
2. Let $X$ and $Y$ be normally distributed variables with means $\mu_X$ and $\mu_Y$, resp., and variances $\sigma_X^2$ and $\sigma_Y^2$, resp.

a) Show that the random variable

$$Z = X + Y,$$

is normally distributed and find its mean and variance. (Hint: Derive the moment generating function, use that the joint density of $X$ and $Y$ can be written as the product of the conditional density of $X$ given $Y$ and the marginal density of $Y$.)

b) Argue, using the result in part a), that if $X_1, X_2, ..., X_n$ are random variables with means $\mu_1, ..., \mu_n$, and $a_1, a_2, ..., a_n$ are constants then $a_1 X_1 + a_2 X_2 + ... + a_n X_n$ is a normally distributed random variable and state its mean and variance.

c) What is the distribution of the mean $\overline{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$?

3. If

$$\Sigma = \begin{pmatrix} 20 & 10 \\ 10 & 10 \end{pmatrix}$$

verify that

$$\Sigma^{1/2} = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$$

Also find $\Sigma^{-0.5}$ and $\Sigma^{-1}$ and verify that $\Sigma^{-0.5}/\Sigma^{-0.5} = \Sigma^{-1}$. 