CONVOLUTION

DSP for Scientists
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Most Important in DSP

- Combining two (or more) signals to form a third output. (Superposition)

- Signal Filtering (Estimation)
  Function Approximation

- Interpolation

- Prediction (extrapolation)
CONCEPTS

• Delta Function
• Unit Impulse $\delta[n]$

$\delta[n] = 1$, when $n = 0$;
$\delta[n] = 0$, when $n \neq 0$. 
Standard Function

- Standard Function $u[n]$
  - $u[n] = 1$, when $n \geq 0$
  - $u[n] = 0$, when $n < 0$
  
  \[
  \delta[n] = u[n] - u[n - 1]
  \]

  \[
  u[n] = \sum_{k = 0}^{\infty} \delta[n - k]
  \]
• Impulse response \( h[n] \) of linear system \( S \)
  - The signal that exits a system when a Delta function (unit impulse) is the input.

\[
h[n] = S(\delta[n])
\]
Delta Decomposition

- Any discrete sequence $x[n]$ can be represented by weighted Delta function

$$x[n] = \sum_{k=-\infty,\infty} x[k] \delta[n-k]$$

$$x[n] = \sum_{k=-\infty,\infty} x[n-k] \delta[k]$$
Periodic Signal

- \( x[n] = x[n + N] \), N: period

- **Example:** When \( f \in \mathbb{Z} \)

- \( \sin[2\pi f(n + 1)] = \sin[2\pi fn] \)

- \( f \) is normalized frequency,

- minimum period is 1.
• $y[n] = h[n] \ast x[n]$

• $x[n]$ \hspace{2cm} \text{Linear System } h[n] \hspace{2cm} y[n]$

• The \textit{impulse response} of the system
• How the relation between input and output.
Convolution Sum (Linear)

• \( y[i] = \sum_{j=-\infty,\infty} h[j] x[i-j] \)

• \( y[i] = \sum_{j=-\infty,\infty} x[j] h[i-j] \)

• \( y[n] = h[n] \ast x[n] = x[n] \ast h[n] \)
Convolution Calculation

- Keep $x[i]$ invariant
- Symmetry of $h[j]$
  $$g[j] \Rightarrow h[-j]$$
- $y[0] = \sum_{j=-\infty}^{\infty} x[j]g[j]$
  $$= \sum_{j=-\infty}^{\infty} x[j]h[-j]$$
- Shift $g[j]$ Left to obtain $y[i < 0]$
- Shift $g[j]$ Right to obtain $y[i > 0]$