r) > 0.
\]

(48)

Because \( p_{ad} = p(\tau^r) = p_f \) under constant elasticity, the left hand side of (48) is

\[
D(p^i_{ad}, p_f)(p^*_i - p^i_{ad}) - D(p^i(\tau^r), p_f)\tau^r p^i_f \frac{1}{1 - \tau^r}.
\]

But

\[
D(p^i_{ad}, p_f)(p^*_i - p^i_{ad}) < D(p^i_{ad}, p_f)(p^*_f - p^i(\tau^r)),
\]

where the inequality follows from \( p^i_{ad} > p^i(\tau^r) \) and \( p^*_i < p^*_f \). A sufficient condition for (48) to hold is then

\[
D(p^i_{ad}, p_f)(p^*_f - p^i(\tau^r)) > D(p^i(\tau^r), p_f)\tau^r p^i_f \frac{1}{1 - \tau^r},
\]

or equivalently,

\[
(p^*_f - p^i(\tau^r)) > \tau^r p^i_f \frac{1}{1 - \tau^r},
\]

which reduces to

\[
\varepsilon^* > \frac{\varepsilon}{3\varepsilon - 1} \frac{\varepsilon + 1}{3\varepsilon - 1}.
\]

The right hand side is increasing in \( \varepsilon \) for \( \varepsilon > 1 \). Letting \( \varepsilon^*_w = \frac{\varepsilon + 1}{3\varepsilon - 1} > 1 \) and noting that \( \varepsilon^* < \frac{\varepsilon(1-\lambda(p_{ad}))}{2(\varepsilon - 1)} \), the proposition follows.

**Proof of Proposition 11.** Welfare functions under the two regimes can be written, using constant elastic demand functions, as

\[
W_{ad} = \frac{2 - \varepsilon}{\varepsilon - 1} p^{i\gamma - \varepsilon}_{ad} + \frac{N\varepsilon + (\varepsilon - 1)}{N\varepsilon(\varepsilon - 1)} p^{1-\varepsilon-\gamma}_{ad} p^i_{ad} + p^{i\gamma}_{ad} p^*_i
\]

\[
W(\tau^r) = \frac{2 - \varepsilon}{\varepsilon(\varepsilon - 1)} (p^i(\tau^r))^{1-\varepsilon} + \frac{N\varepsilon + (\varepsilon - 1)}{N\varepsilon(\varepsilon - 1)} (p(\tau^r))^{1-\varepsilon-\gamma} (p^i(\tau^r))^{\gamma}
\]

28
Set \( p^i(\tau^r) = p_{ad}^i \). Then, everything else constant, it must be true that
\[
\frac{2 - \varepsilon}{\varepsilon - 1} p_{ad}^{i-\varepsilon} + \frac{N\varepsilon + (\varepsilon - 1)}{N\varepsilon(\varepsilon - 1)} p_{ad}^{1-\varepsilon} p_{ad}^{i-\varepsilon} + p_{ad}^{i-\varepsilon} p_{ad} < \frac{2\varepsilon - 1}{\varepsilon - 1} p_{ad}^{i-\varepsilon} + \frac{N\varepsilon + (\varepsilon - 1)}{N\varepsilon(\varepsilon - 1)} (p(\tau^r))^{1-\varepsilon} p_{ad}^{i-\varepsilon},
\]
which yields
\[
\frac{p_{ad}^{i-\varepsilon}}{p_{ad}^{i}} < \frac{\varepsilon + 1}{\varepsilon},
\]
which holds when the prices are replaced by their explicit forms. This implies that the tariff regime provides a higher welfare when the import price is set equal to the import price under the anti-dumping duty regime and keep all other variables at their current optimal levels. It is, however, already known that \( p_{ad}^i > p_{ad}(\tau^r) \). Therefore a sufficient condition for the proposition to hold is welfare be decreasing with the import price at \( p^i(\tau^r) = p_{ad}^i \). The derivative of the welfare function with respect to the import price at \( p^i(\tau^r) = p_{ad}^i \) is
\[
-\frac{1}{N\varepsilon(\varepsilon - 1)} \left( N(1 - 3\varepsilon + 2\varepsilon^2) + p_{ad}^{i-\varepsilon} (p_{ad}^i)^{\varepsilon + \gamma - 1} \gamma(1 - \varepsilon - N\varepsilon) \right).
\]
Therefore a sufficient condition is
\[
N(1 - 3\varepsilon + 2\varepsilon^2) + \left( \frac{p_{ad}^i}{p_{ad}} \right)^{\varepsilon + \gamma - 1} \gamma(1 - \varepsilon - N\varepsilon) > 0.
\]
Substituting explicit values of \( p_{ad}^i \) and \( p \), and rearranging yields
\[
\frac{c^*}{c} < \left( \frac{N(2\varepsilon^2 - 3\varepsilon + 1)}{\gamma((N + 1)\varepsilon - 1)} \right) \frac{1}{\varepsilon + \gamma - 1} \left( \frac{N(\varepsilon - 1)}{N\varepsilon - 1} \right) \left( \frac{2\varepsilon^* - 2(1 - \lambda(p_{ad})))}{(2\varepsilon^* - (1 - \lambda(p_{ad})))} \right).
\]

\[\blacksquare\]

Proof of Corollary 1. By definition \( W(\tau^o) \geq W(\tau^r) \). Moreover, when the above condition holds \( W(\tau^r) > W_{ad} \). Therefore it must be true that \( W(\tau^o) > W_{ad} \). \[\blacksquare\]

B Uniqueness of equilibrium

In this appendix, the uniqueness of equilibrium under the three regimes is shown when the demand functions are constant-elastic or linear.

B.1 Constant-elastic demand

The free trade prices under constant-elastic demand are
\[
p_f^* = c^* \left( \frac{\varepsilon^*}{\varepsilon^* - 1} \right), \quad p_f^i = c^* \left( \frac{\varepsilon}{\varepsilon - 1} \right), \quad p_f = c \left( \frac{N\varepsilon}{N\varepsilon - 1} \right),
\]
which are uniquely determined given \( \varepsilon^*, \varepsilon \) and \( N \). In the anti-dumping duty regime, the first order conditions (15) and (16) yield
\[
p^i = \left( \frac{\alpha p^\gamma}{Ap^* - \varepsilon^* (1 - \varepsilon^* p^* - c)} \right)^{1/\varepsilon}, \quad \text{(49)}
\]
\[ p^i = \left( \frac{\varepsilon}{2(\varepsilon - 1)} \right) p^* + \left( \frac{\varepsilon}{2(\varepsilon - 1)} \right) c^*. \]  

(50)

Given any domestic price level \( p \), the intersection of these two trajectories in the \((p^i, p^*)\) plane yields the equilibrium values of \( p^i \) and \( p^* \) as a function of \( p \). It is easy to verify that (49) and (50) both have positive slopes and they intersect at a unique point \((p^i_{ad}, p^*_{ad})\) in the \((p^i, p^*)\) plane satisfying the equilibrium constraint \( p^*_{ad} \geq p^i_{ad} \). The domestic price is given by

\[ p_{ad} = c \left( \frac{N \varepsilon}{N \varepsilon - 1} \right). \]

Therefore, the equilibrium is also unique in this case. Finally, in the tariff regime, the prices are

\[ p^*(\tau) = c^* \left( \frac{\varepsilon^*}{\varepsilon^* - 1} \right), \]

\[ p^i(\tau) = c^* \left( \frac{\varepsilon}{\varepsilon - 1} \right) \frac{1}{1 - \tau}, \]

\[ p(\tau) = c \left( \frac{N \varepsilon}{N \varepsilon - 1} \right), \]

which are again unique given \( \varepsilon^*, \varepsilon, N, \) and \( \tau \).

**B.2 Linear demand**

**B.2.1 Free Trade**

The free trade prices are

\[ p^*_f = \frac{1}{2B} (A + Bc^*), \]

\[ p^i_f = c^* \left( \frac{bp^i_f}{2bp^i_f - a - dp^i_f} \right), \]

\[ p_f = c \left( \frac{Nbp^i_f}{(N + 1)bp^i_f - a - dp^i_f} \right), \]

Note that \( p^*_f \) is unique. Explicit solutions for \( p^i_f \) and \( p_f \) are

\[ p^i_f = \frac{1}{2(N + 1)b^2 - d^2} \left( (N + 1)b^2 c^* + (N + 1)ab + ad + Nbcd \right), \]

\[ p_f = \frac{1}{2(N + 1)b^2 - d^2} \left( 2ab + ad + 2Nb^2 c + bde \right), \]

which are also unique.

**B.2.2 Antidumping Regime**

From the first order conditions, prices under the anti-dumping duty are

\[ p^*_ad = \frac{bp^i_ad - dp^i_ad + Bc^* - (A - a)}{2B} \]

\[ (51) \]

\[ p^i_ad = \frac{b(p^*_ad + c^*) + 2(dp^i_ad + a)}{4b} \]

\[ (52) \]

\[ p_{ad} = \frac{dp^i_ad + bcN + a}{(N + 1)b}. \]

\[ (53) \]

It is seen from equation (53) that for each \( p^i_{ad} \) there is a unique \( p_{ad} \). Similarly, from equation (51), one can see that there is a unique \( p^*_ad \) for each pair \( \{p^i_{ad}, p_{ad}\} \). Thus if there exists a unique \( p^i_{ad} \), then \( p_{ad} \) and \( p^*_ad \) must also be unique. Solving the system of 3 equations simultaneously gives
\[ p_{ad}^i = \frac{b(4B - b)dcN}{(N + 1)(8B - b)b^2 - (4B - b)d^2} + \frac{(4B - b)da}{(N + 1)(8B - b)b^2 - (4B - b)d^2} \\
+ \frac{3B(N + 1)b^2c^*}{(N + 1)(8B - b)b^2 - (4B - b)d^2} + \frac{(N + 1)b^2(A - a)}{(N + 1)(8B - b)b^2 - (4B - b)d^2} \\
+ \frac{4B(N + 1)ba}{(N + 1)(8B - b)b^2 - (4B - b)d^2}, \]

which is unique.

**B.2.3 Tariff Regime**

In the case of tariff regime foreign price is independent of tariff and hence is the same as that under free trade. For the domestic market, using the first order conditions one obtains

\[ p^i(\tau) = \frac{c^*}{(1 - \tau)^2} + \frac{a + dp(\tau)}{2b}, \]

\[ p^i(\tau) = b(N + 1)p(\tau) - cbN - a. \]

Solving for \( p(\tau) \) gives

\[ p(\tau) = \left( \frac{dc^*}{2(1 - \tau)} + cbN \right) \frac{2b}{(2b^2(N + 1) - d^2)} + \frac{da + 2ba}{(2b^2(N + 1) - d^2)}. \]

Given \( \tau \), \( p(\tau) \) is unique, so is \( p^i(\tau) \).