Homework 5. Wednesday October 15. Due Wednesday October 22.

1. Practice problem 5.1 in Ramanathan, p. 84.

2. Assume you roll two dice. Let $X$ be the number of times you observe 1 or 3 eyes and let $Y$ be the number of times you observe a 3. Derive
   a) the joint probability distribution $f(x, y)$ (as in example 5.1).
   b) $f_X(x)$, the marginal probability function for $X$.
   c) $f_Y(y)$, the marginal probability function for $Y$.
   d) $P(X < Y)$.
   e) $P(Y = 2X)$.
   f) $P(X + Y = 2)$. (We will soon cover how to do this soon more systematically, but for now you should find the probability of the set of $(X, Y)$ pairs that sum to 2.)
   g) Are $X$ and $Y$ independent or dependent?

3. Let $f(x, y) = \frac{3}{16}xy^2 ; 0 < x < 2, 0 < y < 2$, be the joint density function for $X$ and $Y$. Find the marginal density functions $f_X(x)$ and $f_Y(y)$. Find the distribution function (CDF) for $X$. Are the two random variables independent?

4. Let $f(x, y) = \frac{1}{6}e^{-x/2-y/3}$ be the joint density function for $X$ and $Y$. Find the marginal density functions $f_X(x)$ and $f_Y(y)$. Are the two random variables independent?

5. Consider two random variables $X$ and $Y$. Assume they both are discrete and that $X$ can take the values 1, 2, and 4 while $Y$ takes the values 0 and 2. The probabilities for $(X, Y)$ are shown in the following table:

<table>
<thead>
<tr>
<th></th>
<th>X=1</th>
<th>X=2</th>
<th>X=4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y=0</td>
<td>3/24</td>
<td>3/24</td>
<td>6/24</td>
</tr>
<tr>
<td>Y=2</td>
<td>3/24</td>
<td>5/24</td>
<td>4/24</td>
</tr>
</tbody>
</table>

i) Find the marginal probabilities of $X$ and $Y$. Mark clearly which are the marginal probabilities of $X$ and which are the marginal probabilities of $Y$. Explain what the marginal probabilities measure.

ii) Find the means and the variances of $X$ and $Y$. 
iii) Are the events $X = 1$ and $Y = 2$ independent events?
iv) Are the random variables $X$ and $Y$ independent?
v) Find the probability $P(\{X > 1\} \cap \{Y \leq 1\})$
vi) Find the conditional distribution of $X$ given $Y = 2$.
vii) Find the random variable $E(X|Y)$.
viii) Take the mean of the random variable that you derived in vii) and verify that it equals $E(X)$.