Visual Group Normalization Using Gaussian-Lagrange Distributed Approximating Functional Wavelets

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Abstract

Interpolating wavelets are constructed based on Gaussian-Lagrange distributed approximating functionals (DAFs). The utility of DAF wavelets is tested for digital image de-noising in combination with a novel blind restoration technique—Visual Group Normalization.

I. INTRODUCTION

Distributed approximating functionals (DAFs) were introduced as a powerful grid method for quantum dynamical propagations [1]. DAFs can be regarded as scaling functions and associated DAF-wavelets are generated in a number of ways [2]. DAF-wavelets are smooth and decaying in both the time and frequency domains and have been used for numerically solving linear and nonlinear partial differential equations with extremely high accuracy and computational efficiency. Typical examples include simulations of 3D reactive quantum scattering and solution of 2D Navier-Stokes equation with non-periodic boundary conditions. The present work extends the DAF approach to image processing by constructing interpolating DAF-wavelets [3]. An earlier Group Normalization (GN) technique [4] and human vision response [5] are utilized to normalize the equivalent decomposition filters (EDFs) and perceptual luminance sensitivity. The combined DAF Visual Group Normalization (VGN) approaches achieve robust image restoration results.

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II. INTERPOLATING DAF WAVELETS

Interpolating wavelets are particularly efficient for calculations since their multiresolution spaces are identical to the discrete sampling spaces. Adaptive boundary treatments and irregular samplings can be easily implemented using symmetric interpolating solutions. We design the interpolating scaling function as an interpolating Gaussian-Lagrange DAF (GLDAF) [6]

$$\begin{aligned}\phi_M(x) &= W_\sigma(x)P_M(x) \\ &= W_\sigma(x)\prod_{i=-M,i\neq 0}^{M}\frac{x-i}{-i}\end{aligned} \tag{1}$$

where $W_\sigma(x)$ is selected as a Gaussian window since it satisfies the minimum-frame-bound condition in quantum physics. The quantity σ is a width parameter, $P_M(x)$ is the Lagrange interpolation kernel. DAF dual scaling and wavelet functions are easily generted for perfect reconstruction [3]. The Gaussian window efficiently smoothes out the oscillations, which plague many wavelet bases. As shown in Figure 1, both DAF wavelets and their dual partners display excellent smoothness and rapid decay compared with the popular B97 wavelets. The GLDAF low-pass filter response is designed as $h(-2n-1) = h(2n+1) = \{0.310395, -0.093343, 0.045190, -0.022832, 0.010590\}$, $n = 0, 4$. $h(2n) = 0$, $n\neq 0$ and $h(0) = 1/2$.

III. VISUAL GROUP NORMALIZATION

The wavelet transform is implemented by a tree-structure filtering iteration. The coefficients can be regarded as the output of a single *equivalent decomposition filter* (EDF). As shown in Figure 3(a) and Figure 3(b), EDF responses without normalization can exceed one and are therefore not on same level. Clearly, the decomposition coefficients cannot exactly represent the actual signal strength. To adjust for this, the wavelet coefficients, $C_m(k)$, in block m should be multiplied by a magnitude scaling factor, λ_m. We define this factor as the reciprocal of the maximum magnitude of the EDF frequency response [4] (where LC_m is m-th EDF response).

$$\lambda_m = \frac{1}{\sup_{\omega\in\Omega}\{|LC_m(\omega)|\}}, \quad \Omega = [0, 2\pi]. \tag{2}$$

The normalized EDF responses of both the DAF and B97 wavelets are shown in Figure 2(c) and Figure 2(d), respectively. It is obviously that the DAF's possess smaller sidelobes, which induce less frequency leakage distortion.

An image is a human-vision-dependent source. Using a just-noticeable distortion profile, we can efficiently remove the visual redundancy as well as normalize the coefficients with respect to perception importance. A mathematical model for a "perception lossless" matrix Y_m has been presented in [5] and is used as perceptual normalization combined with the EDF magnitude normalization. (Note here that we use λ_m for magnitude normalization and not the wavelet "basis function amplitude" in [5], because the digital image decomposition is completely done using filter banks.) We denote the combination of the two normalization operations as *Visual Group Normalization* (VGN).

IV. Experimental Results

To test our approaches, two benchmark 512×512 Y-component images are employed (Barbara with much texture and high frequency edges, and Lena with large areas of flat response). A thresholding method [7] is utilized for the image restoration. The PSNR results are compared in Table I, while the perceptual quality of the GLDAF-VGN processed Lena and Barbara are shown in Figure 4. It is evident that our technique yields good performance.

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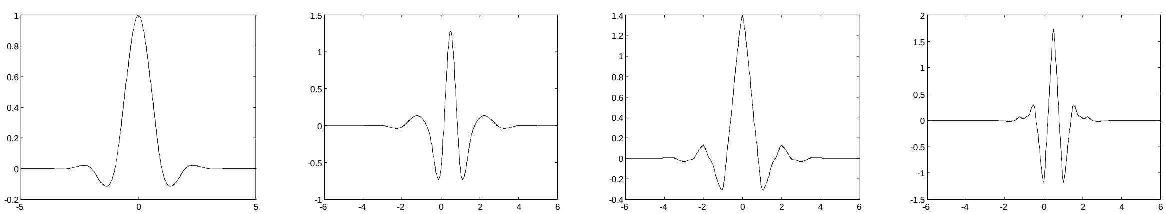
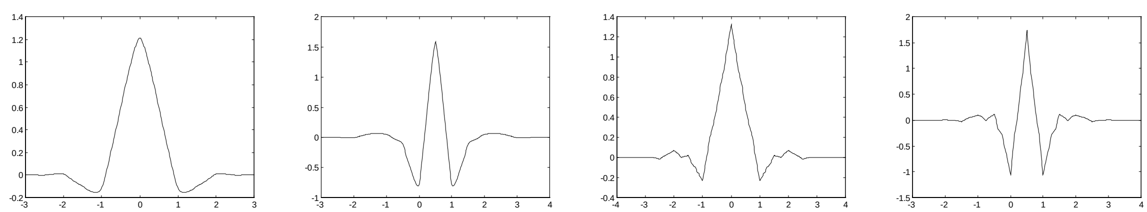


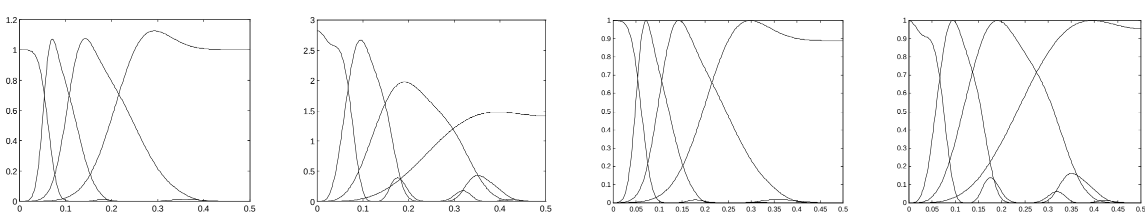
(a) (b) (c) (d)

Figure 1. Lagrange DAF wavelets. (a) Scaling. (b) Wavelet. (c) Dual scaling. (d) Dual wavelet.



(a) (b) (c) (d)

Figure 2. B97 biorthogonal wavelets. (a) Scaling. (b) Wavelet. (c) Dual scaling. (d) Dual wavelet.

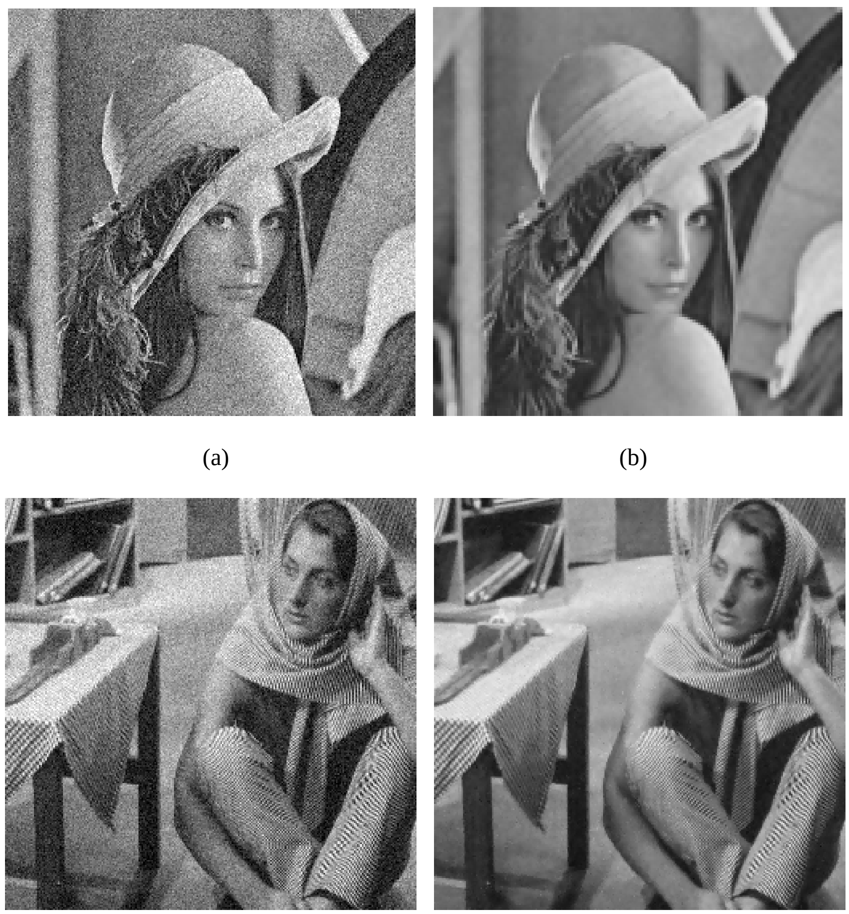


(a) (b) (c) (d)

Figure 3. Frequency response of EDF. (a) Non-normalized response (GLDAF). (b) Non-normalized response (B97). (c) Normalized response (GLDAF). (d) Normalized response (B97).

Table 1. Performance comparison by PSNR (dB)

Noisy Images		Median	B97 (Non-VGN)	B97 (VGN)	GLDAF (VGN)
Barbara	17.60	22.01	22.81	24.27	24.63
	20.30	23.23	24.15	25.67	26.19
	24.50	24.39	27.01	28.38	29.03
Lena	16.46	26.23	23.38	26.74	27.30
	20.07	27.40	25.71	28.51	28.96
	24.47	28.41	28.18	30.98	31.43



(a) (b)

(c) (d)

Figure 4. GLDAF wavelet filtering. (a) Noisy Lena (PSNR=24.47dB). (b) GLDAF restoration (PSNR=31.43). (c) Noisy Barbara (PSNR=24.50dB). (d) GLDAF restoration (PSNR=29.03).