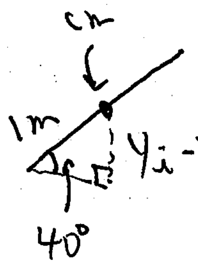
1.) A thin, uniform rod has length 2 meters, and can pivot about a horizontal, frictionless pin through one end. It is released from rest at an angle 40° above the horizontal. What is the angular speed of the rod when it passes through the horizontal position?

Easiest to use conservation of Energy

$$K_i + U_i = K_f + U_f$$

Since $K_i = 0$, $\quad K_f = U_i - U_f$

$U = mg\,y_{cm}$ $\quad$ so $\quad U_i - U_f = mg(y_i - y_f)$



$$y_i - y_f = (1m)\sin 40^\circ$$

so $\quad K_f = mg(1m)\sin 40^\circ = \frac{1}{2} I \omega^2$

I is moment of Inertia around End $\quad I = \frac{1}{3} mL^2$

So $\quad \frac{1}{6} mL^2 \omega^2 = mg(1m)\sin 40^\circ$ $\quad$ or

$$\omega = \sqrt{\frac{g(6m)\sin 40^\circ}{L^2}} = \sqrt{\frac{(9.8\,m/s^2)(6m)\sin 40^\circ}{4m^2}}$$

$$\boxed{\omega = 3.07\ rad/s}$$

2.) A railroad car with mass 2.5×10^4 kg is moving with a speed of 4 m/s. It collides and couples with three other coupled railroad cars, each with the same mass as the single car and moving in the same direction with a speed of 2 m/s. What is the speed of the four cars after the collision?

We must use Conservation of momentum, and Recognize that this is a completely inelastic Collision:

$$m_1 V_{1i} + m_2 V_{2i} = (m_1 + m_2) V_f$$

$$m_2 = 3 m_1, \quad V_{1i} = 4\,m/s \quad V_{2i} = 2\,m/s$$

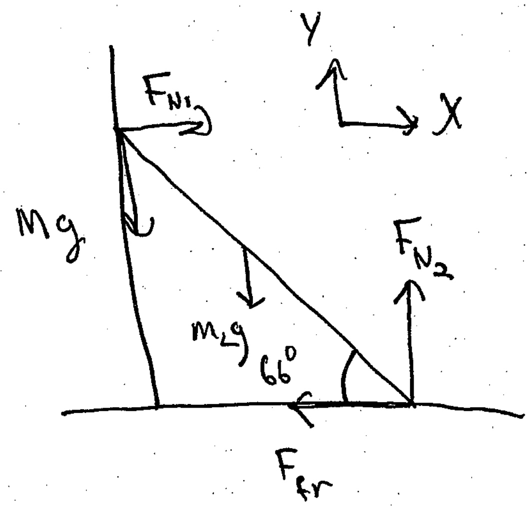
so $$\frac{m_1 V_{1i} + 3 m_1 V_{2i}}{4 m_1} = V_f$$

or $$\frac{(4\,m/s) + 3(2\,m/s)}{4} = V_f$$

$$\boxed{V_f = 2.5\,m/s}$$

3.) A 5 meter long ladder has mass 9.5 kg and is leaning against a frictionless wall, making an angle of 66° with respect to the horizontal. If the coefficient of static friction between the ground and ladder is 0.42, what is the mass of the heaviest person who can safely ascend to the top of the ladder?

This is a static Equilibrium problem, i.e. we want $\sum \vec{F} = 0$, $\sum \tau = 0$



Force Equations

X-comp	Y-comp
$F_{N_1} - F_{fr} = 0$	$-mg - m_L g + F_{N_2} = 0$

Torque Eqn. (Around Bottom of Ladder)

$$F_{N_1} L \sin 66^\circ - MgL \sin 24^\circ - m_L g\left(\frac{L}{2}\right)\sin 24^\circ = 0$$

$F_{fr} = \mu_s F_{N_2}$

Solving 3-equations for m

$$m = \frac{\left(\frac{1}{2}\sin 24^\circ - \mu_s \sin 66^\circ\right)}{\left(\mu_s \sin 66^\circ - \sin 24^\circ\right)} m_L$$

so $\boxed{m = 74.3\ kg}$

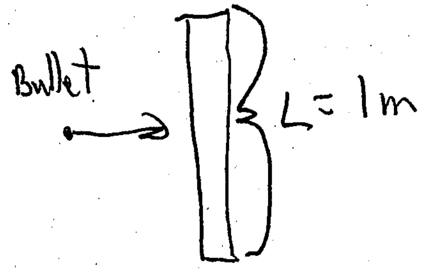
4.) A door 1 meter wide, of mass 15 kg is hinged at one side so it can rotate without friction around a vertical axis. It is unlatched. A police officer fires a bullet with a mass of 10 g and a speed of 400 m/s into the exact center of the door, in a direction perpendicular to the plane of the door. What is the angular speed of the door after the bullet embeds itself in the door?

Need to use Cons. of Angular momentum

$$L_i = L_f$$

initially, Bullet has angular momentum around hinge of

Top View



$$L_i = m_B V\left(\frac{L}{2}\right)$$

final Angular momentum is $L_f = (I_B + I_{Door})\omega_f$

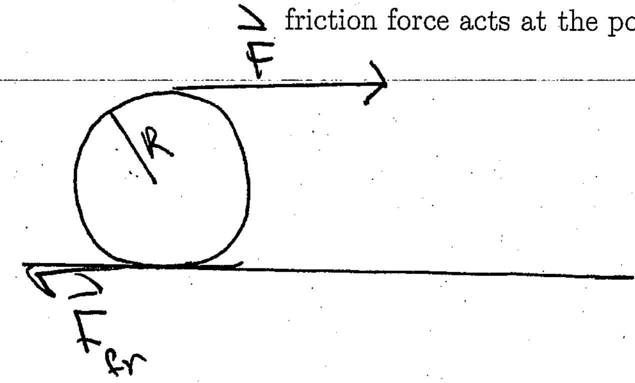
$$L_f = \left(\frac{1}{3} M_{Door} L^2 + m_b\left(\frac{L}{2}\right)^2\right)\omega_f \quad \text{so}$$

$$m_b V\left(\frac{L}{2}\right) = \left(\frac{1}{3} M_{Door} L^2 + m_b\left(\frac{L}{2}\right)^2\right)\omega_f$$

Solving for ω_f

$$\boxed{\omega = \frac{\frac{1}{2} V}{\left(\frac{1}{3}\frac{M_{Door}}{m_b} + \frac{1}{4}\right)L} = 0.4 \text{ Rad/s}}$$

5.) A constant horizontal force **F** is applied to a uniform solid cylinder by fishing line wrapped around the cylinder. The mass of the cylinder is m, and its radius is R, and the cylinder rolls smoothly on horizontal surface. What is the linear acceleration of the center of mass of the disk? (Hint: A static friction force acts at the point of contact between the disk and the surface.)



Need 2 Equations*

$$\vec{F} = m\vec{a}$$

$$\tau = I\alpha \quad \text{with } \alpha = \frac{a}{R}$$

Y-comp

$$F - F_{fr} = ma$$

Taking torque around Center

$$FR + F_{fr}R = I\alpha$$

$$I = \frac{1}{2}MR^2$$

So

$$FR + F_{fr}R = \frac{1}{2}mR^2\left(\frac{a}{R}\right) \quad \text{So}$$

$$F + F_{fr} = \frac{1}{2}ma \quad \text{or}$$

$$F_{fr} = \frac{1}{2}ma - F$$

Put that in here

$$F - \frac{1}{2}ma + F = ma$$

So

$$2F = \frac{3}{2}Ma$$

or

$$\boxed{a = \frac{4}{3}\left(\frac{F}{m}\right)}$$

* If you make a clever choice of the point to find Torque about, you can solve problem using just Torque Equation.