Midterm Exam 2, April 7—4 questions. All sub-questions carry equal weight except where otherwise indicated.

1. (40%) Assume that $y_t$ follows the stationary AR(1) process

$$y_t = 200 + 0.2y_{t-1} + u_t$$

where $u_t$ is white noise with variance 2.

a) (5%) Find the mean and variance of $y_t$.

Now assume that the PIH holds such that $\Delta C_t = \alpha u_t$. Assume the rate of interest is 10%.

b) (10%) Find the value of $\alpha$ (this should be a number).

c) (5%) What is the variance of consumption growth?

Now you are told that $y_2 = 210, y_1 = 200$ and $y_0 = 200$. (This holds for the remaining questions.)

d) (5%) What is $\Delta C_2$?

e) (10%) Assume that the consumer has assets $A_2 = 0$ at the beginning of period 2. What is $C_2$?

f) (5%) What is the conditional expectation $E\{C_3|y_2, y_1, y_0\}$?

2. (17%) Derive the consumption CAPM.

3. (18%) A consumer lives for 3 periods and earns 200$, X$, and 200$ in period 1, 2, and 3 respectively. $X$ is a normally distributed random variable with mean $EX = 200$ and variance $Var X = 5$. The consumer has an exponential utility function $U(C) = -e^{-\theta C}$ and is—in period 1—allowed to freely borrow and lend at an interest rate of 0 (for simplicity). Any borrowing or lending has to be paid back in period 2. For simplicity also assume that the consumer has a 0 rate of time preference ($\beta = 1$). The consumer is not allowed to borrow or lend in period 2 (meaning from period 2 to period 3). Find $C_1$, $C_2$ and $C_3$. 

PLEASE TURN OVER
4. (25%) Consider the CAPM. Assume that there are two states of the economy next year, “A” and “B”; state A happens with probability 0.2 and state B with probability 0.8. Assume the return to the market portfolio is 10% in both states. Now consider assets 1 and 2. For these we know the pay-outs. For asset 1 the payout is 20 in state A and 120 in state B, while for asset 2 the payout is 200 in state A and 5 in state B. Use the CAPM as it was derived in the handout. The safe rate of return is 1%. (Note: it is not really feasible for the market return to have zero variance and a higher rate of return than the safe asset—we make this assumption for simplicity.)

a) What would be the rate of returns of assets 1 and 2?

Now assume that in state A the return to the market portfolio is 10% and in state B the return to the market portfolio is 0%.

b) Find the rate of returns to asset 1 and asset 2 under this assumption.

c) Explain in words why one of the assets (which one) is worth relatively more in this situation compared to the situation where the market return has zero variance.