Midterm Exam 1 — 5 questions. All sub-questions carry equal weight.

1. (30%) Consider a uniform distribution on the closed interval [1, 4]. Assume a random variable $X$ follows this distribution.
   a) What is the Cumulative Density Function (CDF)?
   b) What is the density function (PDF)?
   c) Find the Moment Generating Function.
   d) Find the mean of $X$.
   e) Find the variance of $X$.

2. (24%) A study of college students finds that while 60 percent of college students are male, only 40 percent of college students with an A average are male. In contrast, 15 percent of female students have an A average. Assuming these results are accurate answer the following questions.
   a) Are “being a male student” and “having an A average” independent? Why?
   b) What is the probability that a randomly selected student has an A average?
   c) What is the probability that a randomly selected male student has an A average?

3. (24%) Assume that $X$ follows a standard exponential distribution with density $e^{-x}$ for $x > 0$.
   a) What is the density function for $Y$ if $Y = 2X$?
   b) Find $P(X < 1)$.
   c) Find the 10% upper percentile for $X$.
   d) Now assume that you are told that $X < 2$. Given that, what is $P(X < 1)$?

4. (12%) a) Define “excess kurtosis.”
   b) State Chebychev’s inequality.
   c) What is the formula for the probability of an event $A$ conditional on an event $B$?

5. (10%) For a random variable $X$, a constant $c$, and two functions $g()$ and $h()$ prove that
   a) $E[g(X) + h(X)] = E[g(X)] + E[h(X)]$.
   b) $E[cg(X)] = cE[g(X)]$. 
