Winter 2005 Micro Comprehensive Exam Solutions

(1) We have

$$\begin{aligned} Q^d &= Q^d(p) && (1)\\ Q^s &= Q^s(p,\hat{p},\gamma) && (2)\\ Q^d &= Q^s && (3)\\ \hat{Q}^d &= \hat{Q}^d(\hat{p}) && (4)\\ \hat{Q}^s &= \hat{Q}^s(p,\hat{p}) && (5)\\ \hat{Q}^d &= \hat{Q}^s. && (6) \end{aligned}$$

(a) Taking first total derivatives, we get:

$$\begin{aligned} dQ^d &= Q_p^d dp && (1^*)\\ dQ^s &= Q_p^s dp + Q_{\hat{p}}^s d\hat{p} + Q_\gamma^s d\gamma && (2^*)\\ dQ^d &= dQ^s && (3^*)\\ d\hat{Q}^d &= \hat{Q}_{\hat{p}}^d d\hat{p} && (4^*)\\ d\hat{Q}^s &= \hat{Q}_p^s dp + \hat{Q}_{\hat{p}}^s d\hat{p} && (5^*)\\ d\hat{Q}^d &= d\hat{Q}^s && (6^*). \end{aligned}$$

Then, from (1^*)–(3^*), we get $Q_p^d dp = Q_p^s dp + Q_{\hat{p}}^s d\hat{p} + Q_\gamma^s d\gamma$, or $(Q_p^d - Q_p^s)dp = Q_{\hat{p}}^s d\hat{p} + Q_\gamma^s d\gamma$, and from (4^*)–(6^*), we get $\hat{Q}_{\hat{p}}^d d\hat{p} = \hat{Q}_p^s dp + \hat{Q}_{\hat{p}}^s d\hat{p}$, or $(\hat{Q}_{\hat{p}}^d - \hat{Q}_{\hat{p}}^s)d\hat{p} = \hat{Q}_p^s dp$.

(b) The last equation above can be written $d\hat{p} = \frac{\hat{Q}_p^s}{\hat{Q}_{\hat{p}}^d - \hat{Q}_{\hat{p}}^s}dp$; plugging this into the equation found from (1^*)–(3^*), we get $(Q_p^d - Q_p^s)dp = Q_{\hat{p}}^s\left(\frac{\hat{Q}_p^s}{\hat{Q}_{\hat{p}}^d - \hat{Q}_{\hat{p}}^s}dp\right) + Q_\gamma^s d\gamma$. Simplifying yields

$$\left(Q_p^d - Q_p^s - \frac{Q_{\hat{p}}^s \hat{Q}_p^s}{\hat{Q}_{\hat{p}}^d - \hat{Q}_{\hat{p}}^s}\right)dp = Q_\gamma^s d\gamma.$$

(c) Multiplying both sides through by p/Q gives us

$$\left(Q_p^d \cdot \frac{p}{Q} - Q_p^s \cdot \frac{p}{Q} - \frac{Q_{\hat{p}}^s \cdot \frac{\hat{p}}{Q} \cdot \hat{Q}_p^s \cdot \frac{p}{\hat{Q}}}{\hat{Q}_{\hat{p}}^d \cdot \frac{\hat{p}}{\hat{Q}} - \hat{Q}_{\hat{p}}^s \cdot \frac{\hat{p}}{\hat{Q}}}\right)dp = Q_\gamma^s \cdot \frac{p}{Q}d\gamma$$

(note that both numerator and denominator of the third term of the dp coefficient have also been multiplied by $\hat{p}/\hat{Q}$), which simplifies to

$$\left(e_d - e_s - \frac{e_{\hat{s}}\hat{e}_s}{\hat{e}_{\hat{d}} - \hat{e}_{\hat{s}}}\right)dp = Q_\gamma^s \cdot \frac{p}{Q}d\gamma.$$

This can be written

$$\frac{dp}{d\gamma} = \frac{Q_\gamma^s \cdot \frac{p}{Q}}{e_d - e_s - \frac{e_{\hat{s}}\hat{e}_s}{\hat{e}_{\hat{d}} - \hat{e}_{\hat{s}}}};$$

multiplying both sides by γ/p gives

$$\frac{dp}{d\gamma} \cdot \frac{\gamma}{p} = \frac{Q_\gamma^s \cdot \frac{p}{Q} \cdot \frac{\gamma}{p}}{e_d - e_s - \frac{e_{\hat{s}}\hat{e}_s}{\hat{e}_{\hat{d}} - \hat{e}_{\hat{s}}}},$$

or equivalently,

$$e_{p,\gamma} = \frac{e_\gamma}{e_d - e_s - \frac{e_{\hat{s}}\hat{e}_s}{\hat{e}_{\hat{d}} - \hat{e}_{\hat{s}}}} = -\frac{e_\gamma}{|e_d| + |e_s| - \frac{e_{\hat{s}}\hat{e}_s}{|\hat{e}_{\hat{d}}| + |\hat{e}_{\hat{s}}|}}.$$

(d) Taking Equations (1*)–(3*) only, and going through a similar (though much simpler) process, yields $e_{p,\gamma} = -\frac{e_\gamma}{|e_d|+|e_s|}$. This is the same as the answer to (c) if and only if $e_{\hat{s}}\hat{e}_s = 0$, which is true if and only if either $e_{\hat{s}}$ or $\hat{e}_s$ is zero. This last condition means that changes in the price of Good 1 do not affect the supply of Good 2, or changes in the price of Good 2 do not affect the supply of Good 1. (This is a sort of supply analogue to the goods' being neither complements nor substitutes, though we don't normally use these terms when talking about supply.)

(e) From (b), we saw that $d\hat{p} = \frac{\hat{Q}^s_p}{\hat{Q}^d_{\hat{p}}-\hat{Q}^s_{\hat{p}}}dp$. Also from (b), we saw that $dp = \frac{Q^s_\gamma}{Q^d_p - Q^s_p - \frac{Q^s_{\hat{p}}\hat{Q}^s_p}{\hat{Q}^d_{\hat{p}}-\hat{Q}^s_{\hat{p}}}}d\gamma$.

Combining these, we get

$$d\hat{p} = \frac{\hat{Q}^s_p}{\hat{Q}^d_{\hat{p}} - \hat{Q}^s_{\hat{p}}} \cdot \frac{Q^s_\gamma}{Q^d_p - Q^s_p - \frac{Q^s_{\hat{p}}\hat{Q}^s_p}{\hat{Q}^d_{\hat{p}}-\hat{Q}^s_{\hat{p}}}}d\gamma.$$

Given the assumptions stated, this expression is nonzero as long as $\hat{Q}^s_p \neq 0$, so changes in γ do indeed affect $\hat{p}$. (This should be intuitively obvious, as changes in γ clearly affect p, which will affect $\hat{p}$ as long as changes in p affect the supply of Good 2.)

(f) Partial–equilibrium analysis ignores the effects of markets upon other markets; general–equilibrium analysis takes them into account. In this problem, a change in γ has more than one effect on p. There is a direct effect, captured in the nonzero value of Q^s_γ (and the corresponding elasticity) and the effect of changes in supply on p. But there is also an indirect effect, due to the facts that changes in p affect the supply of Good 2, and changes in $\hat{p}$ affect the supply of Good 1. Because of this, a change in γ leads to a change in p (the direct effect), which affects the supply of Good 2 and hence $\hat{p}$, which affects the supply of Good 1 and hence p. The feedback across markets is ignored by partial equilibrium analysis. The indirect effect is often (though not always) small compared to the direct effect, which is why partial–equilibrium analysis can still be useful, especially given its computational simplicity relative to general–equilibrium analysis.

(3) Joe's utility function is $U(x_1, x_2) = \alpha \cdot \ln(x_1) + \beta \cdot \ln(x_2)$, where $\alpha + \beta = 1$. This is a standard Cobb–Douglas utility function.

(a) Demand functions can be found by setting up the Lagrangean or by setting the MRS equal to the price ratio, and solving from there. We get $x_1 = \alpha \cdot \frac{\hat{m}}{p_1}$ and $x_2 = \beta \cdot \frac{\hat{m}}{p_2}$. Indirect utility is found by plugging these into the original utility function, so $v = \ln\left(\frac{\alpha^\alpha \beta^\beta M}{p_1^\alpha p_2^\beta}\right) = \ln(\alpha^\alpha\beta^\beta) + \ln(M) - \ln(p_1^\alpha p_2^\beta)$.

(b) Under the assumption $p_1 = p_2 \equiv \hat{p}$, Joe's indirect utility function becomes $v = \ln(\alpha^\alpha\beta^\beta) + \ln(M) - \ln(\hat{p})$.

(c) From the answer to (b), we have $v'(m) = \frac{1}{m}$ and $v''(m) = -\frac{1}{m^2}$. For any $m > 0$, $v''(m) < 0$, so Joe is risk averse in income. So, making income uncertain while keeping the mean the same makes him worse off.

(d) We have $v'(m) = \frac{1}{m}$ and $v''(m) = -\frac{1}{m^2}$, so Joe's coefficient of relative risk aversion in income is

$$-\frac{m \cdot v''(m)}{v'(m)} = +1.$$

(e) We have $v'(p) = -\frac{1}{p}$ and $v''(p) = +\frac{1}{p^2}$. For any $p > 0$, $v''(p) > 0$, so Joe is risk seeking in the price level. So, making the price level uncertain while keeping the mean the same makes him better off.

(4) The monopolist's profit function is $\Pi = p_H - \frac{1}{2}q_H^2 + p_L - \frac{1}{2}q_L^2$.

(a) This is a case of third–degree price discrimination. The monopolist will set prices to extract all surplus from the consumers: $p_H = \theta_H q_H$ and $p_L = \theta_L q_L$. So, its profit function will be $\Pi = \theta_H q_H - \frac{1}{2}q_H^2 + \theta_L q_L - \frac{1}{2}q_L^2$. Maximizing with respect to q_H yields $q_H = \theta_H$, and maximizing with respect to q_L yields $q_L = \theta_L$. Then, $p_H = \theta_H^2$ and $p_L = \theta_L^2$, so profit will be $\Pi = \frac{1}{2}\theta_H^2 + \frac{1}{2}\theta_L^2$. Both consumers will have a consumer surplus of zero.

(b) This is now second–degree price discrimination. The monopolist will choose prices and qualities to maximize its profit, subject to four constraints:

$$\begin{aligned}
\theta_H q_H - p_H &\geq 0 && (1)\\
\theta_L q_L - p_L &\geq 0 && (2)\\
\theta_H q_H - p_H &\geq \theta_H q_L - p_L && (3)\\
\theta_L q_L - p_L &\geq \theta_L q_H - p_H. && (4)
\end{aligned}$$

The first two constraints are Hugo and Louise's participation constraints (respectively); the last two are their incentive–compatibility constraints (in the same order).

(c) Notice that if the monopolist is maximizing profit, at least one of (1) and (2) must be satisfied with equality (otherwise, the monopolist could raise p_H and p_L by equal small amounts, increasing profit while still satisfying all constraints). Similarly, at least one of (1) and (3) must be satisfied with equality (otherwise, the monopolist could raise p_H by a small amount). (This is also true of (2) and (4), though we don't need this for the problem.) But it can not be that (1) is satisfied with equality; if it were, then we would have

$$\begin{aligned}
0 &= \theta_H q_H - p_H \quad \text{(by assumption)}\\
&\geq \theta_H q_L - p_L \quad \text{(by (3))}\\
&> \theta_L q_L - p_L \quad (\text{since } \theta_H > \theta_L)
\end{aligned}$$

(that is, $\theta_L q_L - p_L < 0$), which violates (2) above. Since (1) cannot be satisfied with equality, it must be that (2) and (3) are.

(d) Since (2) is satisfied with equality, we have $p_L = \theta_L q_L$; since (3) is, we have $p_H = \theta_H q_H - \theta_H q_L + p_L = \theta_H q_H - \theta_H q_L + \theta_L q_L$. Plugging into the monopolist's profit function yields an expression with only quality variables: $\Pi = \theta_H q_H - \theta_H q_L + \theta_L q_L - \frac{1}{2}q_H^2 + \theta_L q_L - \frac{1}{2}q_L^2$.

(e) Maximizing this profit function with respect to q_H gives $q_H = \theta_H$; maximizing with respect to q_L gives $q_L = 2\theta_L - \theta_H$. (Since $\theta_H < 2\theta_L$, this is positive.) Then, $p_H = \theta_H(\theta_H) - \theta_H(2\theta_L - \theta_H) + \theta_L(2\theta_L - \theta_H) = 2\theta_H^2 - 3\theta_L\theta_H + 2\theta_L^2$, and $p_L = \theta_L(2\theta_L - \theta_H) = 2\theta_L^2 - \theta_L\theta_H$. Profit is

$$\begin{aligned}
\Pi &= 2\theta_H^2 - 3\theta_L\theta_H + 2\theta_L^2 - \frac{1}{2}\theta_H^2 + 2\theta_L^2 - \theta_L\theta_H - \frac{1}{2}(2\theta_L - \theta_H)^2\\
&= \theta_H^2 - 2\theta_H\theta_L + 2\theta_L^2.
\end{aligned}$$

Louise's consumer surplus is zero; Hugo's is $\theta_H q_H - p_H = (\theta_H - \theta_L)(2\theta_L - \theta_H)$, which is positive since $2\theta_L > \theta_H > \theta_L$.

(f) These results show that in this situation, moving from third–degree to second–degree price discrimination has no effect on q_H or Louise's consumer surplus. It raises Hugo's consumer surplus, and lowers profit, q_L, and both prices. All of these are trivial to show except p_H and profit. For p_H, note that $p_H(2nd) - p_H(3rd) = \theta_H^2 - 3\theta_L\theta_H + 2\theta_L^2 = (\theta_H - \theta_L)(\theta_H - 2\theta_L)$; the first factor is positive and the second is negative, so $p_H(2nd) - p_H(3rd) < 0$. Similar steps can be used for profit.