8 Chapter 4 - Probability and Probability Distributions

1. Role of probability in economics.

   (a) In your first set of economics courses, the models assume that there is certainty (or complete information) in economic markets. Consumers know the prices of goods, firms know output and factor prices. In more advanced course work, the certainty assumption is relaxed. Consumers face markets where prices are not perfectly known. These economic models are called models of uncertainty or stochastic economic models. For each event (or state of the world), probability is used to assign a likelihood that the event will occur.

   (b) The relative frequencies of observations can be approximated or represented mathematically through probability density functions (PDFs). In this course, we will learn about a set of continuous and discrete probability functions. If data can be represented by a function such as the normal probability density function, the researcher has a large number of statistical tools to her disposal. Because of this, it is important to know if the relative frequencies of a data set can be represented by a PDF.

   (c) The reliability of the sample, when the population is not completely known, is assessed using probability tools.

2. Definitions of terms used in this section.

   (a) **Simple Event** - a potential outcome. In economics, these are referred to as a state of the world. A person searching for a job knows that there are two states of the world. After a day of job searching, the person is either A) employed or B) not employed. A & B are events or two states of the world.

   (b) **Mutually Exclusive Events** - describes the relationship among the set of events. A set of events that are mutually exclusive are ones that cannot occur together. Certain combinations of weather (e.g., snow blizzard and an air temperature of 90 degrees) cannot occur at the same time so they are mutually exclusive.

   (c) **Event/Sample space/Event space** - ways of describing a collection of events. In economics, this also may be called the states of world or state space.

3. How to obtain probability values.

   (a) From a distribution of data. The relative frequency values and cummulative relative frequency values can be interpreted as probabilities.

(b) Using the permutation and combination functions, you can determine the frequency of an event given the the total number of possible events.

(c) Using a probability distribution functions or probability density functions like poisson or normal (if the set of events can be represented using one of these functions).

4. Calculating probabilities

   (a) If a data set consist of a collection of mutually exclusive events (Categories) A_1, A_2,A_3, ...,A_n, the probability $P(A_i)$ that an event is event A_i is given as:

   $$P(A_i) = \lim_{n \to \infty} \frac{Freq.\ of\ event\ A_i}{n}$$

   (b) Probability is the limiting value of the relative frequency as the value of n increases.

   (c) Properties of $P(A_i)$

      i. $P(A_i)$ lies between 0 and 1.
      ii. If $P(A_i) = 0 \rightarrow$ the event never occurred.
      iii. If $P(A_i) = 1 \rightarrow$ only event A_i has occurred.
      iv. $\sum_{i=1}^{n} P(A_i) = 1 \quad \rightarrow$ the sum of all of the probabilities in the collection of events (or in the event space) equals 1. (recall: the sum of the relative frequencies equal 1.)

5. Counting Rules-how does one count the number of events and the number of possible arrangement of events

   (a) Counting the maximum number of events.

      i. One stage - one event from a set of n possible events. we are only looking at one time period. If there are n possible events, the maximum number of events is n.
      ii. Two stage - one event is expected to occur in period one. This event is an element of the set of events (m). In period two, another event from another set of events (n) occurs. Rule $\rightarrow$ If the first stage can be accomplished in m ways and the second stage can be accomplished in n ways, there are mn possible outcomes.
      iii. Multiple stages (Extended mn rule) $\rightarrow$ if there are k steps, each with n_k steps, the total number of possible outcomes are

      $$n_1 n_2 n_3 \ldots n_k$$

(b) Number of ways we can arrange n objects, taking them r at a time

i. Permutation - each arrangement of objects is treated as unique, even different arrangements of the same group of objects.

$$P_r^n = \frac{n!}{(n-r)!}$$

ii. Combination -different arrangements of the same objects are treated as the same.

$$C_r^n = \frac{n!}{r!(n-r)!}$$

(c) Examples

6. Event Composition and Event Relationship as a way to compute probabilities.

(a) Probability with a single set of events is simple. it is computed as

$$P(A_i) = \frac{\text{Frequency of a}}{\text{Total number of events}}$$

i. Properties of $P(A_i)$ are the same as the properties listed in 3.a.iii.

(b) Now the problem is determine the outcome when two events are considered. The textbook describes the case where a person has two coins. The person considers two possible events (A & B).

A. The coins are tossed and at least one is head.

B. The coins are tossed and at least one is tail.

(c) The sample or event space for 2 coin are the following. Each has a probability of 0.25 of occurring.

i. HH

ii. HT

iii. TH

iv. TT

(d) Probability of A occurring is the probability that either HH, HT, or TH happens (P(A) = 0.75). Probability of B occurring is the probability that either TT, HT, or TH happens (P(B) = 0.75).

(e) The intersection of A and B describes those events where A *and* B are the outcomes. These are HT and TH. $P(A \cap B)$=0.5

(f) The union of A and B describes those events where A *or* B are the outcomes. These are HH, TT, HT and TH. $P(A \cup B)$=1.0

(g) Disjoint events are two or more events that are mutually exclusive. In the coin example above, two disjoint events (C & D) are:

C. The coins are tossed and both are head. (HH)

D. The coins are tossed and both are tail. (TT)

(h) If the set of events are disjointed:

i. $P(C \cap D)$=0

ii. $P(C \cup D)$=P(C) + P(D) = 0.5

(i) If two events are complements, all of the events that are not in one event are found in the other. For the coin example, two complementary events (E & F) are:

E. The coins are tossed and both are head. (HH)

F. The coins are tossed and the coins are not both heads (TT, HT, TH). (F is also called E^c or the complement of E)

(j) If the events are complements.

i. $P(E \cap F)$=0

ii. $P(E \cup F)$=P(E) + P(F) = 1.0

iii. P(E) = 1-P(F)

(k) Conditional Probability - there exists a dependency of one event (A) with another (B)

i. Given that $P(A) \neq 0$, conditional probability of A given B has occurred (or $P(A|B)$)

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

ii. Given that $P(B) \neq 0$, conditional probability of B given A has occurred (or $P(B|A)$)

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

iii. Independence - two events (A & B) are independent if and only if:

A. $P(A|B)$ = P(A) or

B. $P(B|A)$ = P(B)

iv. Additive Rule - Given two events (A & B)

A. Probability of their union

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

B. If A and B are mutually exclusive, $P(A \cap B) = 0$, therefor,

$$P(A \cup B) = P(A) + P(B)$$

v. Multiplicative Rule - Given two events (A & B)

A. Probability of their intersection

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$$

B. If A and B are independent,$P(B|A) =$ P(B) and $P(A|B) =$ P(A), therefor

$$P(A \cap B) = P(A)P(B)$$

7. Discrete Random Variable and their probability distributions

(a) Previously, we introduced how probability tools allow us to describe the occurrence of events. When probability drives the outcome of events, we say that the events are random. Variables that capture random events are called random variables.

(b) Each distribution that is random can be described by its probabilities and has a probability distribution.

i. Probability distributions have a measure of center and a measure of dispersion, just like the previous distributions discussed in class.

A. Mean or the Expected value of the random variable x

$$\mu = E(x) = \sum_{i=1}^{N} x_i * p(x_i)$$

where $p(x_i)$ is the probability that x is x_i.

B. Variance of the random variable x

$$\sigma^2 = E[(x - \mu)^2] = \sum_{i=1}^{N} (x_i - \mu)^2 * p(x_i)$$

ii. There are two additional properties that come out of our earlier discussion of probability

A. The range of values in a probability distribution $\rightarrow$ ($0 \leq$ p(x) ≤ 1), where p(x) is the probability of random variable x.

B. $\sum$ p(x) $= 1$

iii. Finally, earlier distributions were described using relative frequencies tables and cumulative relative frequencies tables. Probability distribution can also be describe in this way. The description of a probability distribution that corresponds to a relative frequencies table is its probability density function (or PDF). The cumulative version of the probability density function is the CDF.

A. In general, PDF measures the probability that x is one category ($P(x = A_i)$

B. In general, CDF measures an interval (e.g., $P(x \leq A_i)$