

Econ 4349 Final exam solutions

1.

		Firm 2	
		A	B
Firm 1	A	-11, -9, -7	-1, 13 , 3
	B	15 , 1 , 3	3 , 1 , 3

Firm 3: A

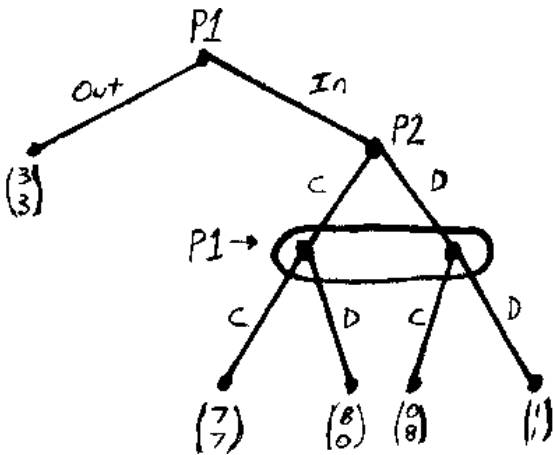
		Firm 2	
		A	B
Firm 1	A	-1, 1 , 11	11 , 1 , -1
	B	3 , 13 , -1	-7, -9, -11

Firm 3: B

a. Boldface payoffs correspond to best responses to pure strategies.

b. The pure-strategy Nash equilibria are (B, A, A) and (B, B, A).

2.



a. The subgame perfect equilibrium is found as follows. First, solve the (only) proper subgame: the portion of the game tree starting at Player 2's decision node. This subgame can be written in strategic form as:

		Player 2	
		C	D
Player 1	C	7,7	0,8
	D	8,0	1,1

This is just the Prisoner's Dilemma, and as always, has one Nash equilibrium: (D, D) with payoffs (1, 1). Moving back one step to Player 1's first decision, she can choose Out and get 3, or In and get 1 (given what we just found for the subgame). She will choose Out. So, the subgame perfect equilibrium is ((Out, D), D).

b. A strategic form for this game is:

		Player 2	
		C	D
Player 1	Out, C	3,3	3,3
	Out, D	3,3	3,3
	In, C	7,7	0,8
	In, D	8,0	1,1

Boldface payoffs correspond to best responses to pure strategies, and Nash equilibria are $((Out, C), D)$ and $((Out, D), D)$.

c. Yes, the subgame perfect equilibrium is indeed one of the Nash equilibria.

3.

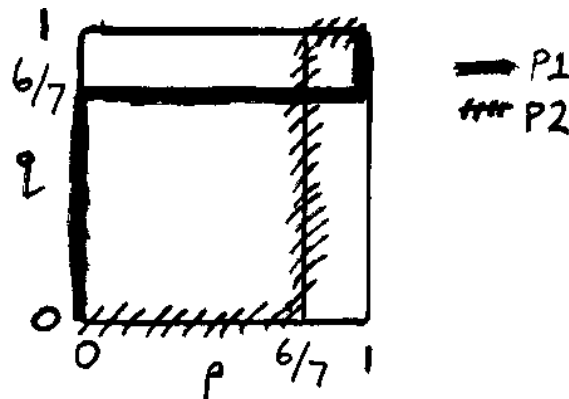
		Player 2	
		Left	Right
Player 1	Up	1,1	0,0
	Down	0,0	6,6

As usual, let p be the probability of Player 1's choosing Up, and q the probability of Player 2's choosing Left. For Player 1, the payoff to choosing Up is q , and the payoff to choosing Down is $6(1-q)=6-6q$. Up is Player 1's best response when $q > 6-6q$, which happens when $q > 6/7$. Then, Down is her best response when $q < 6/7$, and all pure and mixed strategies are best responses when $q = 6/7$. So, when $q > 6/7$, she should choose Up (that is, $p=1$), when $q < 6/7$, she should choose Down (that is, $p=0$), and when $q = 6/7$, she can choose any strategy (that is, $p \in [0,1]$).

The game is symmetric, so Player 2's decision is similar. When $p > 6/7$, he should choose Up (that is, $q=1$), when $p < 6/7$, he should choose Down (that is, $q=0$), and when $p = 6/7$, he can choose any strategy (that is, $q \in [0,1]$).

A best-response diagram is shown below. Nash equilibria are:

- $((1,0), (1,0))$ or equivalently (U, L)
- $((0,1), (0,1))$ or equivalently (D, R)
- $((6/7, 1/7), (6/7, 1/7))$



4. The preferences of the professor and the student are as follows:

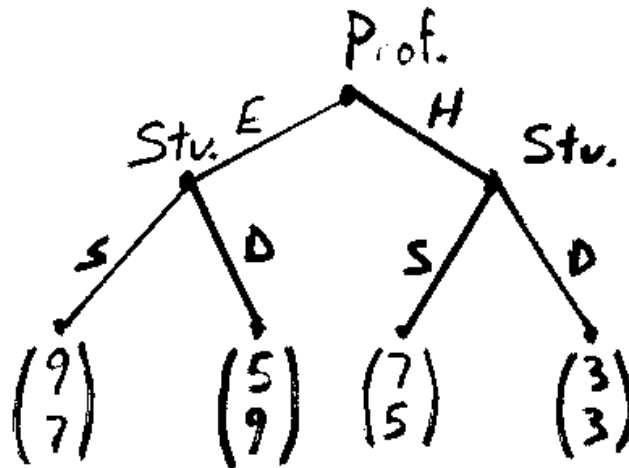
	<u>Professor</u>	<u>Student</u>	<u>Utility</u>
Favorite outcome:	Easy test, student studies	Easy test, student does not study	9
2 nd -favorite outcome:	Hard test, student studies	Easy test, student studies	7
3 rd -favorite outcome:	Easy test, student does not study	Hard test, student studies	5
Least-favorite outcome:	Hard test, student does not study	Hard test, student does not study	3

a. If the professor and student make their decisions *simultaneously*, then the game has this strategic form:

		Student	
		Study (S)	Don't Study (D)
Professor	Hard (H)	7,5	3,3
	Easy (E)	9,7	5,9

b. The relevant solution is Nash equilibrium. Boldface payoffs correspond to best responses to pure strategies, and the Nash equilibrium is (E, D). This game can also be solved by iterated removal of dominated strategies, as E dominates H for the professor, and once this is removed, D dominates S for the student.

c. If instead, the professor chooses first and the student observes this choice before making her own choice, we have a *game with sequential moves*. An extensive form is shown below.



d. Now, the relevant solution is subgame perfect equilibrium (or equivalently here, rollback equilibrium). The student prefers D if the professor has chosen E (left node) and S if the professor has chosen H (right node). So, the professor will choose H. The solution is therefore (H, DS).

e. When the professor does not announce his decision to the student, we are in the simultaneous-move case of (a) and (b), and payoffs are 5 for the professor and 9 for the student. When he does announce his decision, we are in the sequential-move case of (c) and (d), with payoffs 7 for the professor and 5 for the student. So the professor prefers to announce his decision, while the student would rather he kept it a secret.