
1. (From Midterm 1, Spring 2004, counted 20%) Suppose we have some observations of Texans and Californians. The probability of observing a Texan is 1/3 and the probability of observing a Californian is 2/3. Now assume the following (made up numbers), namely that the probability that a Texan is a republican is 40% (so the probability that he is a democrat is 60%, we assume), and the probability that a Californian is a republican is 50% (so the probability that a Californian is a democrat is also 50%).
   a) If you select one person from the population according to these probabilities, what is the probability that you will observe a republican from Texas? (Explain how you arrive at you answer)
   b) In the model described for Californians and Texans, are the events A: \{A person is a democrat\} and the event B: \{A person is from California\} independent events? (Explain how you find the answer).
   c) If you select 5 people randomly from the Texans. What is the expected number of republicans?

2. Assume that a random variable \(X\) is uniformly distributed on the interval \([1, 6]\).
   a) What is the probability that \(X < 3\)? And the probability that \(X > 5\)?
   b) What is the probability that \(7 + 3X \geq 15\)?
   c) If \(f(x) = 7 + 3x\), what is the density for the random variable \(Y = f(X)\)?
   d) What is the distribution function
   e) If \(f(x) = e^x\), what is the density and distribution function of the random variable \(Y = f(X)\)?
   You have to be explicit about the support (the area where the density for \(Y\) non-zero).

3. Ramanthan, Practice Problem 3.7.


5. (28% of Midterm 1, 2005) Consider a uniform distribution on the closed interval \([0, 1]\). Assume a random variable \(X\) follows this distribution.
   a) Find the mean of \(X\).
b) Find the distribution of $Y = \log(X)$. (Be specific about all details of the distribution.)
c) Find $P(Y < -0.5)$.
d) Find $E(Y)$. 