Economic integration, industrial specialization, and the asymmetry of macroeconomic fluctuations

Sebnem Kalemli-Ozcan\textsuperscript{a}, Bent E. Sørensen\textsuperscript{b,}\textsuperscript{*}, Oved Yosha\textsuperscript{c}

\textsuperscript{a}Department of Economics, University of Houston, Houston, TX 77204, USA
\textsuperscript{b}Federal Reserve Bank of Kansas City, Economic Research Department, 925 Grand Boulevard, Kansas City, MO 64198-0001, USA
\textsuperscript{c}Berglas School of Economics, Tel Aviv University, Tel Aviv, 69978, Israel

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text

Abstract

We show empirically that regions with a more specialized production structure exhibit output fluctuations that are less correlated with those of other regions (less ‘symmetric’ fluctuations). Combined with the causal relation running from capital market integration to regional specialization found in an earlier study, this finding supports the idea that higher capital market integration leads to less symmetric fluctuations. This mechanism counter-balances the effect of lower trade-barriers on the symmetry of fluctuations quantified by Frankel and Rose (1998). Deriving a simple closed form expression for the gains from risk sharing for CRRA utility is an independent contribution of the present article. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Much of the debate on the desirability of economic integration centers on the degree of synchronization (symmetry) of macroeconomic fluctuations across...
countries. It has been noted that economic integration itself will affect the symmetry of macroeconomic fluctuations. Frankel and Rose (1998) argue that removal of trade barriers will entail more correlated business cycles, since a higher level of trade will allow demand shocks to more easily spread across national borders. They further mention that economic integration will render policy shocks more correlated and that knowledge and technology spillovers will increase (Coe and Helpman, 1995). Krugman (1993), on the other hand, claims that lower barriers to trade will induce countries to specialize more rendering output fluctuations less, not more, symmetric. Fig. 1 summarizes these effects visually.

Frankel and Rose (1998) provide empirical evidence for the mechanism they propose by regressing the pairwise correlation of business cycles on bilateral trade intensity instrumented by distance for a sample of OECD countries. They obtain a positive and significant coefficient which suggests that even if the effect proposed by Krugman is present in the data, it is dominated by the mechanism they describe.

Our goal here is two-fold. First, we want to draw attention to yet another mechanism: economic integration will lead to better income insurance through greater capital market integration which will, ceteris paribus, induce higher specialization in production and more trade rendering fluctuations less symmetric across countries. Second, we establish empirically that higher specialization in production indeed translates into less symmetry of output fluctuations; see Fig. 1.

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1In recent years, the discussion of European monetary integration has dominated the scene. It is argued that the cost of joining a monetary union and giving up independent monetary policy will be low if countries have highly synchronized (symmetric) business cycles. See De Grauwe and Vanhaverbeke (1993) for an exposition of the main issues. Naturally, this debate builds on Mundell’s (1961) classic analysis of Optimum Currency Areas.

2These additional mechanisms should also contribute to fluctuations becoming more symmetric following economic integration.

3Krugman corroborates his argument with the observation that US states are more specialized in production than European countries.

4It is well established empirically that trade volume increases with geographical proximity; see Table 1 in Frankel and Rose (1998).

5The effect suggested by Krugman operates via inter-industry trade while that proposed by Frankel and Rose applies mainly to intra-industry trade. In their analysis, Frankel and Rose use the total volume of trade instrumented by distance. Since distance affects both inter- and intra-industry trade, the positive relation between trade volume and business cycle correlation indicates that the effect suggested by Krugman is not the dominant one. Rose (2000) adds another empirical building block to the Frankel/Rose mechanism by providing cross-sectional country- and regional-level evidence that a common currency enhances the volume of trade. Canova and Dellas (1993) also study the relation of trade interdependencies and business cycles. They focus on the transmission across countries of business cycle fluctuations and obtain mixed results. They do not discuss the potential endogenous response of country-level business cycles to economic integration.
Fig. 1. The effects of economic integration on fluctuations asymmetry. Notes: "Coe and Helpman (1995)." This channel is mentioned by Frankel and Rose (1998). 'Frankel and Rose (1998) estimate the overall effect on fluctuations asymmetry of lowering trade barriers. They instrument by distance (a trade barrier). Krugman (1993) stresses the effect of lower trade barriers on specialization. "Kalemli-Ozcan et al. (1999) estimate the effect of greater inter-regional income insurance on industrial specialization. In the current paper, we estimate the effect of greater industrial specialization on fluctuations asymmetry. Typically, more intra-industry trade. Typically, more inter-industry trade.

The claim that economic integration will induce higher specialization in production through better cross-country income insurance has been substantiated empirically by Kalemli-Ozcan et al. (1999). Here we find that higher specialization in production is associated with less symmetry of output fluctuations. Together,
these findings substantiate an effect of income insurance on industrial specialization which, other things equal, results in less symmetric output fluctuations.\(^6\)

There is no contradiction between our empirical findings and those reported by Frankel and Rose (1998) since the mechanism we suggest (better opportunities for income diversification) is independent of barriers to trade.\(^7\) Our papers thus isolate distinct potential effects of economic integration on fluctuations asymmetry that are part of a rich menu of economic mechanisms that will jointly shape post-integration patterns of GDP fluctuations.\(^8\) Which effect will dominate in the European Monetary Union remains an open empirical question.

To establish empirically that higher specialization in production is associated with less symmetry of output fluctuations, we calculate measures of asymmetry in GDP fluctuations for OECD countries and US states and regress them on industrial specialization indices. The regressions control for relevant economic and demographic variables and yield positive and significant coefficients for the specialization indices.

A natural measure of asymmetry is one that quantifies the potential loss of welfare due to asymmetric GDP fluctuations in the absence of risk sharing mechanisms. (Of course, we want an asymmetry measure that is independent of the amount of risk sharing actually obtained). To construct such a measure we use a simple model of risk sharing among countries inhabited by representative agents. First, we evaluate the welfare that each country would obtain if it were constrained to consume its own GDP. Next, we evaluate the welfare that each country would obtain if output were pooled across the entire OECD. The difference represents potential gains from risk sharing that we here regard as a measure of fluctuations asymmetry. The logic is that the more a country can gain from sharing risk with other countries in a group, the more asymmetric are its GDP shocks relative to the group.\(^9\) The derivation of a simple closed form expression for the gains from risk sharing is an independent contribution of the present article.

\(^6\)Our finding in this paper also serves as partial corroboration for the mechanism suggested by Krugman (1993). But to our knowledge, the positive effect of lower trade barriers on industrial specialization has not yet been established by systematic empirical analysis.

\(^7\)In particular, the main instrument used by Frankel and Rose – geographical distance – seems to be orthogonal to the amount of income diversification across regions and countries. Sørensen and Yosha (1998) find that the amount of insurance across OECD countries (including Japan, Canada, and the US) is very similar to the amount of insurance across European Union countries and Sørensen and Yosha (2000) find that the amount of insurance within different regions of the US is very similar to the amount of insurance within the US as a whole. It seems that the amount of insurance among regions and countries is determined by institutional factors (for instance, the legal and financial environment); see Kalemli-Ozcan et al. (1999).

\(^8\)See Giannetti (1999) for yet another such mechanism. She argues that industrial composition determines who benefits from knowledge spillovers. In her model, high-productivity rich regions will become richer and even more specialized in the high-productivity sector relative to poor regions as a result of economic integration.

\(^9\)An analogous reasoning holds for US states.
In the next section, we review relevant conceptual issues. In Section 3, we sketch a stylized model of fluctuations in order to provide a framework for interpreting our findings. In Section 4, we present our measure of fluctuations asymmetry which is derived in detail in Appendix A. In Section 5, we define the specialization indices that we use and in Section 6, we describe our data and report the empirical results. Section 7 concludes the paper.

2. Conceptual issues

In the presence of production risk and with no markets for insuring it, countries that specialize in the production of a small number of goods may suffer a loss in economic welfare due to the high variance of GDP. But if international financial markets and goods markets are integrated, countries are able to insure against asymmetric shocks through diversification of ownership and can therefore ‘afford’ to have a specialized production structure. The central empirical implication of this idea is that better insurance among countries should be associated with higher country-level specialization in production. (An analogous logic holds for regions within countries). This was confirmed empirically by Kalemli-Ozcan et al. (1999) who established a causal link running from risk sharing (income insurance), facilitated by a developed and reliable financial system, to specialization in production.

Financial integration will likely lead to more specialization since entrepreneurs will be less reluctant to ‘put more eggs in the same basket.’ This is because a greater fraction of their (or their investors’) income will be derived from other sources, such as internationally diversified investment funds. Further, foreign investors will be buying shares in domestic firms since they themselves will be seeking to diversify their portfolios internationally. It is also likely that governments will insist less on subsidizing diversity within national borders.

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10See Brainard and Cooper (1968); Kemp and Liviatan (1973); and Ruffin (1974).
11See Helpman and Razin (1978a,b) and Feeney (1994). The idea that insurance induces specialization has made an impact in the economic growth and development literature; see Greenwood and Jovanovic (1990); Saint-Paul (1992); Acemoglu and Zilibotti (1997); and Feeney (1999). Closely related to the topic of this paper is Obstfeld (1994a). In his model, countries choose the investment mix in risky (high return) projects and safe (low return) projects. International capital market integration provides insurance, inducing countries to shift investment towards high return projects promoting faster growth.
12To address the possibility of endogeneity bias, they used instrumental variables which are exogenous to the degree of specialization but are likely to be correlated with the extent of observed inter-regional risk sharing. These include quantitative indicators of the ‘legal environment’ that are likely to have an impact on the amount of cross-regional ownership of assets, for example, the degree of protection of investor rights (La Porta et al., 1998).
What are the implications for the European Monetary Union? Today, there is little risk sharing between countries, but capital market integration is bound to increase with further economic integration. First, there is some indication that a change is already taking place in Europe. Liebermann (1999) has replicated the Sørensen and Yosha (1998) study, extending the sample period to include the 1990s. She finds significantly higher cross-country insurance via capital markets during the period 1992–1997 which indicates that capital markets in Europe are integrating. Second, the high degree of cross-regional ownership in the US, documented by Asdrubali et al. (1996), suggests that economic and monetary unification will indeed induce a greater geographical spread of ownership across Europe.

We expect the main impact on specialization to occur in manufacturing where corporate ownership is most prevalent. At the 1-digit level, production patterns are determined to a large extent by exogenous circumstances, most notably the existence of natural resources such as oil, minerals, or fertile land. However, cross-border insurance should have an impact on specialization even at the 1-digit level – at least at the margin. To illustrate, with insurance against asymmetric fluctuations it would be less risky for the Italian Riviera regions to further specialize in tourism and for Norway to further specialize in oil production.

If countries indeed specialize more as a result of international capital market integration, the opportunities to insure within countries will be reduced. Therefore, higher specialization in production should render country-level GDP fluctuations less symmetric.14

We use data for the 50 US states and a sample of OECD countries to test empirically whether countries and states that are more specialized (at the 1-digit level and in manufacturing) are subject to less symmetric fluctuations. We repeat the analysis for the sample of US states alone – this may be more informative about conditions in an economic union – and obtain similar results.

It is worth stressing that although the effect of capital market integration on the asymmetry of macroeconomic fluctuations is expected to happen over time, there is no compelling need to formulate and test a dynamic model using time-series data. Paraphrasing Rose (2000, p. 11), one can perfectly well exploit cross-sectional variation to trace the effects of capital market integration on the asymmetry of fluctuations.

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13 See French and Poterba (1991) and Tesar and Werner (1995) who document ‘home bias’ in portfolio holdings, Backus et al. (1992) who compare cross-country GDP correlations and consumption correlations, and Sørensen and Yosha (1998) and Arreaza (1998) who carry out cross-country variance decompositions of shocks to GDP for EC/OECD and Latin American countries, respectively. All these studies point to negligible risk sharing through cross-country ownership of assets.

14 Interestingly, Kenen (1969) points out that well diversified countries suffer less from asymmetric GDP fluctuations and should be more inclined to join a monetary union. He does not, however, take the further step of arguing that joining a monetary union will itself affect the degree of industrial specialization through the mechanism described above.
3. A stylized model of fluctuations

In order to focus the discussion, we make use of a simple stylized model. We follow Frankel and Rose (1998) and express the per capita GDP growth process of countries $i$ and $j$ as:

$$\Delta \log gdp_i^t = \sum_s \alpha_s^i u_{s,t}^i + \beta^i \nu_i^t$$
$$\Delta \log gdp_j^t = \sum_s \alpha_s^j u_{s,t}^j + \beta^j \nu_j^t$$

The variables on the left hand side should be regarded as generic expressions for per capita GDP fluctuations in each country (whether measured as log-differences of GDP at the 1-year frequency or as HP-filtered GDP, etc.).\footnote{For brevity, we will often omit the adjective ‘per capita.’} The variable $u_{s,t}^i$ represents a time $t$ sector-specific shock to the output in sector $s$ which is common to both countries. It reflects technological changes, sudden changes in the prices of inputs that are more heavily used in some sectors and changes in the composition of demand. The variables $\alpha_s^i$ and $\alpha_s^j$ are the weights of sector $s$ in the total output of countries $i$ and $j$ – they are not indexed by $t$ to indicate that they do not change from year to year.\footnote{It is well known that industrial composition is not constant over time. Kim (1995), for example, documents how industrial specialization in the US gradually changed over the past century, while Imbs and Wacziarg (2000) formalize the idea that countries experience ‘stages of diversification’ as they develop and grow. Our empirical analysis (as well as the analysis in Frankel and Rose, 1998) is cross-sectional, and uses data from a relatively short time period. The formulation in Eq. (1) is, therefore, appropriate for our purposes even if the constants $\alpha_s^i$ and $\alpha_s^j$ slowly change over time.}

The variables $\nu_i^t$ and $\nu_j^t$ represent the time $t$ country-specific GDP shocks that are common to all the sectors in each economy and are interpreted as country-wide policy shocks. The variables $\beta^i$ and $\beta^j$ represent the weights in each country of the country-wide (as opposed to sector-specific) shocks.

The variables $u_{s,t}^i$, $\nu_i^t$, and $\nu_j^t$ are assumed to be identically distributed random variables with mean zero and unit variance. They are further assumed to be independently distributed over time. The sector-specific shocks, $u_{s,t}^i$, are assumed to be independently distributed of $\nu_i^t$ and $\nu_j^t$, but the latter variables have a time-invariant correlation coefficient denoted $\rho^{ij}$. It is further assumed that the $U_{s,t}$ terms are independent of each other, and that $\sum (\alpha_s^i)^2 = 1$.

The correlation of the country-specific GDP shocks, $\rho^{ij}$, has two interpretations. It captures the common element in the shocks themselves, e.g. the extent to which major strikes are likely to occur in the same year in both countries. It also captures the common response of GDP in both countries to shocks that occur in only one of the countries, e.g. the response of aggregate demand in both countries to increased government spending in one country (a ‘Keynesian’ demand spillover effect).

Consider the first term on the right hand side of each of the equations in Eq. (1),
\( \Sigma_i \alpha_i' u_{i,t} \) and \( \Sigma_i \alpha_i' v_{i,t} \). Their correlation is \( \Sigma_i \alpha_i' \alpha_i' \). We predict that the distribution of the sector shares, \( \alpha_i \), will become more dissimilar across countries as a result of capital market integration with various countries specializing in specific sectors. As a result, the correlation across countries of sector-specific shocks, \( \Sigma_i \alpha_i' \alpha_i' \), will decrease.\(^{17}\) Therefore, the correlation of GDP fluctuations (i.e. of \( \Delta \log gdp_i \) and \( \Delta \log gdp_j \)) will also decrease.

The stylized model also illustrates that our analysis is complementary to that of Frankel and Rose (1998). They concentrate on the second term on the right hand side in each of the equations in Eq. (1), \( \beta_i' \nu_i' \) and \( \beta_j' \nu_j' \), representing country-specific shocks. The correlation of these terms (\( \rho \)) will increase as a result of economic integration through lower trade barriers and increased intra-industry trade, and so will the correlation of GDP fluctuations. Of course, lowering of trade barriers is also likely to make the distribution of sector shares more dissimilar as predicted by Krugman (1993); but according to the results of Frankel and Rose (1998) this effect is dominated by increased correlation of country specific shocks. In order to predict the total effect of economic integration on fluctuations asymmetry one needs to know the elasticities of the sector shares, \( \alpha_i \), of the demand shock correlation, \( \rho \), and of the weights, \( \beta \), with respect to all the relevant variables that will change as a result of economic and financial integration. This should be high on the research agenda of scholars interested in the economics of monetary unification.

4. Measuring the asymmetry of fluctuations

Academic research on the asymmetry of economic shocks, at the regional and national levels, dates back at least to Cohen and Wyplosz (1989) and Weber (1991) who studied output growth rate correlations for European countries, and to Stockman (1988) who distinguished between country-specific and industry-specific shocks.\(^{18}\) This literature generated a debate,\(^{19}\) and there is no consensus regarding the ‘correct’ statistical model for country-level (or regional-level) GDP. We, therefore, opt for a more ‘structural’ approach that builds on economic theory:

\(^{17}\)To illustrate, if country \( i \) produces only good \( s \) and country \( j \) produces only good \( s' \), this correlation is zero.


\(^{19}\)Some studies suggest that economic integration will result in less symmetric shocks (De Grauwe and Vanhaverbeke, 1993) or that the degree of asymmetry will not change (Forni and Reichlin, 1997) while others conclude that economic integration will result in more symmetric shocks (Clark and van Wincoop, 1999; Frankel and Rose, 1998). These studies typically attempt to identify the full stochastic process for regional output (as in Forni and Reichlin) or concentrate directly on output correlations (as in Clark and van Wincoop and Frankel and Rose).
we calculate the increase in utility obtained from consuming a fraction of aggregate GDP rather than actual GDP for the representative consumer of each country. More precisely, in the framework of a simple model of optimization and general equilibrium, we evaluate the increase in per capita discounted expected utility that would be achieved by moving from financial autarky (each country consumes the value of its GDP) to full insurance (each country consumes a fixed fraction of aggregate GDP). The fraction of aggregate GDP that a country consumes under full insurance is the fraction that would accrue to it in a perfect risk sharing general equilibrium. We interpret this utility gain as a measure of fluctuations asymmetry. The more a country can gain from sharing country-specific risk with other countries in a group, the more asymmetric are its GDP fluctuations relative to the group.

4.1. A utility-based measure of fluctuations asymmetry

Our proposed measure builds on the following counter-factual thought experiment. Consider a group of countries inhabited by risk averse agents (consumers) who derive utility from consumption of a homogeneous non-storable good. This group constitutes a ‘stochastic endowment economy’ in the sense that the GDP of these countries is regarded as exogenous and stochastic by consumers. Securities markets in this economy are complete, permitting cross-country insurance. Consumers within each country are identical ex-ante as well as ex-post: all have the same utility function, the same rate of time preference, $d$, and are subject to the same realization of uncertainty. It is well known that under commonly used assumptions – symmetric information, no transaction costs, CRRA utility, identical rate of time preference for all countries – perfect risk sharing among the countries in the group implies that $c_i = \frac{k}{gdp}$. Here $c_i$ is the per capita consumption in country $i$, $gdp$ is the aggregate per capita GDP of the group of countries under consideration, and $k$ is a country-specific constant that does not vary across ‘states of the world’ or over time.

For each country, we compare the expected utility of consuming $\frac{k}{gdp}$, with that of consuming the endowment, $gdp_i$. To quantify these gains we must make distributional assumptions. Let the natural logarithm of the per capita GDP of the

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20 Most of the time, we will refer only to countries, but our analysis applies equally well to states within the US.

21 We, thus, focus on fluctuations asymmetry between countries, ignoring potential symmetry within countries.

22 Under perfect risk sharing, each country consumes a fixed fraction of the aggregate gross product every period regardless of the realization of GDP shocks. The constant $k'$ represents the strength of country $i$’s claim in the risk sharing arrangement. See Huang and Litzenberger (1988) for a derivation for CRRA utility. In the literature, the perfect risk sharing condition is often expressed in terms of aggregate consumption, but since output is assumed to be non-storable in our model, $gdp_i$ is equal to aggregate consumption.
group and the per capita GDP of each country be random walks with linear trend. Further suppose that, conditional on \( gdpi \) and \( gdpj \), the joint distribution of the log-differences of these processes is stationary and normal: \( \Delta \log gdp_i \sim N(\mu, \sigma^2) \), \( \Delta \log gdp_j \sim N(\mu, \sigma^2) \), and \( \text{cov}(\Delta \log gdp_i, \Delta \log gdp_j) = \sigma^2 \) for all \( t \). With these assumptions we obtain closed form solutions for the gains from risk sharing and, in the process, for the equilibrium shares in aggregate consumption (the \( k_i \)'s).

To the best of our knowledge, this has not been accomplished before in the literature on risk sharing.

In the derivation (is presented in full detail in Appendix A) we distinguish between CRRA utility, \((1 / 1 - \gamma) c^{1-\gamma} \) \((\gamma \neq 1)\), and log-utility which yields simple and intuitive expressions but is more restrictive.

The utility gains from risk sharing will be substantial only if shocks have a cumulative effect over longer horizons. If gross product were not highly persistent, these gains would be small as pointed out by Obstfeld (1994b). Indeed, the random walk assumption is important for our derivation. If the actual GDP growth rate of countries is stationary, the random walk assumption will cause under-estimation of the gains if the actual GDP growth rate is more persistent than a random walk. Since our regression results depend only on the relative magnitude of the gains from risk sharing, it is not crucial for our purpose to pin down the level of these gains.

It is economically more meaningful to express the gains from risk sharing in terms of consumption certainty equivalence. We do so by calculating the permanent percentage increase in the level of consumption that would generate an equivalent increase in expected utility. More precisely, the gain in utility (of

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23This assumption involves an approximation since the aggregate GDP cannot, in general, be strictly log-normally distributed if each country’s GDP is log-normally distributed.
24After the final version of this article was completed we became aware of Kim et al. (2000). Using a quite different approach, they obtain analytical solutions for gains from risk sharing in a 2-country framework that allows for more general dynamics than we do.
25The approximation that aggregate GDP is log-normally distributed may introduce minor bias of unknown direction. For example, the calculated shares of aggregate GDP that each state or country would consume under perfect risk sharing do not sum precisely to 1 in our calculations (see Appendix A). In this study the shares sum to a number very close to one (between 1.00 and 1.01), which indicates this bias is negligible.
26As it is for van Wincoop’s (1994) estimation of non-exploited welfare gains from risk sharing.
27For US states, we performed state-by-state Augmented Dickey–Fuller tests for a unit root in state gross product and were never able to reject a unit root. These tests, based on relatively short samples, have low power against near unit root alternatives, and indeed the question of whether typical macroeconomic series contain unit roots is still open. Nevertheless, as shown in the Appendix to Obstfeld (1992), welfare gains are substantial when shocks to gross product are persistent whether or not the process contains an exact random walk. Another issue is that our measure may under-estimate gains from insurance since we do not use preferences with separate parameters for risk aversion and intertemporal elasticity of substitution. Obstfeld (1994b) shows that welfare gains estimates are typically higher with such utility functions.
28We follow van Wincoop (1994) in this respect.
moving from autarky to perfect risk sharing) equals the gain in utility that would be achieved by increasing consumption permanently from GDP$_0$ to GDP$_0(1 + G')$. $G'$ is our country-by-country measure of fluctuations asymmetry and, for log-utility, is given by:  

$$G' = \frac{1}{\delta} \left( \frac{1}{2} \sigma_i^2 + \frac{1}{2} \sigma_i^2 - \text{cov} \right),$$  

where $\delta$ is the intertemporal discount rate. The intuition for this formula is straightforward. First, the gain from sharing risk is higher for countries with a lower covariance between $\Delta \log gdp$ and $\Delta \log gdp_i$. The interpretation is that countries with `countercyclical' output are compensated for providing insurance to other countries by stabilizing aggregate output. Second, the higher the variance of country $i$'s GDP the more it contributes to smoothing shocks in other countries, other things equal, and the more it receives in exchange for this service. Third, the higher the variance of the aggregate gross product of the group, keeping the variance of country $i$'s GDP constant, the more other countries would be willing to `pay' country $i$ for joining the risk sharing arrangement. The interpretation of the formula for CRRA utility is similar, although less transparent). We regard $G_i$ as a reasonable and intuitive country-by-country measure of fluctuations asymmetry: the more a country can gain from sharing idiosyncratic risk with other countries in a group, the more asymmetric are its shocks relative to the group.

There is nothing novel in characterizing the equilibrium allocation of an Arrow–Debreu exchange economy, but to the best of our knowledge, a closed form solution for the equilibrium sharing rule and the gains from risk sharing for CRRA utility has not been explicitly worked out before.  

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29See Appendix A for a derivation for both CRRA and log-utility.

30And $e^{-\delta t}$ is the intertemporal discount factor.

31Of course, $\sigma_i$, the variance of the growth rate of aggregate GDP, cannot change without any of the $\sigma_i^2$'s changing. The distributional approximation regarding aggregate GDP allows us to treat $\sigma_i^2$ as a parameter (that can be estimated from aggregate GDP data) rather than as a complicated function of the country-by-country $\sigma_i^2$'s.

32Obstfeld (1994b) provides a closed form solution for the welfare gains due to a reduction in consumption variability in a partial equilibrium setting, whereas van Wincoop (1994) computes welfare gains from risk sharing in a general equilibrium model but relies on approximation techniques. Of course, our work builds on these papers which were the first to compute and estimate welfare gains from risk sharing taking into account the persistence of shocks to GDP; see also Tesar (1995). van Wincoop (1994) calculates potential gains from risk sharing using consumption data, measuring how much further gains from risk sharing can be achieved by moving from the observed consumption allocation (in the data) to the perfect risk sharing consumption allocation. (That is, he computes non-exploited gains from risk sharing). The potential gains from risk sharing that we calculate have a different interpretation, as they are based on a countercfactual thought experiment: moving from autarkic (rather than actual) consumption to perfect risk sharing. The calculation of this measure uses only GDP data rendering it more appropriate as a measure of GDP fluctuations asymmetry. Of course, the techniques developed here can also be used to calculate the non-exploited gains from risk sharing using consumption data as in van Wincoop (1994).
In the empirical implementation, the parameters $\sigma^2$, $\sigma_i^2$, and $\text{cov}_i$ are estimated using country-level (or state-level) and aggregate GDP data. A natural measure of output is GDP deflated by the Consumer Price Index (CPI). We stress the logic of deflating by the CPI rather than by a GDP-deflator: since our measure is utility based, we want measured output to reflect consumption in autarky (with countries consuming the value of their GDP). Thus, we want to translate GDP to the amount of consumption that it can buy which is obtained by deflating using the CPI. Note, also that our fluctuations asymmetry measure focuses entirely on the value of GDP (in terms of consumption) and its volatility, not on the composition of GDP.

5. Measuring specialization in production

Each specialization index is computed annually (for every country) for the relevant sample years and averaged over time. The 1-digit specialization index for country $i$ is

$$\text{SPEC}_{i} = \sum_{s=1}^{S} \left( \frac{\text{GDP}_{i}^s}{\text{GDP}_i} - \frac{1}{J-1} \sum_{j \neq i} \frac{\text{GDP}_j^s}{\text{GDP}_j} \right)^2,$$

where $\text{GDP}_{i}^s$ is the gross product of (1-digit) sector $s$ in country $i$, $\text{GDP}_i$ is the total GDP of this country, $S$ is the number of sectors, and $J$ is the number of countries in the group. The index represents the distance between the vector of sector shares in country $i$, $\text{GDP}_{i}^s / \text{GDP}_i$, and the vector of average sector shares in the countries other than $i$. It measures the extent to which country $i$ differs from the other countries in terms of industrial composition. Similarly, the manufacturing (2-digit) specialization index for country $i$ is

$$\text{SPEC}_{iM} = \sum_{s=1}^{S} \left( \frac{\text{GDP}_{i}^s}{\text{GDP}_i^M} - \frac{1}{J-1} \sum_{j \neq i} \frac{\text{GDP}_j^s}{\text{GDP}_j^M} \right)^2,$$

where $\text{GDP}_{i}^s$ is the gross product of manufacturing sector $s$ in country $i$, and $\text{GDP}_i^M$ is the total manufacturing gross product of this country. Alternatively, we use the indices

To illustrate, consider Alaska and suppose that it produces only oil. Suppose now that physical production of oil remains fixed from period $t$ to period $t+1$ but that the price of oil doubles, whereas the CPI is unchanged. Deflating by the GDP-deflator would yield no change in the real value of Alaska’s output, whereas deflating by the CPI would yield a doubling of the value of output. The latter makes more sense since Alaskans consume approximately the same basket of goods as the rest of the nation and they therefore become ‘richer’ when oil prices increase. In sum, when using a utility based measure of fluctuations asymmetry, output must be measured in consumption-equivalent terms.
6. Empirical analysis

6.1. Data used

United States: Gross state product (GSP) data are from the Bureau of Economic Analysis (BEA). Washington DC is very atypical and is omitted. The sample period for GSP by sector (used for computing specialization indices) is 1977–1994 while for total GSP (used for computing the fluctuations asymmetry measure) it is 1963–1994. We transform all gross product magnitudes to per capita terms using population by state, also obtained from the BEA. We use BEA data for ISIC 1-digit industries and utilize BEA data for 21 manufacturing sub-sectors, which we aggregate to nine ISIC 2-digit levels. High school enrollment in percent of total population (1990) and total land mass are from the 1997 Statistical Abstract of the United States. The data are transformed to constant prices using the US aggregate CPI. All the data are annual.

OECD: We use data from the OECD National Accounts 1996, Volume 2. The countries in our sample are Belgium, Denmark, France, Netherlands, West Germany, Austria, Canada, Finland, New Zealand, Norway, and the US. We restrict attention to this sample due to missing sectoral GDP data for other OECD countries. Data for Greece are available, but it was omitted a priori since during the sample period it was at a substantially lower level of economic development than the rest of the countries, with a very high dependence on agricultural production. The sample period for sectoral GDP (used for computing specialization indices) is 1977–1990 for the nine ISIC 2-digit manufacturing sectors and 1980–1990 for the ISIC 1-digit industries. The sample period for total GDP (used for computing the fluctuations asymmetry measure) is 1963–1993. GDP is transformed to per capita terms using population data from the National Accounts, and is further converted into constant dollars using the CPI for each country (from the National Accounts) and 1990 (end of year) exchange rates (from the IMF International Financial Statistics database). Land area is from the 1997 Statistical

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\[ \text{SPEC}_1 = \sum_{i=1}^{s} \left[ \frac{\text{GDP}_i'}{\text{GDP}_i} - \frac{1}{J-1} \sum_{j \neq i} \frac{\text{GDP}_j'}{\text{GDP}_j} \right], \]

\[ \text{SPEC}_{2M} = \sum_{i=1}^{s} \left[ \frac{\text{GDP}_i'}{\text{GDP}_i^M} - \frac{1}{J-1} \sum_{j \neq i} \frac{\text{GDP}_j'}{\text{GDP}_j^M} \right], \]

for 1-digit and manufacturing specialization, respectively.\(^{34}\)

---

\(^{34}\)These alternative indices put less weight on very specialized sectors.

\(^{35}\)The BEA official GSP series start in 1977 and we have combined these series with older series. The BEA advises against using the older data at the sectoral level.
Abstract of the United States. All the data are annual (from now on we refer to West Germany as Germany).

6.2. Asymmetry measures and specialization indices

Tables 1 and 2 display the variance of real per capita GDP, its covariance with aggregate GDP, the asymmetry measures for logarithmic and CRRA utility, and the specialization indices for US states and OECD countries, respectively. The variance of state-level GSP (Table 1) is typically higher than that of country-level GDP (Table 2). The gross product of oil-rich states and countries typically exhibits a low covariance with aggregate gross product (Alaska, Wyoming, Norway) – even negative in the case of Alaska.36

The third columns of Tables 1 and 2 provide the estimated measures of fluctuations asymmetry for log-utility. These numbers represent the permanent percentage increase in initial GDP (autarkic consumption in the initial period) that would generate the same increase in discounted expected utility as moving from autarky to perfect risk sharing. The numbers are calculated using the expression in Eq. (2) multiplied by 100.37

For log-utility, the average (population weighted) gain from sharing risk across the 50 US states is 1.27, while for OECD countries it is 0.67. For CRRA utility \( (\gamma = 3) \), the average gains are 1.55 and 0.62, respectively, so the sensitivity of these measures to the risk aversion parameter is not substantial.38 The estimated gains from risk sharing are quite large, but we reiterate that pinning down their level is difficult since the estimation strongly depends on the persistence of GDP shocks and on the chosen discount rate. Discount rates are usually estimated very imprecisely in econometric work and empirical measures of persistence are well known to be sensitive to model specification. Nevertheless, our analysis of

36A careful inspection of the notes to Table 1 reveals that the variance of US real per capita percent GDP growth is about 8 but is reported as about 6 in Table 2. This discrepancy is due to minor differences in the underlying data. The BEA US state-level data and the OECD data are internally consistent but they are obviously not quite consistent between them.

37They are computed as follows: \( (1/100^*\delta) \) times [one half the variance of \( 100^*\Delta\log GDP \) + one half the first column – the second column] the discount rate set at \( \delta = 0.02 \). The variance of \( 100^*\Delta\log GDP \) is 8.39 for the US and 4.08 for the aggregate OECD sample.

38To compare our estimates for OECD countries to those reported in van Wincoop (1994), consider for instance Belgium and the US – the first and the last entries in column 3, Table 2. Our estimates are 1.37 and 0.26 whereas van Wincoop’s are 1.1 and 0.6, respectively. The samples differ somewhat in the number of countries included and in the time period selected and if we had used a discount rate of 0.01 like van Wincoop, our numbers would have been 2.74 for Belgium and 0.52 for the US. A priori, the gains from risk sharing as defined in this paper should be larger since we measure potential gains (using GDP data) while van Wincoop measures non-exploited gains (using consumption data). However, risk sharing among OECD countries is quite low (Sørensen and Yosha, 1998) so all in all one should expect numbers of roughly the same order of magnitude, which is what we find. It is reassuring that the estimates obtained from two very different approaches, using different data, are quite similar.
Table 1
Fluctuations asymmetry and industrial specialization: US States

<table>
<thead>
<tr>
<th>States</th>
<th>(1) Variance (GSP)</th>
<th>(2) Covariance (GSP, GDP)</th>
<th>(3) Asymmetry index (log)</th>
<th>(4) Asymmetry index (CRRA spec. index)</th>
<th>(5) 1-Digit spec. index</th>
<th>(6) 2-Digit spec. index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alabama</td>
<td>11.57</td>
<td>9.05</td>
<td>0.46</td>
<td>0.54</td>
<td>0.63</td>
<td>3.69</td>
</tr>
<tr>
<td>Alaska</td>
<td>171.19</td>
<td>−9.42</td>
<td>49.60</td>
<td>79.36</td>
<td>13.10</td>
<td>22.60</td>
</tr>
<tr>
<td>Arizona</td>
<td>16.70</td>
<td>9.41</td>
<td>1.56</td>
<td>1.83</td>
<td>0.59</td>
<td>7.89</td>
</tr>
<tr>
<td>Arkansas</td>
<td>15.97</td>
<td>10.37</td>
<td>0.91</td>
<td>1.07</td>
<td>0.81</td>
<td>1.11</td>
</tr>
<tr>
<td>California</td>
<td>8.35</td>
<td>7.57</td>
<td>0.40</td>
<td>0.46</td>
<td>0.64</td>
<td>3.23</td>
</tr>
<tr>
<td>Colorado</td>
<td>5.06</td>
<td>5.05</td>
<td>0.84</td>
<td>0.98</td>
<td>0.53</td>
<td>2.42</td>
</tr>
<tr>
<td>Connecticut</td>
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<td>7.99</td>
<td>0.80</td>
<td>0.93</td>
<td>1.40</td>
<td>7.37</td>
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<td>18.57</td>
<td>8.61</td>
<td>2.43</td>
<td>2.85</td>
<td>3.16</td>
<td>21.88</td>
</tr>
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<td>0.70</td>
<td>0.81</td>
<td>1.47</td>
<td>1.13</td>
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<td>0.53</td>
<td>0.63</td>
<td>0.36</td>
<td>5.40</td>
</tr>
<tr>
<td>Hawaii</td>
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<td>4.16</td>
<td>2.56</td>
<td>2.99</td>
<td>4.14</td>
<td>20.61</td>
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<td>Idaho</td>
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<td>8.90</td>
<td>2.33</td>
<td>2.73</td>
<td>0.61</td>
<td>9.42</td>
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<tr>
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<td>0.32</td>
<td>0.49</td>
<td>1.24</td>
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<td>1.45</td>
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<td>3.08</td>
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<td>2.80</td>
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<td>2.49</td>
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<td>1.54</td>
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<td>6.36</td>
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<td>19.57</td>
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<td>1.07</td>
<td>1.25</td>
<td>0.40</td>
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<td>0.53</td>
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<td>1.11</td>
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<td>0.96</td>
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<td>1.20</td>
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<td>4.18</td>
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<td>0.95</td>
<td>0.36</td>
<td>2.89</td>
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<td>22.61</td>
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<td>1.69</td>
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<td>1.69</td>
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<td>5.46</td>
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<td>7.77</td>
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<td>4.25</td>
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<td>1.89</td>
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<td>0.75</td>
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<td>6.87</td>
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<td>Pennsylvania</td>
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<td>0.31</td>
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<td>1.14</td>
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<tr>
<td>Rhode Island</td>
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<td>0.80</td>
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</tr>
<tr>
<td>South Carolina</td>
<td>15.38</td>
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<td>0.61</td>
<td>0.73</td>
<td>1.49</td>
<td>8.87</td>
</tr>
<tr>
<td>South Dakota</td>
<td>35.85</td>
<td>11.56</td>
<td>5.28</td>
<td>6.41</td>
<td>2.34</td>
<td>4.73</td>
</tr>
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<td>Tennessee</td>
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<td>0.79</td>
<td>0.94</td>
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</tr>
<tr>
<td>Texas</td>
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<td>3.06</td>
<td>3.58</td>
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<td>3.28</td>
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<td>1.05</td>
<td>1.23</td>
<td>0.44</td>
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</table>
Table 1. Continued

<table>
<thead>
<tr>
<th>States</th>
<th>(1) Variance (GSP)</th>
<th>(2) Covariance (GSP, GDP)</th>
<th>(3) Asymmetry index (log)</th>
<th>(4) Asymmetry index (CRRA spec. index)</th>
<th>(5) 1-Digit spec. index</th>
<th>(6) 2-Digit spec. index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vermont</td>
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<td>9.55</td>
<td>1.10</td>
<td>1.28</td>
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<td>0.59</td>
<td>0.85</td>
<td>3.16</td>
</tr>
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<td>8.15</td>
<td>0.96</td>
<td>1.11</td>
<td>0.36</td>
<td>2.79</td>
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<td>5.43</td>
<td>1.53</td>
<td>1.79</td>
<td>1.60</td>
<td>15.37</td>
</tr>
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<td>Wisconsin</td>
<td>11.39</td>
<td>9.17</td>
<td>0.36</td>
<td>0.42</td>
<td>1.62</td>
<td>3.03</td>
</tr>
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<td>Wyoming</td>
<td>34.70</td>
<td>0.82</td>
<td>10.36</td>
<td>12.50</td>
<td>14.35</td>
<td>19.32</td>
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<tr>
<td>Average</td>
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<td>8.53</td>
<td>1.27</td>
<td>1.55</td>
<td>1.23</td>
<td>4.17</td>
</tr>
</tbody>
</table>

Notes: GSP is gross state product per capita. GDP is aggregate US gross domestic product per capita. The first four columns are calculated for 1963–1994 and the last two columns are for 1977–1994. Average numbers are population weighted. Column 1 is $10^4\sigma^2$, where $\sigma^2 = \text{var}(\Delta \log \text{GSP})$ [in other words, it is var($100\Delta \log \text{GSP}$)]. Column 2 is $10^4\text{cov}$, where $\text{cov} = \text{cov}(\Delta \log \text{GSP}, \Delta \log \text{GDP})$. Column 3 is $10^4(1/\delta) (\frac{1}{2} \sigma^2 + \frac{1}{2} \sigma^2 - \text{cov})$, where $\delta = 0.02$ (discount rate) and $10^4\sigma^2 = 8.39$ [var($100\Delta \log \text{GDP}$)]. Column 4 is $10^4[\log(\delta - (1 - \gamma)\mu - \frac{1}{2} (1 - \gamma)^2 \sigma^2) - \log((\mu - \gamma + \frac{1}{2} \sigma^2 + \frac{1}{2} \gamma^2 \sigma^2 - \gamma \text{cov})(1/1 - \gamma)) + (1/1 - \gamma) \log(\delta - (1 - \gamma)\mu - \frac{1}{2} (1 - \gamma)^2 \sigma^2) - (1/1 - \gamma) \log(\delta - (1 - \gamma)\mu - \frac{1}{2} (1 - \gamma)^2 \sigma^2)]$, where the risk aversion parameter is $\gamma = 3$ and the US GDP growth rate is $\mu = 0.020$. Specialization indices are defined in the text. The displayed indices are multiplied by 100.

Specialization and asymmetry depends only on the relative value of the asymmetry measure across countries, and this is unlikely to be very sensitive to the persistence of GDP shocks and the discount rate.\textsuperscript{39}

The third columns of Tables 1 and 2 reveal that the oil-rich states and countries (e.g. Alaska, North Dakota, Wyoming, Norway) exhibit high asymmetry measures, and it appears that small states and countries have relatively high asymmetry measures. Finland has the highest asymmetry measure among the OECD countries which is most likely due to the sharp recession experienced after the collapse of the Soviet Union. The asymmetry measures calculated with CRRA utility ($\gamma = 3$) are displayed in column 4 of Tables 1 and 2. In general, the ranking of states and countries is the same as for log-utility (column 3). For the US the asymmetry measures are higher for CRRA utility while the opposite is true for OECD countries.\textsuperscript{40}

\textsuperscript{39}Moreover, the log-utility measure is proportional to $1/\delta$, which renders the $t$-statistics in the regressions fully independent of the size of $\delta$.

\textsuperscript{40}We verified empirically that this is due to different growth rates of OECD countries and US states during the sample period. The asymmetry measure for log-utility is independent of growth rates, but for CRRA utility we cannot fully disentangle this effect from ‘pure’ risk sharing (see Appendix A).
Table 2
Fluctuations asymmetry and industrial specialization: OECD countries

<table>
<thead>
<tr>
<th>Countries</th>
<th>(1) Variance (GDP)</th>
<th>(2) Covariance (GDP,GDP)</th>
<th>(3) Asymmetry index (log)</th>
<th>(4) Asymmetry index (CRRA)</th>
<th>(5) 1-Digit spec. index</th>
<th>(6) 2-digit spec. index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>7.65</td>
<td>3.11</td>
<td>1.37</td>
<td>1.28</td>
<td>3.28</td>
<td>1.07</td>
</tr>
<tr>
<td>Denmark</td>
<td>7.62</td>
<td>3.58</td>
<td>1.13</td>
<td>1.05</td>
<td>1.05</td>
<td>1.14</td>
</tr>
<tr>
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<td>0.49</td>
<td>0.39</td>
<td>2.85</td>
</tr>
<tr>
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<td>6.60</td>
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<td>0.85</td>
<td>0.79</td>
<td>0.64</td>
<td>2.59</td>
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<td>8.38</td>
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<td>1.20</td>
<td>1.12</td>
<td>2.51</td>
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<td>0.85</td>
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<td>10.60</td>
<td>4.69</td>
<td>1.33</td>
<td>1.24</td>
<td>0.41</td>
<td>1.69</td>
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<tr>
<td>Finland</td>
<td>21.67</td>
<td>4.36</td>
<td>4.26</td>
<td>4.05</td>
<td>0.90</td>
<td>3.89</td>
</tr>
<tr>
<td>New Zealand</td>
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<td>4.08</td>
<td>2.36</td>
<td>2.22</td>
<td>0.95</td>
<td>26.30</td>
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<td>Norway</td>
<td>7.03</td>
<td>0.98</td>
<td>2.29</td>
<td>2.14</td>
<td>2.80</td>
<td>31.67</td>
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<tr>
<td>United States</td>
<td>5.88</td>
<td>4.46</td>
<td>0.26</td>
<td>0.23</td>
<td>0.97</td>
<td>5.40</td>
</tr>
<tr>
<td>Average</td>
<td>6.75</td>
<td>4.07</td>
<td>0.67</td>
<td>0.62</td>
<td>1.20</td>
<td>5.04</td>
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</tbody>
</table>

Notes: GDP is gross domestic product per capita of each country. GDPp is the total gross domestic product per capita of the 11 OECD countries in the sample. The first four columns are calculated for 1963–1993. The fifth column displays average values for 1980–1990 and the sixth column displays average values for 1977–1990. Average numbers are population weighted. Column 1 is $10^4\sigma_i^2$, where $\sigma_i^2 = \text{var}(\Delta \log GDP)$ [in other words, it is var(100*$\Delta \log GDP$)]. Column 2 is $10^4\text{cov}$, where $\text{cov} = \text{cov}(\Delta \log GDP, \Delta \log GDP_p)$. Column 3 is $10^4(1/\delta) \sigma_i^2 + \frac{1}{2} \sigma_i^2 - \text{cov}$, where $\delta = 0.02$ (discount rate) and $10^4\sigma_i^2 = 4.08$ [var(100*$\Delta \log GDP_p$)]. Column 4 is $10^4[\log(\delta - (1 - \gamma)\mu - \frac{1}{2}(1 - \gamma)^2\sigma^2) - \log(\delta - (\mu - \gamma\mu + \frac{1}{2} \gamma^2\sigma^2 - \gamma \text{cov}) + (1/1 - \gamma) \log(\delta - (1 - \gamma)\mu - \frac{1}{2}(1 - \gamma)^2\sigma^2) - (1/1 - \gamma) \log(\delta - (1 - \gamma)\mu - \frac{1}{2}(1 - \gamma)^2\sigma^2)]$, where the risk aversion parameter is $\gamma = 3$ and the growth rate of the aggregate GDP of the OECD countries (GDPp) is $\mu = 0.023$. Specialization indices are defined in the text. The displayed indices are multiplied by 100.

The specialization indices are displayed in the last two columns of Tables 1 and 2. The numerical value of the indices are not easily interpreted, although a value of zero means that the state or country has sector shares identical to the average sector shares of the remaining states or countries. In the US, Alaska and Wyoming are very specialized (in oil) and Nevada is quite specialized (in services) at the 1-digit level. Specialization at the 2-digit manufacturing level is reported in column 6. Some US states have a small and highly specialized manufacturing sector; for example Alaska (food), Montana (wood), and Hawaii (food). The manufacturing sector is, however, also very specialized in Delaware, Louisiana, and West Virginia (all in chemical industry). The set of states with high asymmetry indices is extremely similar to the set of states that Del Negro (1999) identifies as asymmetric using an econometric factor model to estimate asymmetry – it seems that the identification of asymmetric states is very robust to the method used. Among OECD countries, Belgium is the most specialized at the 1-digit level (in services) and Norway and New Zealand are extremely specialized at the 2-digit
manufacturing level (both in food processing).\textsuperscript{41} Norway and New Zealand are both more specialized than any US state.

The population weighted 1-digit and 2-digit manufacturing specialization indices for US states are 1.2 and 4.2 whereas for OECD countries they are 1.2 and 5.0, suggesting that US states and OECD countries are approximately equally specialized.\textsuperscript{42}

6.3. Regression analysis

Table 3 reports the central results of our paper. We present ordinary least squares (OLS) and instrumental variables (IV) regressions of the asymmetry measures on the specialization indices.\textsuperscript{43} These regressions use the pooled sample of OECD countries and US states. We control for country (and state) size using population since small countries may exhibit very asymmetric GDP fluctuations due to few opportunities for within-country diversification. We choose a square root specification which produces the best fit. We control for (log-transformed) shares of mining and agriculture in GDP since the previous tables showed that oil-rich countries might be outliers. The log-transformation is chosen based on inspection of the data (some countries have sector shares that are extremely small relative to other countries, so the raw shares have a highly skewed distribution).\textsuperscript{44} We further included a dummy variable for countries. The regressions are weighted by log-population, and the dependent variable is the logarithm of the fluctuations asymmetry measure.\textsuperscript{45} Similarly, the specialization indices are log-transformed.

The main result is that higher specialization induces greater asymmetry. Both 1-digit and manufacturing specialization are significant (at the 5% level) in all the specifications displayed in Table 3. For each regression we calculate the partial $R^2$ as the $R^2$ of the full regression minus the $R^2$ of a regression where both

\textsuperscript{41}Specialization is not necessarily driven by one sector. The sectors reported in the text, in parentheses, are mentioned for illustration only and are obtained as follows. Consider Belgium, for example. It is most specialized in services relative to other OECD countries in the sense that

$$\frac{\text{GDP}_s}{\text{GDP}_i} = \frac{1}{J-1} \sum_{j=1}^{J} \frac{\text{GDP}_j}{\text{GDP}_i}$$

is largest (over all sectors $s$) for services in Belgium (country $i$).

\textsuperscript{42}If the two strong outliers, New Zealand and Norway, are removed, the population weighted 1-digit and 2-digit manufacturing specialization indices for OECD countries are 1.2 and 4.6. Kalemli-Ozcan et al. (1999) found that, in general, regions within countries are more specialized than countries. Their sample includes regions of Italy and the UK, Japanese prefectures, Canadian provinces, communities of Spain, US states, and OECD countries.

\textsuperscript{43}All the regressions include a constant (not reported).

\textsuperscript{44}If the mining and agriculture sector shares are dropped the results are extremely similar.

\textsuperscript{45}The logarithmic transformation of the asymmetry measure makes it less likely to be dominated by outliers like Alaska and Wyoming.
Table 3
Determinants of GDP fluctuations asymmetry: ordinary least squares and instrumental variables regressions

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>(1) OLS asym. index (log)</th>
<th>(2) IV asym. index (log)</th>
<th>(3) OLS asym. index (CRRA)</th>
<th>(4) IV asym. index (CRRA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regressors:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log 1-digit specialization (SPEC index)</td>
<td>0.43 (4.13)</td>
<td>0.68 (2.48)</td>
<td>0.44 (4.16)</td>
<td>0.73 (2.57)</td>
</tr>
<tr>
<td>log Manuf. specialization (SPEC_{1,M} index)</td>
<td>0.30 (3.41)</td>
<td>0.57 (2.20)</td>
<td>0.30 (3.37)</td>
<td>0.58 (2.15)</td>
</tr>
<tr>
<td>Population^{1/2}</td>
<td>-0.14 (4.36)</td>
<td>-0.12 (2.84)</td>
<td>-0.14 (4.31)</td>
<td>-0.12 (2.70)</td>
</tr>
<tr>
<td>log Agriculture GDP share</td>
<td>0.28 (2.36)</td>
<td>0.40 (2.48)</td>
<td>0.29 (2.35)</td>
<td>0.42 (2.50)</td>
</tr>
<tr>
<td>log Mining GDP share</td>
<td>0.14 (2.92)</td>
<td>0.09 (1.50)</td>
<td>0.14 (2.91)</td>
<td>0.09 (1.41)</td>
</tr>
<tr>
<td>Country dummy</td>
<td>0.33 (1.46)</td>
<td>0.22 (0.79)</td>
<td>0.10 (0.42)</td>
<td>-0.03 (0.10)</td>
</tr>
<tr>
<td>R^2</td>
<td>0.68 (1.30)</td>
<td>0.56 (0.30)</td>
<td>0.68 (0.13)</td>
<td>0.54 (0.16)</td>
</tr>
<tr>
<td>Partial R^2</td>
<td>0.30 (0.68)</td>
<td>0.13 (0.56)</td>
<td>0.30 (0.68)</td>
<td>0.16 (0.54)</td>
</tr>
</tbody>
</table>

Notes: The sample consists of the 50 US states and 11 OECD countries. The OECD countries are Belgium, Denmark, France, Germany, Netherlands, Austria, Finland, Canada, New Zealand, Norway, US. ‘Agriculture GDP Share’ is the average over time (1977–1994 for US and 1980–1990 for OECD) of the GDP share of this sector in each country or state. Analogously for Mining. The instruments are, for each country or state: FIRE GDP share (computed in the same manner and for the same time periods as Agriculture and Mining GDP shares), land mass, log-population density averaged over time (1977–1994 for US and 1977–1990 for OECD), percent high school enrollment (1990), GDP per capita averaged over time (1977–1994 for US and 1977–1990 for OECD), and an interaction variable of the Agriculture and Mining GDP shares averaged over time. The country dummy takes a value of 1 for countries and 0 for states. The specialization indices SPEC_{i} and SPEC_{1,M} are defined in the text. All variables in all regressions are weighted by log-population. The dependent variable is log-transformed in all regressions. t-values in parentheses. The Partial R^2 is the R^2 reported for the full regression minus the R^2 obtained when both specialization indices are left out.

specialization indices are left out.\textsuperscript{46} It reflects the fraction of the variance of the left-hand side variable explained by the two specialization indices. It appears that specialization explains a large fraction of the variation in the asymmetry index.\textsuperscript{47}

\textsuperscript{46}The R^2 is calculated as \(1 - \sum e_i^2 / \sum (Y - \bar{Y})^2\), where \(e_i = Y_i - X_i \hat{\beta}\), \(X_i\) and \(Y_i\) are the unweighted left- and right-hand side variables, and \(\hat{\beta}\) is the vector of parameters estimated in the weighted regression.

\textsuperscript{47}Since the left-hand side variable is log-asymmetry, the coefficients of the log-specialization indices represent elasticities, but it is difficult to interpret their magnitude.
We cannot rule out that specialization is affected by fluctuations asymmetry. As an example, imagine that the manufacturing output of a country has a particularly high variance relative to other countries for reasons that are not related to industrial structure. Since its manufacturing production is very variable, the country is likely to decrease manufacturing production, thus affecting the specialization index (downwards if the country was specialized in manufacturing to begin with, and upwards if not). We therefore also estimate the regressions using IV methods with the following instruments for the specialization indices: land mass, the logarithm of average population density, percent high school enrollment in 1990, average GDP level, share of the Finance, Insurance, Real Estate (FIRE) sector in GDP, and the product of the log-agriculture and log-mining shares in GDP. There is some difference in the estimated coefficients between the OLS and IV regressions, with the coefficient of specialization being higher in the IV regressions, but the important fact is that all the regressions in Table 3 have highly significant t-statistics for both specialization indices.

Table 4 focuses on robustness. US states are not separated by national borders and might exhibit different patterns of specialization and fluctuations asymmetry. The first column of Table 4 shows that the results for US states alone are qualitatively similar to those in Table 3. In column 2, Table 4, we report the results of regressions using alternative specialization indices based on the absolute value of the differences between sector shares (see Section 5). The signs of the estimated parameters are the same and the t-statistics are similar to those in Table 3. Column 3 experiments further with regression specifications. Including (real per capita) GDP and human capital as regressors has little impact on the results. Oil-rich countries and states seemed to be outliers in Tables 1 and 2 and it may not be a sufficient remedy to include the mining share as a regressor. We, therefore, show in columns 4 and 5 results of regressions that leave out countries and states for which the GDP share of mining exceeds 10%. This has little effect on the estimated coefficients of the specialization indices. (We also tried the regression in column 5 further leaving out New Zealand which has a highly specialized manufacturing sector. That only increased the t-statistics).

As a final robustness test, we estimated a regression similar to the one reported in the first column of Table 3, but using specialization measures calculated for

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48 Kalemli-Ozcan et al. (1999) show that FIRE is highly correlated with risk sharing and, therefore, an effective instrument for specialization.

49 We assume that when the log-level of these sector shares are included as regressors, their product does not directly affect the degree of asymmetry. Including this instrument increases the significance of 1-digit specialization but not of manufacturing specialization.
Table 4
Sensitivity analysis: specialization measure, oil-rich countries and states, US states vs. pooled sample

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asym. index</td>
<td>Asym. index</td>
<td>Asym. index</td>
<td>Asym. index</td>
<td>Asym. index</td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>pooled</td>
<td>pooled</td>
<td>US (no oil)</td>
<td>pooled (no oil)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Regressors:</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log 1-digit specialization</td>
<td>0.39</td>
<td>–</td>
<td>0.37</td>
<td>0.31</td>
<td>0.31</td>
</tr>
<tr>
<td>(SPEC index)</td>
<td>(3.45)</td>
<td>–</td>
<td>(3.56)</td>
<td>(2.87)</td>
<td>(3.23)</td>
</tr>
<tr>
<td>log Manuf. specialization</td>
<td>0.40</td>
<td>–</td>
<td>0.28</td>
<td>0.30</td>
<td>0.27</td>
</tr>
<tr>
<td>(SPEC index)</td>
<td>(3.72)</td>
<td>–</td>
<td>(3.25)</td>
<td>(3.06)</td>
<td>(3.41)</td>
</tr>
<tr>
<td>log 1-digit specialization</td>
<td>–</td>
<td>1.03</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>(SPEC index)</td>
<td>–</td>
<td>(4.35)</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>log Manuf. specialization</td>
<td>–</td>
<td>0.58</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>(SPEC index)</td>
<td>–</td>
<td>(3.05)</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Population$^{1/2}$</td>
<td>–0.17</td>
<td>–0.14</td>
<td>–0.15</td>
<td>–0.23</td>
<td>–0.13</td>
</tr>
<tr>
<td></td>
<td>(1.90)</td>
<td>(4.26)</td>
<td>(4.53)</td>
<td>(2.77)</td>
<td>(4.74)</td>
</tr>
<tr>
<td>log Agriculture GDP share</td>
<td>0.26</td>
<td>0.29</td>
<td>0.35</td>
<td>0.33</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>(2.04)</td>
<td>(2.43)</td>
<td>(2.89)</td>
<td>(2.93)</td>
<td>(3.45)</td>
</tr>
<tr>
<td>log Mining GDP share</td>
<td>0.17</td>
<td>0.16</td>
<td>0.13</td>
<td>0.04</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>(3.36)</td>
<td>(3.36)</td>
<td>(2.67)</td>
<td>(0.63)</td>
<td>(0.23)</td>
</tr>
<tr>
<td>Country dummy</td>
<td>–</td>
<td>0.26</td>
<td>0.41</td>
<td>–</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>–</td>
<td>(1.16)</td>
<td>(1.87)</td>
<td>–</td>
<td>(2.37)</td>
</tr>
<tr>
<td>GDP per capita</td>
<td>–</td>
<td>–</td>
<td>0.58</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>–</td>
<td>–</td>
<td>(2.16)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Human capital</td>
<td>–</td>
<td>–</td>
<td>0.01</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>–</td>
<td>–</td>
<td>(0.51)</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

| $R^2$                            | 0.72 | 0.69 | 0.73 | 0.65 | 0.63 |

Notes: Asym. index is the log-utility asymmetry index, ‘Pooled’ refers to US states and OECD countries (the sample used in Table 3), in the last two columns, states and countries with a Mining GDP share exceeding 10% are excluded (Alaska, Louisiana, New Mexico, Oklahoma, Texas, Wyoming, Norway). Human capital is the percentage of high school enrollment in the population in 1990, GDP per capita is the average over time (1977–1994 for US and 1977–1990 for OECD), the Country dummy and Agriculture and Mining GDP shares are defined in Table 3, the specialization indices SPEC$_1$, SPEC$_{1,4}$, SPEC$_2$, and SPEC$_{2,4}$ are defined in the text, the dependent variable is log-transformed in all regressions, all variables in all regressions are weighted by log-population. t-values in parentheses.

If changes in fluctuations asymmetry feed back quickly (within a few years) into industrial specialization, this alternative regression would potentially
exhibit different results than those reported in previous tables. Yet, the results for this regression are very similar to those reported in Table 3. The specialization measure changes little over time so this is about as far as our data allow us to go in terms of ‘dynamics.’

Our regressions demonstrate that asymmetry as measured by the utility based measure significantly (and robustly) increases with industrial specialization and that specialization in manufacturing has an impact on fluctuations asymmetry beyond that of 1-digit specialization. The instrumental variables regressions provide support for the notion that there is an effect running from industrial specialization to the asymmetry of GDP fluctuations.

6.4. Regressions using a pairwise correlation measure of fluctuations asymmetry

Most of the empirical papers in the literature on asymmetric shocks perform the analysis using country pairs as the unit of observation. For robustness, we perform a similar analysis. Following Frankel and Rose (1998), we compute pairwise correlations of country-level GDP (or state-level GSP) detrended by first differencing or Hodrick–Prescott (HP) filtering. We do not calculate correlations for mixed state-country pairs. As our ‘pairwise specialization measure’ we use the index suggested by Krugman (1993). For example, the 1-digit specialization index for countries \(i\) and \(j\) is

\[
\text{SPEC}_{1}^{ij} = \sum_{s=1}^{S} \left| \frac{\text{GDP}_{s}^{i}}{\text{GDP}_{s}^{j}} - \frac{\text{GDP}_{s}^{j}}{\text{GDP}_{s}^{i}} \right|,
\]

and the manufacturing specialization index for countries \(i\) and \(j\), \(\text{SPEC}_{2M}^{ij}\), is defined analogously.

We regress the pairwise asymmetry measures on the pairwise specialization measures, controlling for the same variables as in previous tables (taking the average of each pairwise variable over the sample period), and including a dummy variable for country pairs (as opposed to pairs of US states). The results are

\footnote{In the order of the rows in Table 3, the parameters estimated from this regression are 0.43, 0.25, -0.14, 0.15, 0.14, and 0.45, respectively. The t-statistics are also extremely similar to those reported in the first column of Table 3, with the most notable difference being that the log-agriculture GDP share is not significant.}

\footnote{A referee pointed out that although the estimated degree of specialization in manufacturing for US states and OECD countries is roughly the same, the fluctuations asymmetry measure is higher for US states on average. This is a potential indication that US states are more specialized within 2-digit manufacturing categories.}
Table 5
Sensitivity analysis: regressions using pairs of countries and pairs of US states

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pairwise GDP correlation</td>
<td>Pairwise GDP correlation</td>
</tr>
<tr>
<td>Detrending method:</td>
<td>Difference</td>
<td>HP</td>
</tr>
<tr>
<td>Regressors:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>log 1-digit specialization</td>
<td>-0.28</td>
<td>-0.36</td>
</tr>
<tr>
<td>(22.30)</td>
<td>(17.61)</td>
<td></td>
</tr>
<tr>
<td>log Manuf. specialization</td>
<td>-0.08</td>
<td>-0.04</td>
</tr>
<tr>
<td>(6.81)</td>
<td>(2.11)</td>
<td></td>
</tr>
<tr>
<td>Population^{1/2}</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>(3.10)</td>
<td>(1.31)</td>
<td></td>
</tr>
<tr>
<td>log Agriculture GDP share</td>
<td>0.05</td>
<td>0.07</td>
</tr>
<tr>
<td>(5.64)</td>
<td>(4.91)</td>
<td></td>
</tr>
<tr>
<td>log Mining GDP share</td>
<td>-0.08</td>
<td>-0.14</td>
</tr>
<tr>
<td>(18.68)</td>
<td>(19.47)</td>
<td></td>
</tr>
<tr>
<td>Pairs of countries dummy</td>
<td>-0.32</td>
<td>-0.27</td>
</tr>
<tr>
<td>(9.35)</td>
<td>(5.02)</td>
<td></td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.65</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Notes: The sample consists of all pairs of OECD countries and pairs of US states in the sample used in Table 3. ‘Pairwise GDP correlation’ is the correlation of the log of real GDP per capita between two countries or two US states. Real log-GDP per capita is detrended with two different methods: first-differencing or Hodrick–Prescott filtering. The pairwise specialization indices are defined in the text. The other regressors are averaged over time for pairs of countries or states. For example, log Agriculture GDP share is the average over time (for the same period as in previous tables) of the log of the average Agriculture GDP share of countries \( i \) and \( j \). The pairs of countries dummy is 1 for pairs of countries and 0 for pairs of US states. \( t \)-values in parentheses.

Our results are, thus, robust to different measures of asymmetry in GDP fluctuations which indicates that the empirical relation between specialization in production and fluctuations asymmetry holds in the data both at the short (yearly) frequency and at the (longer) business cycle frequency.

As in Frankel and Rose (1998), the standard errors and \( t \)-statistics are approximate since correlations between the error terms are not controlled for.
7. Conclusion

We demonstrated that OECD countries and US states with higher industrial specialization exhibit output shocks that are less correlated on average with aggregate OECD output and US output, respectively. We argued that this constitutes evidence in support of an economic mechanism that (partly or fully) offsets the one studied by Frankel and Rose (1998). The mechanism is one where countries (and states) choose to specialize in production after having spread the risk of specialization in the international (or nation-wide) capital market so that increased variability of output will not have as large an effect on the variability of income.

This should not be taken as an argument against economic integration. On the contrary, it is an argument in support of integration which will lead, true, to more asymmetric output shocks, but not necessarily to more asymmetric income shocks. As a consequence of extensive cross-country ownership of productive assets, income shocks may actually become more symmetric despite the greater asymmetry of output shocks.

Acknowledgements

We thank three referees and Andy Rose for extremely helpful comments on earlier drafts as well as participants in several seminars and conferences. The paper previously circulated as ‘Industrial Specialization and the Asymmetry of Shocks across Regions.’ The views expressed herein are solely those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Kansas City or the Federal Reserve System.

Appendix A. A utility-based measure of fluctuations asymmetry

We derive the fluctuations asymmetry measure for CRRA utility (and log-utility as a special case). Let countries be indexed by \( i \). Consumers within country \( i \) are identical ex-ante and ex-post: all have the same utility function and produce the same non-storable, homogeneous, stochastic gross product. The representative consumer of country \( i \) chooses a consumption plan in period \( t = 0 \), solving the problem

\[
\text{max}_{\{c_t^i, gdp_t^i\}} \int_0^\infty e^{-\delta t} \left( \sum_{\omega} \pi_{\omega} u(c_t^i) - \sum_{\omega} \pi_{\omega} gdp_t^i \right) \, dt
\]

subject to

\[
\sum_{\omega} \pi_{\omega} p_{\omega} c_t^i \, dt \leq \int_0^\infty \sum_{\omega} \pi_{\omega} gdp_t^i \, dt
\]

where \( c_t^i \) and \( gdp_t^i \) are per capita consumption and gross product in country \( i \) in state of nature \( \omega \) which occurs with probability \( \pi_{\omega} \). \( p_{\omega} \) is the price in period 0 of

\[\text{Integrals are assumed to be convergent.}\]
a period \( t \), state \( \omega \), contingent unit of consumption, and \( \delta \) is the common intertemporal discount rate. Since securities markets in period 0 are complete, each country faces a single budget constraint. Let \( u(c) = (1/1 - \gamma)\mathrm{e}^{\gamma} \) \( (\gamma \neq 1) \). (We address the log-utility case as we proceed). The first order condition with respect to \( c_{t} \) can be written as \( p_{t} / \pi_{t} = e^{-\delta t}(1/\lambda t)(c_{t}^{\lambda t})^{-\gamma} \) where \( \lambda t \) is a Lagrange multiplier. Market clearing implies \( \Sigma_{t} c_{t} - \Sigma_{t} \delta_{t} \) for all \( \omega \) where \( n' \) is country \( i \)'s population. Prices are normalized so that \( \Sigma_{t} \delta_{t} = 1 \). Letting \( \delta_{t} = n' \delta_{t} \), we have \( c_{t}^{i} = k \delta_{t} \). From the budget constraint: 

\[
(\sum_{t} \delta_{t}^i / n) = 1 \quad \text{where} \quad n = \sum_{n'} n'.
\]

To compute \( k' \), multiply and divide by \( \pi_{t} \) inside the summation operator on both sides of the budget constraint (which binds at an optimum) and substitute for \( p_{t} / \pi_{t} \) using the first order condition to obtain \( (\lambda t \mathrm{terms} \cancel{\text{cancel}}) \)

\[
\int_{0}^{\infty} e^{-\delta t} E_{0}(gdp_{t})^{1-\gamma} dt = \int_{0}^{\infty} e^{-\delta t} E_{0} k \delta_{t}^{i} dt,
\]

rearranging, we obtain the share of aggregate consumption that would accrue to country \( i \) in a perfect risk sharing equilibrium:

\[
k' = \left[ \int_{0}^{\infty} e^{-\delta t} E_{0}(gdp_{t})^{1-\gamma} dt \right]^{-1} \int_{0}^{\infty} e^{-\delta t} E_{0} k \delta_{t}^{i} dt. \tag{A.1}
\]

This also holds for log-utility yielding:

\[
k' = \delta \int_{0}^{\infty} e^{-\delta t} E_{0} k \delta_{t}^{i} dt. \tag{A.2}
\]

The interpretation is simple: the strength of country \( i \) in the risk sharing arrangement (the share of aggregate gross product that country \( i \) consumes) is proportional to its discounted expected share in aggregate gross product.

The analysis so far has been independent of the nature of the joint stochastic process governing the gross product of the countries sharing risk. In order to quantify gains from risk sharing we make distributional assumptions (see Section 4) that allow us to express the constant \( k' \) in an even simpler and economically intuitive manner. Recalling that for \( z \sim N(\eta, \phi^2) \), \( Ez^\theta = e^{\eta \theta + \frac{1}{2} \phi^2 \theta^2} \), we have:\hfill

\[\text{See Huang and Litzenberger (1988) for a derivation for CRRA utility. The derivation for log-utility is much simpler and is provided in Sorensen and Yoshia (1998).}\]

\[\text{Let } z = (1 - \gamma) \log gdp_{t} - \log gdp_{0}, \text{ then } Ez = (1 - \gamma) \mu t \text{ and } \text{var} z = (1 - \gamma) \mu \text{var}(\Delta \log gdp_{t}, t) = (1 - \gamma) \sigma^2. \text{ Then, } E z = (\mu' - \gamma \mu) t \text{ and var } z = \text{var}(\Delta \log gdp_{t}, - \gamma \Delta \log gdp_{t}, t) = (\sigma^2 + \gamma^2 \sigma^2 - 2 \gamma \sigma^2). \text{ Then, } E y_{i} = (\mu' - \gamma \mu) t \text{ and var } y_{i} = \text{var}(\Delta \log gdp_{t}, - \gamma \Delta \log gdp_{t}, t) = (\sigma^2 + \gamma^2 \sigma^2 - 2 \gamma \sigma^2). \text{ Then, } E y_{i} = (\mu' - \gamma \mu) t \text{ and var } y_{i} = \text{var}(\Delta \log gdp_{t}, - \gamma \Delta \log gdp_{t}, t) = (\sigma^2 + \gamma^2 \sigma^2 - 2 \gamma \sigma^2). \text{ Then, } E y_{i} = (\mu' - \gamma \mu) t \text{ and var } y_{i} = \text{var}(\Delta \log gdp_{t}, - \gamma \Delta \log gdp_{t}, t) = (\sigma^2 + \gamma^2 \sigma^2 - 2 \gamma \sigma^2). \]
\[ k' = \left[ \int_0^\infty e^{-\delta t} E_0 e^{(1-\gamma) \log gdp_t} \, dt \right]^{-1} \int_0^\infty e^{-\delta t} E_0 e^{(1-\gamma) \log gdp_t} \, dt, \]

\[ = e^{(1-\gamma) \log gdp_0} \int_0^\infty e^{-\delta t} (1-\gamma) \mu t + \frac{1}{2} (1-\gamma) \sigma^2 t^2 \, dt \]

\[ \times \int_0^\infty e^{-\delta t} (\sigma_i - \mu_0 + \frac{1}{2} \sigma_i^2 + \frac{1}{2} \sigma^2 + \gamma \text{cov}) \, dt \]

\[ = \left( \frac{gdp_i}{gdp_0} \right) \frac{\delta - (1-\gamma) \mu - \frac{1}{2} (1-\gamma) \sigma^2}{\delta - (\mu_i - \mu + \frac{1}{2} \sigma_i^2 + \frac{1}{2} \sigma^2 - \gamma \text{cov})}. \] (A.3)

Setting \( \gamma = 1 \) yields

\[ k' = \left( \frac{gdp_i}{gdp_0} \right) \frac{\delta}{\delta - (\mu_i - \mu + \frac{1}{2} \sigma_i^2 + \frac{1}{2} \sigma^2 - \text{cov})}. \]

for log-utility. Here the intuition is more transparent: the risk sharing arrangement allocates a higher share of aggregate output to countries with a larger initial share in aggregate output, and to countries with a lower covariance between \( \Delta \log gdp_i \) and \( \Delta \log gdp_t \), reflecting a higher insurance value of country \( i \) for the other regions. The higher the variance of country \( i \)'s GDP, other things equal, the more it can contribute to smoothing shocks in other countries; the higher the variance of the aggregate gross product of the group, keeping the variance of country \( i \)'s GDP constant, the more other countries would be willing to 'pay' country \( i \) for joining the risk sharing arrangement.\(^{56}\)

As a technical note, the population weighted \( k' \) coefficients in Eq. (A.3) do not sum to one due to the distributional approximation made (that aggregate GDP is log-normally distributed). The size of the bias depends on the estimated parameters \( \sigma^2, \sigma_i^2, \text{cov} \) and on the value of \( \delta \) chosen. For our chosen value of \( \delta = 0.02 \), and our sample of US states and OECD countries the bias is negligible with the population weighted sum deviating by less than 0.01 from one.

The term \( \mu_i - \mu \), the deviation of country \( i \)'s trend growth from average trend growth (see the denominator in the last line of Eq. (A.3)), reflects inter-temporal consumption smoothing considerations. A high trend growth of country \( i \), relative to other countries, induces a high consumption share due to the high future

\(^{56}\)Of course, \( \sigma^2 \), the variance of the growth rate of aggregate GDP, cannot change without any of the \( \sigma_i^2 \)'s changing. The distributional approximation regarding aggregate GDP thus allows us to treat \( \sigma^2 \) as a parameter (that can be estimated from aggregate GDP data) rather than as a complicated function of the country-by-country \( \sigma_i^2 \)'s.
share in aggregate output relative to the low initial share in aggregate output.\footnote{For log-utility, we are able to fully disentangle the gains from intertemporal smoothing and the gains from insurance.}

We turn to the calculation of the gains from risk sharing. If there is perfect risk sharing, the discounted expected utility of country $i$ as a function of $gdp_i$ is:

$$U^F(gdp_i) = \frac{1}{1 - \gamma} \int_0^\infty e^{-\delta t} E_0(k_i gdp_i)^{1 - \gamma} dt$$

$$= \frac{1}{1 - \gamma} \left( \frac{\delta - (1 - \gamma) \mu - \frac{1}{2} (1 - \gamma)^2 \sigma_i^2}{\delta - (\mu' + \gamma \mu + \frac{1}{2} \gamma^2 \sigma_i^2)} \right)^{1 - \gamma}$$

$$E_0(gdp_i gdp_o)^{1 - \gamma}$$

$$= \frac{1}{1 - \gamma} \left( \frac{\delta - (1 - \gamma) \mu - \frac{1}{2} (1 - \gamma)^2 \sigma_i^2}{\delta - (\mu' + \gamma \mu + \frac{1}{2} \gamma^2 \sigma_i^2)} \right)^{1 - \gamma}$$

$$= \frac{1}{1 - \gamma} \left( \frac{\delta - (1 - \gamma) \mu - \frac{1}{2} (1 - \gamma)^2 \sigma_i^2}{\delta - (\mu' + \gamma \mu + \frac{1}{2} \gamma^2 \sigma_i^2)} \right)^{1 - \gamma}$$

$$= \frac{1}{1 - \gamma} \left( \frac{\delta - (1 - \gamma) \mu - \frac{1}{2} (1 - \gamma)^2 \sigma_i^2}{\delta - (\mu' + \gamma \mu + \frac{1}{2} \gamma^2 \sigma_i^2)} \right)^{1 - \gamma}$$

(A.4)

The discounted expected utility of country $i$ in autarky is:

$$U^A(gdp_i) = \frac{1}{1 - \gamma} \left[ \int_0^\infty e^{-\delta t} \mu i \left( \frac{\delta - (1 - \gamma) \mu - \frac{1}{2} (1 - \gamma)^2 \sigma_i^2}{\delta - (\mu' + \gamma \mu + \frac{1}{2} \gamma^2 \sigma_i^2)} \right)^{1 - \gamma} dt \right]$$

$$= \frac{1}{1 - \gamma} \left( \frac{\delta - (1 - \gamma) \mu - \frac{1}{2} (1 - \gamma)^2 \sigma_i^2}{\delta - (\mu' + \gamma \mu + \frac{1}{2} \gamma^2 \sigma_i^2)} \right)^{1 - \gamma}$$

(A.5)

We want to express the gain $U^F(gdp_o) - U^A(gdp_o)$ as the permanent percentage increase in the level of autarkic consumption that would increase discounted expected utility by the same amount. We thus calculate $G^i$ that satisfies:

$$U^A(gdp_o(1 + G^i)) = U^F(gdp_o).$$

Taking logs, using Eqs. (A.4), (A.5), and the approximation $\log(1 + G^i) = G^i$, and setting $\mu' = \mu$ we obtain:\footnote{To focus on gains from risk sharing, we want to disregard as much as possible gains from intertemporal substitution. We, therefore, set $\mu' = \mu$ (van Wincoop, 1994 makes the same assumption).}

$$G^i = \log \left( \frac{\delta - (1 - \gamma) \mu - \frac{1}{2} (1 - \gamma)^2 \sigma_i^2}{\delta - (1 - \gamma) \mu - \frac{1}{2} \gamma^2 \sigma_i^2 + \gamma \sigma_i^2} \right)$$

$$- \log \left( \frac{\delta - (1 - \gamma) \mu - \frac{1}{2} \gamma^2 \sigma_i^2 + \gamma \sigma_i^2}{\delta - (1 - \gamma) \mu - \frac{1}{2} \gamma^2 \sigma_i^2} \right)$$

$$+ \frac{1}{1 - \gamma} \log \left( \frac{\delta - (1 - \gamma) \mu - \frac{1}{2} (1 - \gamma)^2 \sigma_i^2}{\delta - (1 - \gamma) \mu - \frac{1}{2} \gamma^2 \sigma_i^2} \right)$$

$$- \frac{1}{1 - \gamma} \log \left( \frac{(1 - \gamma) \mu - \frac{1}{2} (1 - \gamma)^2 \sigma_i^2}{\delta - (1 - \gamma) \mu} \right)$$

(A.6)
For log-utility, the derivation is considerably more elegant. The discounted expected utility gain to country $i$ of moving from autarky to perfect risk sharing is (using the approximation $\log(1+x) \approx x$):

$$
G^i = \int_0^\infty e^{-\delta t} E_0 \log[k^i GDP_i] \, dt - \int_0^\infty e^{-\delta t} \log GDP_i \, dt
$$

$$
= \int_0^\infty e^{-\delta t} \log \frac{\delta}{\delta - (\mu^i - \mu + \frac{1}{2} \sigma^2 + \frac{1}{2} \sigma_i^2 - \text{cov})} \, dt + \int_0^\infty e^{-\delta t} (\mu^i - \mu) \, dt
$$

$$
= -\int_0^\infty e^{-\delta t} \log \left( 1 - \frac{1}{\delta} \left( \mu^i - \mu + \frac{1}{2} \sigma^2 + \frac{1}{2} \sigma_i^2 - \text{cov} \right) \right) \, dt
$$

$$
+ \frac{1}{\delta} (\mu^i - \mu)
$$

$$
= \int_0^\infty e^{-\delta t} \frac{1}{\delta} \left( \frac{1}{2} \sigma^2 + \frac{1}{2} \sigma_i^2 - \text{cov} \right) \, dt + \frac{1}{\delta} (\mu^i - \mu) - \frac{1}{\delta} (\mu^i - \mu).
$$

(A.7)

The third term in the last line of Eq. (A.7) is the discounted expected utility gain or loss from initially being a lender or a borrower. A low trend growth of country $i$ relative to other countries entails a utility gain reflecting the compensation for initially being a ‘net lender’ to other countries. A high trend growth relative to the average entails a utility loss reflecting the ‘payment’ to other countries for initially being a ‘net borrower.’ The second term in the last line of Eq. (A.7) originates from the denominator of the expression for $k^i$. A high trend growth of country $i$ relative to other countries entails a high consumption share for this region due to the high future share in aggregate output relative to the low initial share in aggregate output, and therefore, a high utility gain from risk sharing. This term is an order of magnitude larger than the third (off-setting) term discussed above. In the empirical analysis for log-utility, we ignore both terms since we want to focus on the gains from ‘pure’ risk sharing, i.e. on the first term in the last line of Eq. (A.7). The logarithmic utility specification allows us to study (and estimate) these gains without confounding them with gains from intertemporal substitution. The first term in Eq. (A.7) is the discounted expected utility gain of moving from no risk sharing to perfect risk sharing. Integrating, we obtain

$$
\frac{1}{\delta^2} \left( \frac{1}{2} \sigma^2 + \frac{1}{2} \sigma_i^2 - \text{cov} \right).
$$
We prefer, however, to express the gains from risk sharing using the term inside the integral in the last line of Eq. (A.7), which corresponds precisely to $G^i$ above. Thus, for log-utility,

$$G^i = \frac{1}{\delta} \left( \frac{1}{2} \sigma^2 + \frac{1}{2} \sigma^2_i - \text{cov} \right).$$

The intuition for this expression is provided in the main text.

References


