Econ 4349 Midterm #4 solutions

1. (30 points)

Player 3: A

Player 1 \ Player 2	A	B
A	**9,9,9**	0,1,0
B	1,0,0	**1**,**1**,0

Player 3: B

Player 1 \ Player 2	A	B
A	0,0,1	0,**1**,**1**
B	**1**,0,**1**	**3,3,3**

a. (6 points) In the strategic form above, boldface payoffs correspond to pure-strategy best responses. Pure-strategy Nash equilibria are (A,A,A) and (B,B,B).

b. (24 points) Let (p,1-p), (q,1-q), and (r,1-r) be the three players' mixed strategies. For Player 1, the expected payoff to choosing A is

$$9qr+0(1-q)r+0q(1-r)+0(1-q)(1-r) = 9qr,$$

and the expected payoff to choosing B is

$$1qr+1(1-q)r+1q(1-r)+3(1-q)(1-r) = 3-2q-2r+2qr.$$

Player 1 will be willing to choose a mixed strategy only if the payoffs to A and B are equal, which is true if $9qr = 3-2q-2r+2qr$. This equality simplifies to $7qr+2q+2r-3=0$.

Similar arguments for Player 2 and Player 3 give the conditions for their playing mixed strategies:

$$7pr+2p+2r-3=0$$
$$7qr+2q+2r-3=0$$

We looking for a symmetric equilibrium, so that p=q=r. Then, each of these equations simplifies to $7p^2+4p-3=0$. This quadratic has two solutions: p=-1 and p=3/7. Since probabilities must be between 0 and 1, we can eliminate p=-1. Then, since p=q=r, we have q=3/7 and r=3/7, so the mixed-strategy Nash equilibrium is ((3/7,4/7), (3/7,4/7), (3/7,4/7)).

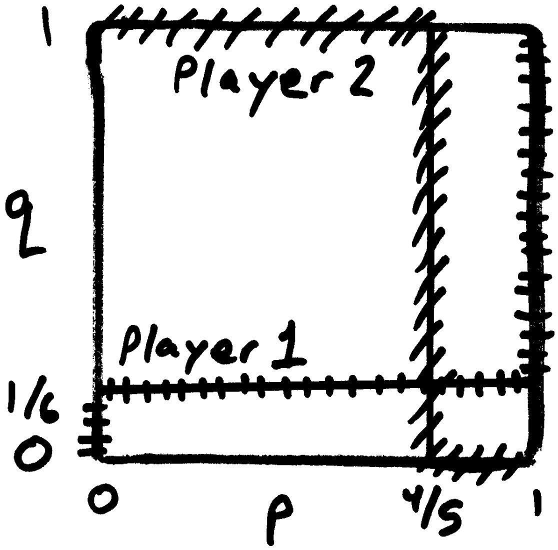
2. (25 points)

Player 1 \ Player 2	Y	Z
A	9,-1	-1,1
B	-1,7	1,-1

a. (20 points) Let (p,1-p) and (q,1-q) be the players' mixed strategies. Then for Player 1, the expected payoff to choosing A is $9q+(-1)(1-q)=10q-1$, and the expected payoff to choosing B is $-1q+1(1-q)=1-2q$. So, A is her best response if $10q-1>1-2q$, or equivalently when $q>1/6$. B is her best response when $q<1/6$, and she is willing to choose any pure or mixed strategy when $q=1/6$.

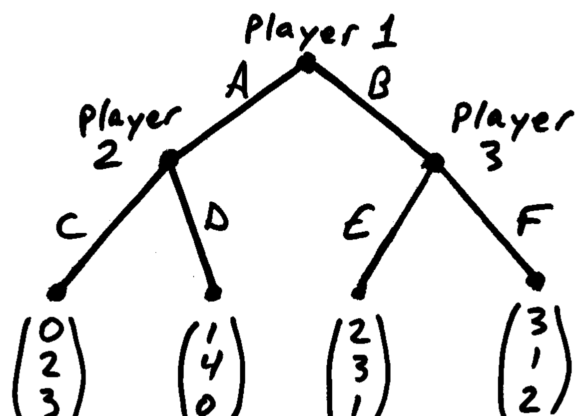
For Player 2, the expected payoff to choosing Y is $-1p+7(1-p)=7-8p$, and the expected payoff to choosing Z is $1p+(-1)(1-p)=2p-1$. So, Y is her best response if $7-8p>2p-1$, or equivalently when $p<4/5$. Z is her best response when $p>4/5$, and she is willing to choose any pure or mixed strategy when $p=4/5$.

A best-response diagram is shown below.



b. (5 points) The only Nash equilibrium is ((1/6,5/6), (4/5,1/5)).

3. (25 points)



a. (9 points) At Player 2's decision node, he will choose D (for a payoff of 4) instead of C (for a payoff of 2). At Player 3's decision node, she will choose F (for a payoff of 2) instead of E (for a payoff of 1). Based on these choices, Player 1 will choose B (for a payoff of 3) instead of A (for a payoff of 1). So, the rollback equilibrium is (B,D,F).

b. (10 points) A strategic form is shown below.

Player 3: E

		Player 2: C	Player 2: D
Player 1	A	0,2,**3**	1,**4**,**0**
	B	**2**,**3**,1	**2**,**3**,1

Player 3: F

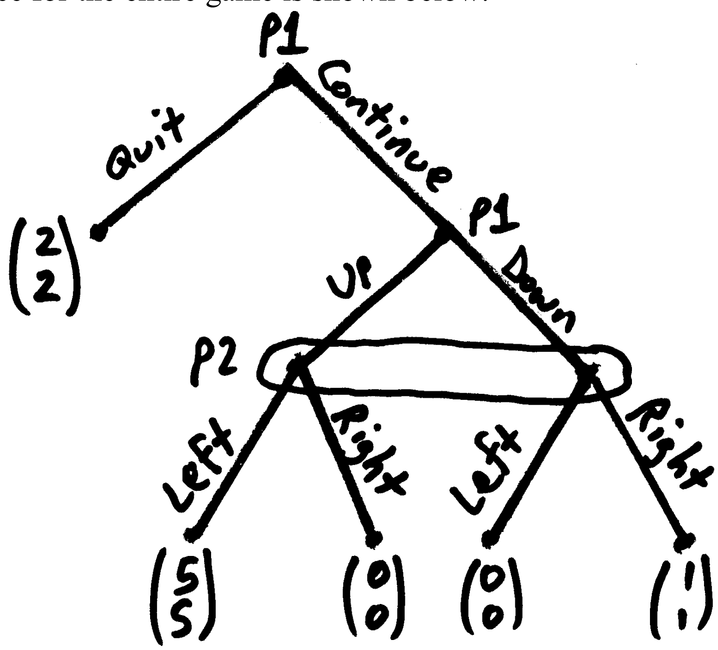
		Player 2: C	Player 2: D
Player 1	A	0,2,**3**	1,**4**,**0**
	B	**3**,**1**,**2**	**3**,**1**,**2**

c. (6 points) Boldface payoffs above correspond to pure-strategy best responses. Pure-strategy Nash equilibria are (B,C,F) and (B,D,F).

4. (25 points) First, Player 1 chooses whether to Quit or Continue. If Player 1 chooses Quit, the game ends and both players receive 2. If she chooses Continue, they play the game below.

		Player 2	
		Left	Right
Player 1	Up	5,5	0,0
	Down	0,0	1,1

a. (9 points) A game tree for the entire game is shown below.



b. (8 points) Player 1's pure strategies are (Quit,Up), (Quit,Down), (Continue,Up), and (Continue,Down). Player 2's pure strategies are Left and Right.

c. (8 points) This game has two subgames: the entire game and the portion starting after Player 1 has chosen Continue. This proper subgame is equivalent to the game matrix above, and has two pure-strategy Nash equilibria: (Up,Left) with payoffs (5,5), and (Down,Right) with payoffs (1,1). To solve the entire game, we use the two solutions of the proper subgame.

Case 1: (Up,Left) in subgame.
Then, Player 1 will choose Continue, so the subgame perfect equilibrium will be ((Continue,Up), Left).

Case 2: (Down,Right) in subgame.
Then, Player 1 will choose Quit, so the subgame perfect equilibrium will be ((Quit,Down), Right).

These are the only pure-strategy subgame perfect equilibria.

5. (15 points) The game tree with information sets in the correct places is shown below.