On Justifications for the ad hoc Black-Scholes Method of Option Pricing

Jeremy Berkowitz
Department of Finance
University of Houston

May 23, 2009

Abstract: One of the most widely used option valuation procedures among practitioners is a version of Black-Scholes in which implied volatilities are smoothed across strike prices and maturities. A growing body of empirical evidence suggests that this *ad hoc* approach performs quite well. It has previously been argued that such a procedure works because it amounts to a sophisticated interpolation tool. We show that this is the case in a formal, asymptotic sense. In addition, we conduct some simulations which allow us to examine the importance of the sample size, the order of the polynomial, and the recalibration frequency in controlled settings. We also apply the ABS approach to daily S&P 100 index options to show that the procedure outperforms the Black-Scholes formula in valuing actual option prices out-of-sample.

Acknowledgements: I am grateful to Nai-fu Chen, Peter Christoffersen, Jeff Fleming, Kris Jacobs, Philippe Jorion, Praveen Kumar, Bruce Mizrach, Matt Pritsker, Stuart Turnbull and Robert Whaley who provided many helpful suggestions. Address correspondence to Bauer College of Business, 334 Melcher Hall, University of Houston, Houston, TX 77204-6021, (713) 743-4764, jberkowitz@uh.edu. Any errors or inaccuracies are solely the responsibility of the author.

On Justifications for the ad hoc Black-Scholes Method of Option Pricing

Abstract: One of the most widely used option valuation procedures among practitioners is a version of Black-Scholes in which implied volatilities are smoothed across strike prices and maturities. A growing body of empirical evidence suggests that this *ad hoc* approach performs quite well. It has previously been argued that such a procedure works because it amounts to a sophisticated interpolation tool. We show that this is the case in a formal, asymptotic sense. In addition, we conduct some simulations which allow us to examine the importance of the sample size, the order of the polynomial, and the recalibration frequency in controlled settings. We also apply the ABS approach to daily S&P 100 index options to show that the procedure outperforms the Black-Scholes formula in valuing actual option prices out-of-sample.

1. Introduction

One of the most widely used option-valuation techniques among traders and other practitioners is an ad hoc procedure in which Black-Scholes (1973) implied volatilities are smoothed across strike prices and maturities and then plugged back into the Black-Scholes formula. This "ad hoc Black-Scholes" approach has become something of a benchmark for evaluating the forecast accuracy of option pricing models because of its consistently impressive empirical performance. Dumas, Fleming and Whaley (1998), for example, find that the ad hoc Black-Scholes (ABS) model forecasts option prices out-of-sample more accurately than Black-Scholes and the deterministic volatility function (DVF) model of Derman and Kani (1994) and Rubinstein (1994). Brandt and Wu (2002) focus on cross-sectional forecast accuracy and show that the ad hoc approach outperforms DVF models in valuing FTSE 100 index options. Heston and Nandi (2000) show that the ad hoc approach compares favorably with a close-form GARCH option pricing model. Christoffersen and Jacobs (2004) find that, with daily updating of parameters, the ABS model outperforms Heston's (1993) theoretical model both in and out of sample.

At the same time, several authors have suggested an explanation for why the ad hoc approach performs so well. Davis (2001) notes that the smoothing of implied volatilities may be viewed as a sophisticated interpolation tool used to ensure that an option is priced consistently with the market prices of other traded options. In this sense, estimation of an implied volatility surface is analogous to fitting the observed set of option prices itself. Davis (2001) and Figlewski (2002) argue that this kind of interpolation technique can be particularly useful for valuing options which are illiquid or which do not trade on an exchange.

An implication of this argument is that other interpolation schemes similar to the ABS approach should provide similar approximation accuracy. For example, Davis (2001) argues that any interpolation approach with a reasonably smooth map from volatility to price should give similar answers. Similarly, Hull (2008) states that if traders switched from Black-Scholes to another plausible model, then arguably dollar prices would not change appreciably.

The incremental contribution of the present paper is to formalize the intuitive arguments of Davis (2001), Figlewski (2002) and Hull (2008). We show that the ABS procedure can be used to provide arbitrarily accurate asymptotic approximations of a true but unknown option pricing formula. This is established through a fairly simple application of the Weierstrass Approximation Theorem, which states that any continuous function can be approximated arbitrarily well by a suitable polynomial. We also confirm that the Black-Scholes formula does not play a crucial role in the interpolation scheme in an asymptotic sense and could indeed be replaced by a wide variety of families of continuous function. As long as the model is used as an interpolation tool and is recalibrated frequently, the approximation argument holds.

In addition, we conduct some simple Monte Carlo simulations to get a feel for the kind of polynomials required to reasonably fit option prices in an environment where the Black-Scholes formula is not applicable. The simulations also allow us to examine the importance of the sample size, the order of the polynomial, and the recalibration frequency in controlled settings. Lastly, we apply the ABS approach to daily S&P 100 index options to show that the procedure outperforms the Black-Scholes formula in valuing actual option prices out-of-sample.

Before outlining the primary results, we briefly describe the ABS model closely following Dumas, Fleming and Whaley (1998). Traders collect Black-Scholes implied volatilities from a cross-section of options on a single underlying asset of interest. Implied volatilities are sometimes available through commercial data services or may be inverted from observed option prices.

The cross-section of implied volatilities is regressed against a polynomial function of the option strike prices and option maturities,

$$\sigma_n^{IV} = \beta_0 + \beta_1 Strike_n + \beta_2 Maturity_n + \varepsilon_n . \tag{1}$$

Fitted values $\hat{\sigma}_n$ from this regression do not have a sensible interpretation – at any given time, there should be one volatility implied by options on the same underlying asset. Nevertheless, equation (1) forms the basis of the ad hoc Black-Scholes approach.

Suppose we wish to value an option with strike *K* and maturity *T*, given a cross-section of observed option prices on the same underlying asset. The option is valued using the Black-Scholes formla

$$BS\left(K, T, S_t, r, \hat{\sigma}_n(K, T)\right) \tag{2}$$

where S_t is the price of the underlying asset, r is the risk-free rate, and volatility is set equal to the fitted value $\hat{\sigma}_n(K,T)$ from regression (1). Formula (2) differs from Black-Scholes only in that each option has its "own volatility" – a free parameter which captures deviations from Black-Scholes across concurrent options.

Regression equation (1) is continually re-estimated as new data becomes available, typically once or twice daily. This results in a new implied volatility surface and new fitted values. Traders thus take the Black-Scholes model -- conceived as a static model -- and continually recalibrate it by updating the unknown parameter.

To understand the efficacy of this approach, suppose there is some true, but unknown, option pricing formula. The implied volatility surface is fit to a large cross-section of observed option prices. As the cross-section grows large, the surface becomes an increasingly accurate approximation of the true but unknown model. When plugged into the Black-Scholes formula, the resulting option prices are increasingly accurate estimates of the true option prices.

While the above procedure can be used to approximate to the cross-section at a given, there is no reason to expect the approximation to hold over time. This is remedied by frequent re-calibration of the volatility surface, which produces continually new approximations. We show that if data generating process is a stochastic volatility model or a stochastic volatility model with jumps, one-step ahead forecast errors approach zero as the calibration becomes more frequent and the forecast horizon approaches zero.

As a byproduct of our analysis, we show that our asymptotic argument can be made valid for American options if the volatilities are inverted from American options. However, the asymptotic error does not necessarily disappear for American options if the implied volatilities are calculated from European options. This may help elucidate some recent empirical findings. In particular, Hull and Suo (2003) find that the ABS model based on European options does not necessarily perform well in pricing other kinds of options.

The remainder of the paper is organized as follows. Section 2 describes the context and the cross-sectional forecasting problem. Section 3 extends the basic framework to time-series forecasting. Section 4 describes a simple simulation study and the results are applied to a set of S&P 100 index options in Section 5. Section 6 concludes.

2. Cross-Sectional Option Pricing

Suppose the true option pricing formula is an unknown function,

$$C_t = \phi(K, T, S_t, r_t, \sigma_t) \quad (3)$$

where K is the strike, T is the time to maturity, S_t is the price of the underlying, r_t is the instantaneous risk free rate and σ_t is the volatility. We assume that option prices are continuous in the arguments, (K, T, S_t, r_t, σ_t). This kind of model can easily be generalized further to include other contemporaneous variables but equation (3) already includes most common theoretical models.

In reality the true option pricing function is unknown and must be replaced with a theoretical valuation model, $\hat{\phi}(\cdot)$, such as Black-Scholes. At the same time, current volatility is unobserved. This means the forecasting problem involves modeling the option pricing function simultaneously with estimating one of the arguments -- volatility.

The problem caused by volatility being unobserved is closely related to the model specification problem. If the model were known, the volatility could be inverted from just a single observed option price. Unfortunately, the model is not known. Denote the implied volatility calculated from a particular option, say (K_i, T_i), as

$$\hat{\sigma}_{i,t} = \hat{\phi}^{-1}(C_{i,t}, K_i, T_i, S_t, r_t). \quad (4)$$

Given a cross-section of N option values $\{C_{1,t}, ..., C_{N,t}\}$ at time t, the estimated volatilities inverted from Black-Scholes display a "smirk" or "smile" pattern across strike prices and across maturities. Such implied volatilities should be identical across concurrent option contracts. Since they are not, the Black-Scholes model is clearly misspecified.

In order to value an unobserved N+1[st] option, we must specify both $\hat{\phi}(\cdot)$ and the volatility $\hat{\sigma}_t$ such that:

$$C_{i,t} \approx \hat{\phi}(\xi_{i,t}, \hat{\sigma}_t) \quad (5)$$

where for notational simplicity $\xi_{i,t} = (K_i, T_i, S_t, r_t)$. Exchanges, for example, often require such valuations so that they can set margin requirements. Broker-dealers similarly need to value option contracts for which market prices are unavailable. Since many other option prices are observed, this is essentially a cross-sectional forecasting problem.

More generally, we may be interested in forecasting option values at future dates as well. For example, at large commercial banks traders forecast the value of their option positions at a daily frequency (Berkowitz, O'Brien (2002)).

In the present paper, we characterize an approach to problems of this form commonly referred to as the *ad hoc Black-Scholes* (ABS) model. Dumas, Fleming and Whaley (1998) consider specifications such as

$$\begin{aligned}\hat{\sigma}_i &= \beta_0 + \beta_1 K_i + \beta_2 T_i + \beta_3 K_i^2 + \beta_4 T_i^2 + \varepsilon_i \\ &= \beta' Z_i + \varepsilon_i \qquad (6)\end{aligned}$$

where $\beta = (\beta_0,...,\beta_4)'$ are unknown parameters, Z_i are the explanatory variables and ε_i are residuals. The implied volatilities are calculated from Black-Scholes and the coefficients in (6) are estimated by least squares.

Fitted values from (6) are plugged back into Black-Scholes to get the predicted value:

$$\hat{C}_{i,t} = \hat{\phi}(K_i, T_i, S_t, r_t, \hat{\beta}' Z_i) \qquad (7)$$

where $\hat{\beta}$ is the least squares estimate.

The m^{th} order polynomial generalization of (6) is given by

$$\begin{aligned}\hat{\sigma}_i &= \beta_0 + \beta_1 K_i + ... + \beta_m K_i^m + ... + \beta_{2m} T_i^m + \beta_{2m+1} K_i T_i^{m-1} + ... + \varepsilon_i \\ &= \beta' Z_i^m + \varepsilon_i \qquad (8)\end{aligned}$$

where again the left-hand side are the implied volatilities inverted from *N* observed options and Z_i^m is the vector of explanatory variables. We can now delineate a set of assumptions under which the above procedure yields arbitrarily accurate forecasts as the number of regressors *m* grows large.

We begin by stating two technical assumptions regarding the true data generating process.

Assumption 1. Option values $\phi(\cdot)$ are continuous in all arguments on a compact set.

Assumption 2. The time t volatility is a continuous function of some, possibly infinite, set of variables, Z_{it}. Formally, it is a function $\sigma_t = \sigma(Z_{it})$.

We assume that Z_{it} is observable but we do not make any such assumption on the functional form of $\sigma(\cdot)$. We also make the usual assumption that the unknown parameter vector θ is estimated by ordinary least squares.

Assumption 3. The implied volatility equation (8) is estimated by minimizing the sum of squared pricing errors

$$\hat{\theta} = \arg\min_{\theta} \sum_{i=1}^{N} \left(C_{i,t} - \hat{\phi}(K_i, T_i, S_t, r_t, \beta' Z_i^m)\right)^2 . \tag{9}$$

The number of observed options, N, obviously has to exceed the number of parameters to be estimated: $N \geq m^2$.

Under these conditions, the ABS procedure yields an arbitrarily accurate fit of the time-t set of contingent claims. This is true regardless of the unknown functional form of the true model.

Theorem 1. Under assumptions 1-3, the ad hoc model provides arbitrarily accurate option valuations in-sample,

$$C_{i,t} - \hat{\phi}(K_i, T_i, S_t, r_t, \hat{\beta}' Z_i^m) \to 0$$

uniformly in the arguments (K_i, T_i, S_t, r_t) as $m \to \infty$.

Proof. See appendix

Economically, Theorem 1 means the approach provides arbitrarily accurate option values within the cross-sectional sample. Suppose, for example, that ε is \$.10. An implied volatility surface estimated with a polynomial of degree at least m is guaranteed to value the observed options used to calibrate the model as well as any unobserved concurrent options within 10 cents.

It is not necessary for the researcher to know the true function $C_t = \phi(K, T, S_t, r_t, \sigma_t)$. The procedure will work as long as the true option price obeys some basic smoothness conditions. As the number of observed options increases, the maximum pricing error approaches zero. In particular, if there were an unlimited number of options observed at time t, the implied volatility surface can be fit to an unboundedly high degree polynomial. When plugged into a valuation model like Black-Scholes, this translates into unlimited flexibility in fitting the true option pricing relation.

In this sense, the result is an asymptotic argument. In cases of practical interest, the order of the polynomial m and the sample size N are finite numbers. However, empirical studies have generally found that even quadratic polynomials price options well so that the number of parameters in (8) is typically quite small, in the range of 5 or 6 (e.g., Dumas, Fleming and Whaley (1998), Christoffersen and Jacobs (2004)). The simulation results in section 4 of the present paper appear to confirm this.

In terms of sample sizes, the evidence from the Monte Carlo in section 4 suggests that pricing accuracy is virtually unchanged beyond sample sizes of only about $N = 64$. For the many of the more widely traded underlying assets, this is well within the range of available option prices at any given time. For example, in the case of equity index options such as S&P 500 index options studied in Christoffersen and Jacobs (2004) or the S&P100 index options studied in the present paper, the available sample size is typically more than 80 at any given time.

One of the interesting implications of the theorem is that the option pricing formula plays only a trivial role. Suppose we wish to use a model, $\hat{\psi}(\cdot)$, other than Black-Scholes. All that is needed is that the model fulfill the same basic smoothness assumption and not be degenerate in any of its arguments. Under these conditions, we can apply the Weierstrass Theorem which is the essential step in the proof of Theorem 1.

If used in conjunction with a sufficiently flexible implied volatility surface -- as in common practice -- whether traders use Black-Scholes or some other option valuation model is not asymptotically important, precisely as argued by Davis (2001) and Hull (2008). If sufficient data is available and if the technical assumptions are not unreasonable approximation of reality, the predicted option price converges to the true price.

American Options

If the volatility surface is calculated from American options, the ad hoc approach will also succeed in large samples when applied to American options. However, if volatilities are implied from American options, the pricing error does not necessarily vanish asymptotically.

This may shed some light on some recent empirical findings of Hull and Suo (2003) who find that the ABS approach based on European options does not necessarily price other kinds of options accurately.

To show these claims mathematically, write the true but unknown American option price

$$C_{i,t}^{A} = \phi^{A}(\xi_{i,t}, \sigma_t) \,. \tag{10}$$

Suppose the volatility surface is estimated from observed American options:

$$\hat{\beta}_A = \arg\min_{\theta} \left\| C_{i,t}^{A} - \hat{\phi}(\xi_{i,t}, \beta' Z_i^m) \right\| \tag{11}$$

where $\hat{\beta}_A$ is the least squares estimate. When plugged into Black-Scholes, this yields the model $\hat{\phi}(\xi_{i,t}, \hat{\beta}_A{}' Z_i^m)$ and Theorem 1 applies. In particular, together with the continuity assumptions,

$$\left\| C_{i,t}^{A} - \hat{\phi}(\xi_{i,t}, \hat{\beta}_A{}' Z_i^m) \right\| \to 0 \tag{12}$$

uniformly in the arguments as $m \to \infty$.

In practice, however, the volatility surface used for all types of options is the same one used for, and calculated from, European options. The actual estimation procedure is thus:

$$\hat{\beta} = \arg\min_{\theta} \left\| C_{i,t} - \hat{\phi}(\xi_{i,t}, \beta' Z_i^m) \right\|$$

where $C_{i,t}$ is the European option and $\hat{\beta}$ the OLS estimate. The pricing error is therefore given by

$$\left\| C_{i,t}^{A} - \hat{\phi}(\xi_{i,t}, \hat{\beta}' Z_i^m) \right\|$$

and in general $\hat{\beta} \neq \hat{\beta}_A$, so that Theorem 1 does not hold. This is true even if an *unlimited number* of European options are observed because $\hat{\beta}$ will not typically converge to $\hat{\beta}_A$ even asymptotically.

3. Time-Series Forecasting

In practice, the ad hoc model is used not only to price options or other derivatives concurrently but also to forecast future values of options. This practice is particularly important in risk management contexts. For example, the ABS approach is often employed to forecast the risk component of options in calculating portfolio value-at-risk. In this section we consider the problem of forecasting option values one-step ahead when the true option pricing function is unknown and volatility is unobserved.

We begin by considering a fixed observation interval, h. At each time t, our objective is to forecast the option value $C_{i,t+h}$. In practice, traders often need to forecast or hedge their option positions at the daily or with even higher frequency. Below, we will consider what happens if we allow the forecast interval to shrink to zero.

At each time t, the volatility surface $\hat{\sigma}(Z_{it})$ is re-estimated and the one-step ahead forecast is given by $\hat{\phi}(\xi_{i,t}, \hat{\sigma}(Z_{it}))$ which we denote $\hat{\phi}_{i,t}$ for simplicity. The time t forecast error is

$$e(h, m, t) = \phi_{i,t+h} - \hat{\phi}_{i,t+h}. \tag{13}$$

It will be convenient to decompose the error $\phi_{i,t+h} - \hat{\phi}_{i,t+h}$ into

$$\left[\phi_{i,t+h} - \phi_{i,t}\right] + \left[\phi_{i,t} - \hat{\phi}_{i,t}\right] \tag{14}$$

where the first term in square brackets represents the change in the option value and is beyond the control of the user. The second term is the time-t modeling error. In order to study the properties of these two components of the forecast error, we will require two additional assumptions. Let $y_t = (\xi_{i,t}, \sigma_t)$ for notational simplicity.

Assumption 4. The y_t are generated by a continuous time diffusion process,

$$dy_t = \mu(y_t)dt + \Omega(y_t)dw_t$$

where $\mu(y_t)$ and $\Omega(y_t)$ are differentiable drift and diffusion functions and dw_t is a vector Brownian motion.

Assumption 5. The contingent claim possesses a finite theta, $\frac{\partial}{\partial t}\phi(y) < \infty$.

Assumptions 4 and 5 ensure that the value of the contingent claim will itself be a diffusion process. From Ito's lemma, the value $\phi(y_t)$ is the solution to the stochastic differential equation,

$$d\phi_t = \left(\sum_i \frac{\partial}{\partial y_i} \phi_t \mu_i (y_{i,t}) + \frac{\partial}{\partial t} \phi_t + \sum_i \sum_j \frac{\partial^2}{\partial y_i y_j} \phi_t \Omega_i \Omega_j \rho_{ij} \right) dt + \sum_i \frac{\partial}{\partial y_i} \phi_t \Omega_i dw_{i,t} \tag{15}$$

where $dy_{i,t} = \mu_i dt + \Omega_i dw_{i,t}$ and ρ_{ij} is the instantaneous correlation between $dw_{i,t}$ and $dw_{j,t}$.

Given assumptions 4 and 5, the forecast error (14) can be written as

$$e(h,m,t) = \int_t^{t+h} \phi(\xi_s, \sigma_s) ds + \left[\phi(\xi_t, \sigma_t) - \hat{\phi}(\xi_t, \hat{\sigma}(Z_t^m)) \right] \tag{16}$$

which is bounded from below by the first term $\int_t^{t+h} \phi(\xi_s, \sigma_s) ds$. The second term, in square brackets, is the model misspecification error.

We now consider the behavior of the ABS approach as the re-calibration interval, *h*, approaches zero. This might be the more relevant paradigm, for example, if the computational costs of frequent recalibration are small enough to recalibrate the model many times per day.

In moving to the time-series context, we make use of the usual mean-square metric, $\|\Phi\| = \left(E_t \Phi^2\right)^{\frac{1}{2}}$ where $E_t \Phi = E[\Phi \mid I_t]$ is the conditional expectation at time t.

We will show that as $h \to 0$, the forecast error converges to zero, $\left\| C_{i,t+h} - \hat{\phi}_t \right\| \to 0$. This is because the lower error bound will itself converge to zero as $h \to 0$ under very general conditions.

Theorem 2. As the calibration interval shrinks to zero, the continual recalibration model provides arbitrarily accurate forecasts of option prices, $\left\| C_{i,t+h} - \hat{\phi}(K_i, T_i, S_t, r_t, \hat{\beta}' Z_i^m) \right\| \to 0$ as $m \to \infty$ and $h \to 0$.

Proof. See appendix

This result shows that the forecast errors can be made of order *o*(*h*). As the recalibration interval shrinks to zero, the error shrinks to zero. Of course, over a fixed horizon the error is $\int_t^{t+h} \phi(\xi_S, \sigma_S) ds$. If the option valuation model being recalibrated is Black-Scholes, then $\hat{\phi}(\cdot)$ is continuous and possesses a finite theta. But the mathematical development above obviously holds in much more general settings. Continuity and a finite theta – together with an underlying diffusion – are sufficient properties in themselves.

This explains Hull (2008) observation that, under some conditions, the pricing of options is not very sensitive to the particular model used. What is required are some basic smoothness properties and frequent recalibration of the model.

3b. Mixed Jump-Diffusion Models

In this section we consider replacing the assumption that the underlying variables are a pure diffusion process with the following data-generating process.

Assumption 4b. Suppose the underlying asset value is described by a mixed jump-diffusion model,

$$dS_t = (\mu_t - \lambda_t \bar{g})dt + \Omega(S_t)dW_t + g_t dJ_t$$

where g_t is a random variable with finite variance and mean $\bar{g}$ and dJ_t is a Poisson jump process with intensity λ_t. As before dW_t represents a standard Brownian motion and $\mu(S_t)$, $\Omega(S_t)$ are real, continuous drift and diffusion functions.

Applying a generalization of Ito's lemma to mixed jump-diffusion processes we have that the value of a contingent claim $\phi(S_t, t)$ is described by the following stochastic differential equation,

$$d\phi_t = \left[(\mu_t - \lambda_t \bar{g})\phi_s + \dot{\phi}_t + \frac{1}{2}\phi_{SS}\Omega(S_t)\right]dt + \phi_s \Omega(S_t)dW_t + [\phi(S_t + g_t, t) - \phi(S_t, t)]dJ_t . \tag{17}$$

Theorem 3. Under assumptions 1-3, 4b and 5, the continual recalibration model provides arbitrarily accurate forecasts of option prices, $\left\| C_{i,t+h} - \hat{\phi}(K_i, T_i, S_t, r_t, \hat{\beta}' Z_i^m) \right\| \rightarrow 0$ as $m \rightarrow \infty$ and $h \rightarrow 0$.

Proof. See appendix

The proof is very similar to that of theorem 2. We again find that the one-step ahead forecasts given by the ABS model become arbitrarily accurate as the re-calibration frequency shrinks to zero.

However, these results come with an important caveat. Since the accuracy of the forecasts is of order $o(h)$, the error can be made arbitrarily small only over the typically short, recalibration period. For example, suppose parameters are recalibrated daily in order to achieve a particular mean-squared error level that is tolerable. Over longer horizons such as a month, these forecast errors will nevertheless accumulate beyond the daily tolerance level.

4. Simulation study: Stochastic Volatility

For illustration, we conduct a modest Monte Carlo study to get a feeling for the quantitative behavior of the ABS method in practical situations. We assume that the ABS model is used but that the true underlying process displays time-varying volatility. In particular, we follow Bakshi, Cao and Chen (1997), Jones (2002), Christoffersen and Jacobs (2004) and many others in examining a well-known data generating process based on the Heston (1993) stochastic volatility model.

Suppose the price of a stock, S_t, is given by the following mean-reverting stochastic volatility process,

(18) $$dS_t = \mu S dt + \sqrt{\sigma_t} S_t dz_{1,t}$$

(19) $$d\sigma_t = \kappa(\alpha - \sigma_t)dt + \eta\sqrt{\sigma_t} dz_{2,t}$$

where $dz_{1,t}$ and $dz_{2,t}$ are Wiener processes with constant correlation ρ and (μ, α, κ, η) are fixed scalar parameters.

For the simulations, we follow Heston (1993) and set μ=.12 , α=.01, κ=2 and the volatility of volatility η = .11. In order to generate some skewness in the process we set the correlation ρ = -0.6, which is typical of the values found when the process is formally estimated (e.g., Jones (2002)). We set the risk-free rate to .05. The initial stock price S_0 = 41, and the initial volatility is equal to the unconditional mean of the volatility process.

Heston (1993) shows that the price of a call option expiring at time T is given by

(20) $$\phi(K,T,S,r) = SP_1 - Ke^{-r(T-t)}P_2$$

where

$$P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} \mathrm{Re}\left[\frac{e^{-is\log K} f_j(S,T,\sigma,t)}{is}\right] ds$$

and f_j are characteristic functions.

We use the correct formula (20) to predict the value of a set of 4 call options with expirations of T = 130 days and 160 days, and strikes of K = 40 and K = 40.50. We consider four versions of the ABS model:

ABS 1: $\hat{\sigma}_i = \beta_0 + \beta_1 K_i + \beta_2 T_i + \varepsilon_i$

ABS 2: $\hat{\sigma}_i = \beta_0 + \beta_1 K_i + \beta_2 T_i + \beta_3 K_i^2 + \beta_4 T_i^2 + \varepsilon_i$

ABS 3: $\hat{\sigma}_i = \beta_0 + \beta_1 K_i + \beta_2 T_i + \beta_3 K_i^2 + \beta_4 T_i^2 + \beta_5 K_i T_i + \varepsilon_i$

ABS 4: $\hat{\sigma}_i = \beta_0 + \beta_1 K_i + \beta_2 T_i + \beta_3 K_i^2 + \beta_4 T_i^2 + \beta_5 K_i^3 + \beta_6 T_i^3 + \varepsilon_i$.

Specifications 1 through 3 are consistent with the models considered by Dumas, Fleming and Whaley (1998) and Christoffersen and Jacobs (2004). Model 4 is added to examine whether higher order terms than quadratic can improve model performance.

The parameters are estimated by least squares from five different sample sizes ranging from N = 16 to 81. For the smallest sample size N = 16, we generate a sample of options for estimation by creating a grid with strike prices taking 4 equispaced values from 38 to 41, and maturities taking 4 equispaced values 100 to 180 days. For sample size N=25, the grid contains 5 different strike prices from 38 to 41 and 5 different maturities from 100 to 180 days, and so on up to sample size 81.

For each sample, we use the ABS models and the correct option pricing formula to value a set of 4 options which are not in the sample. By comparing these predicted values to the true option prices that should hold, we can calculate cross-sectional out-of-sample pricing errors. We aggregate the 4 pricing errors into a scalar quantity by taking their root mean-squared error.

The results are then averaged across 1000 Monte Carlo simulations and are shown in

Table 1. One would expect estimates of the implied volatility surface, and hence the ABS model, to improve with the sample size N. Indeed this is the case. However, there are virtually no reductions in RMSE beyond a sample size of about 64.

The RMSE across the four versions of the ABS model displays a generally U-shaped pattern indicating that the increasingly complexity of models 1-3 improves performance. The quadratic ABS 2 model leads to an improvement over the linear model ABS 1. Additionally, the inclusion of an interaction term generally reduces RMSE further than model 2. However, model ABS 4 which has higher order terms than quadratic are worse in terms of RMSE and hence the extra terms do not improve the cross-sectional fit. Clearly whether the addition of a particular higher-order term will improve the fit or not is something that has to be checked on a case by case basis.

In Table 2, we display the RMSE from the time series out-of-sample forecast errors. The top panel, labeled “10 days”, shows the 10-day-ahead forecast errors. There is a substantial improvement in performance in going from the linear model ABS 1 to the quadratic model, ABS 2, but again a deterioration with the addition of the higher order terms in ABS 4.

In this case the performance of the estimated models is virtually unchanged beyond a sample size of N=36. Given that the ABS models that perform best here are quadratic, the degrees of freedom accumulate quickly with N. In order to improve the fit, it appears more important to get the right polynomial order m, which empirically need only be quadratic, rather than accumulating observations beyond about 30 or 40.

The bottom panel shows the RMSE from ½-day ahead forecasts. The patterns remain the same, though the RMSE is obviously much smaller for all models. This is consistent with Theorem 2 which shows the accuracy of the forecast is of order $o(h)$. Nevertheless, it is interesting to note one of the empirical results that emerge. In this particular simulation environment there are significant reductions in RMSE when changing from a 5-day to 1-day recalibration interval – on the order of 20 percent. However, further increases in the recalibration frequency from 1 day to a half-day only reduces pricing error by less than 3 percent for most of the models and most samples sizes considered here.

5. Application to Equity Index Options

In this section, we apply the models used in the simulation to study their accuracy in pricing S&P100 call options. We chose this data based on several considerations. First, the set of options written on this index are among the most actively traded contracts. Second, the daily dividend distributions are available for the index. Third, the S&P100 index options are American-style options and hence by using this data we complement the literature on the European-style counterpart, S&P 500 index options, which have been studied extensively by Bakshi, Cao, and Chen (1997) and Dumas, Fleming, and Whaley (1998) among many others.

Our sample contains reported prices of S&P 100 index call options traded on the Chicago Board Options Exchange (CBOE) over the period January 2, 2005 through December 31, 2007 for a total of 753 trading days. The price of the option is set equal to the average of the best bid and best ask at the close of each trading day. We exclude only data for which there is a missing bid or ask quote, or missing strike price or maturity. This yields a total of 211,276 usable call option prices.[1] This sample is quite large primarily due to the wide range of strike prices. In our sample, there are often more than 60 different strike prices and 5 different option maturities observed on a single day. The option's time to expiration is measured as the number of calendar days between the trade date and the expiration date.

To summarize the option pricing data, we divide the sample into several categories according to moneyness and time to expiration. Observations are partitioned into 6 degrees of moneyness, ranging from 10% out-of- the-money (OTM) to 30% in-the-money. For each of these categories, we split the data into 3 maturity lengths, ranging from less than 60 days to over 180 days.

The results are shown in Table 3. For each of the 18 moneyness-maturity categories, we report the average option price, the standard deviation and the number of observations. As expected, the typical pattern emerges in which average call prices increase monotonically with moneyness and with maturity. The data display a very rich degree of variation, with the most in-the-money calls having an average price more than 1000 times that of the most OTM options. It is also interesting to note while the OTM call options are worth only about $0.12 on average for maturities less than 2 months, they average over $14 for maturities greater than 6 months. For options deep in the money, increases in maturity have a much smaller effect on call prices than on similar OTM options. This is also a well-established pattern found, for example, in Bakshi, Cao and Chen (1997).

First we examine the ability of the ABS models to price concurrent option observations cross-sectionally. We calculate the Black-Scholes implied volatilities using the interest rates implied by constant maturity zero-coupon bonds taken from the IvyDB OptionMetrics database. Given the implied volatilities, each day we estimate the ABS models described in Section 5 using the sample of options available that day except for 4 target options.

The target options are given by the options with the 4 lowest strike prices that day. The ABS models are then used to value these 4 options cross-sectionally. The results are compared to the price implied by the Black-Scholes formula which uses the estimated sample volatility of the underlying index over the sample period.

For each model and each option, we calculate the square root of the averaged squared pricing error, where the average is taken over the 4 target options. This daily error is then averaged over all days in the sample to get the average RMSE reported in Table 4. The dollar RMSE are shown in Panel A and the errors in percentage terms in Panel B. The typical Black-Scholes pricing error is in the range of $2.73 or about 18 percent. This is roughly comparable in percentage terms to the magnitude of error typically found for S&P 500 index options (e.g. Dumas, Fleming and Whaley (1998)). The table also indicates that every version of the ABS model delivers lower RMSE pricing errors. Again, this is to be expected given that the ABS model allows for variation in the volatility parameter whereas the Black-Scholes formula does not.

For every target option that we try to value, the ABS 3 model has the smallest errors. It is interesting to note that this model, with quadratic terms and an interaction term but no third-order terms, is becoming something of a standard specification among practitioners and academics studying the ABS model (e.g., Christoffersen and Jacobs (2004)). It is also noteworthy that this is precisely the model which had the lowest pricing errors in the simulations

[1] We also investigated the pricing of S&P 100 puts and find the results to be qualitatively similar.

in Section 4. This suggests that the relative accuracy of this particular specification may be somewhat robust to these trading environments.

We next examine the option pricing errors of the 4 ABS models out-of-sample with 1 day, 2 day, 5 day and 10 day recalibration intervals. A 1 day recalibration interval means that at the close of each day, the cross section of options is used to value an option trading the next day. The results are shown in Table 5. Panel A reports the median dollar pricing errors over all days in the sample. Each day the aggregate pricing error for a given model is given by the square root of the median squared error. The results indicate that all 4 of the ABS models outperform the Black-Scholes model by a significant margin. Again we confirm that the ABS 3 model generally is most accurate among the ad hoc models.

To gauge the economic significance of the lower valuation errors from the ABS model relative to Black-Scholes, Panel B reports the pricing errors in percentage terms. The difference between the ABS 3 model in particular and Black-Scholes is quite large in the sense of being generally beyond the 5 percent range associated with transaction costs (e.g., Bakshi, Cao and Chen (1997)). This is true for every recalibration interval we considered. However, the relative improvement in the ABS models is greatest when the recalibration interval is relatively short at 1 or 2 days.

6. Conclusions

Many market makers and traders have resisted using cutting-edge option pricing formulas. The most widely used valuation procedure among practitioners is a version of Black-Scholes with ad hoc adjustments and frequent updating of parameters. A growing body of empirical evidence suggests that the ad hoc approach performs quite well. It has been argued previously that such a procedure should work well because the ad hoc approach serves as a sophisticated interpolation tool.

Our incremental contribution is to show that with sufficiently frequent updating, the ad hoc approach provides forecasts with errors that vanish in the limit. This provides a rigorous, albeit asymptotic, basis for understanding why the ad hoc approach to option pricing does so well. Our theoretical result also explains the observation made by previous authors that, under some conditions, the pricing of options is not terribly sensitive to the particular model used. As long as the model is used as an interpolation tool and is recalibrated frequently, the approximation argument holds. It is the frequent recalibration, rather than the underlying model, that is of primary importance in high frequency option valuation.

Empirically, we find in a set of simulations that our typical ABS models substantially outperform the Black-Scholes formula. The best performing models require only linear and quadratic terms, so that only a few parameters need to be estimated. Consistent with this, we find that sample sizes beyond about 64 do not significantly reduce pricing errors any further.

Lastly, we conducted a modest application of the ABS models study to three years worth of S&P 100 index options. The same linear-quadratic implied volatility functions that performed best in the Monte Carlo appear to do best when applied to this real data.

Our findings should not, however, be taken as an argument against developing theoretically-derived option pricing formulas. Undoubtedly, the remarkable theoretical advances being made will result in analytical formulas that dominate any current practice. As theoretical improvements are achieved, practitioners will adopt them – possibly again with ad hoc adjustments.

References

Bakshi, C., Cao, C. and Z. Chen (1997), "Empirical Performance of Alternative Option Pricing Models," *Journal of Finance*, 52, 2003-2049.

Bates, D. (1996), "Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options," *Review of Financial Studies*, 9, 69-107.

Berkowitz, J. and J. O'Brien (2002), "How Accurate are Value-at-Risk Models at Commercial Banks," *Journal of Finance*, 57, 1093-1112.

Black, F. and Scholes, M. (1973), "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, 81, 637-654.

Christoffersen, P. F. and K. Jacobs (2004), "The Importance of the Loss Function in Option Pricing," *Journal of Financial Economics*, 72, 291-318.

Davis, M. (2001). Mathematics of Financial Markets. In B. Engquist and W. Schmid (Eds.), *Mathematics Unlimited—2001 and Beyond,* pp. 361-380. Springer: New York.

Davis, P. J. (1975). *Interpolation and Approximation*. New York: Dover Publications.

Derman, E., and I. Kani (1994), "Riding on the Smile," *Risk*, 7, 32-39.

Duan, J. (1995), "The GARCH Option Pricing Model," *Mathematical Finance*, 5, 13-32.

Dumas, B., Fleming, F. and R. Whaley (1998), "Implied Volatility Functions: Empirical Tests," *Journal of Finance*, 53, 2059-2106.

Heston, S. L. (1993), "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options," *Review of Financial Studies*, 6, 327-343.

Heston, S. and S. Nandi (2000), "A Closed-Form GARCH Option Pricing Model," *Review of Financial Studies*, 13, 585-626.

Hull, J. C. (2008). *Options, Futures & Other Derivatives* (7^{th} ed.). Prentice Hall: Upper Saddle River, NJ.

Hull, J. and W. Suo (2003), "A Methodology for Assessing Model risk and its Application to the Implied Volatility Function Model," *Journal of Financial and Quantitative Analysis*, 37(2), p. 297-318.

Melino, A. and S. Turnbull (1990), "Pricing Foreign Currency Options with Stochastic Volatility," *Journal of Econometrics*, 45, 239-265.

Merton, Robert C. (1976), "Option Pricing when Underlying Stock Returns are Discontinuous," *Journal of Financial Economics*, 3, 125-144.

Rubinstein, M. (1994), "Implied Binomial Trees," *Journal of Finance*, 49, 771-818.

Appendix

Proof of Theorem 1. We will show that

$$C_{it} - \hat{\phi}(\xi_{it}, \hat{\sigma}(Z_t^m)) \to 0 \tag{21}$$

uniformly in ξ_{it}, Z_t as $m \to \infty$.

From assumption 1, $\hat{\phi}(\cdot)$ is continuous and we can apply the (multivariate) Weierstrass theorem:

$$\hat{\phi}(\xi_{it}, \hat{\sigma}(Z_t^m)) = \sum_{j=1}^{\infty} \alpha_j \xi_{it}^j \hat{\beta}' Z_t^m \tag{22}$$

where by assumption 3, $\hat{\beta} = \arg\min_{\theta} \sum_{i} \left(C_{it} - \hat{\phi}(K_i, T_i, S_t, r, \beta' Z_t)\right)^2$.

Using assumptions 1 and 2, the true but unknown function $C_{it} = \phi(\xi_{it}, \sigma(Z_t))$ can be written as

$$\phi(\xi, \sigma(Z_t)) = \tilde{\phi}(\xi, Z_t) \tag{23}$$

where $\tilde{\phi}(\cdot)$ is unknown but is continuous in both arguments. Using (22), the pricing error $\tilde{\phi}(\xi, Z_t) - \hat{\phi}(\xi, \hat{\sigma}(Z_t^m))$ is given by

$$\tilde{\phi}(\xi, Z_t) - \sum_{j=1}^{\infty} \alpha_j \xi^j \hat{\beta}' Z_t^m . \tag{24}$$

From Theorem 6.1.1 of Davis (1975) it follows that, for any ε>0, we can find a sufficiently large *m* such that

$$\left(\tilde{\phi}(\xi, Z_t) - \sum_{j=1}^{\infty} \alpha_j \xi^j \hat{\beta}_m'{}' Z_t^m\right)^2 < \varepsilon \tag{25}$$

uniformly in ξ, Z_t. That is, $\tilde{\phi}(\cdot)$ can be uniformly approximated by a polynomial of sufficiently high degree.

It follows immediately that

$$\lim_{m\to\infty} \min_{\beta_m} \left(\tilde{\phi}(\xi\ , Z_t) - \sum_{i=1}^{\infty} \alpha_j \xi^j \beta_m Z_t\right)^2 = 0, \tag{26}$$

as shown, for example in Davis (1975, p.108). ■

Proof of Theorem 2. Write the forecast error as

$$e(h, m, t) = \int_t^{t+h} \phi(\xi_s, \sigma_s) ds + \left[\phi(\xi_t, \sigma_t) - \hat{\phi}(\xi_t, \hat{\sigma}(Z_t^m))\right]$$

$$=\left[\int_t^{t+h} \mu_{\phi,s} ds + \int_t^{t+h} \Omega_{\phi,s} dw_s\right] + \left[\phi(\xi_t,\sigma_t) - \hat{\phi}(\xi_t,\hat{\sigma}(Z_t^m))\right]. \qquad (27)$$

From Theorem 1, $\lim_{m\to\infty}\left\|\phi(\xi_t,\sigma_t) - \hat{\phi}(\xi_t,\hat{\sigma}(Z_t^m))\right\|=0$ and so $\lim_{h\to 0}\lim_{m\to\infty} e(h,m,t)$ simplifies to

$$\lim_{h\to 0}\left\|\int_t^{t+h} \mu_{\phi,s} ds + \int_t^{t+h} \Omega_{\phi,s} dw_s\right\|$$

$$= \lim_{h\to 0} E_t^{1/2}\left(\int_t^{t+h} \mu_\phi(y_t) ds + \int_t^{t+h} \Omega_\phi(y_t) dw_t\right)^2 = \lim_{h\to 0} O(h) = 0$$

which is the desired result. ■

Proof of Theorem 3. From equation (17),

$$\phi_{t+k} = \phi_t + \int_t^{t+h}\left[(\mu_s - \lambda_s \bar{g})\phi_s + \frac{1}{2}\phi_{SS}\Omega(S_s)\right]ds + \int_t^{t+h}\phi_s\Omega(S_s)dW_s + \int_t^{t+h}\left[\phi(S_s+g_s) - \phi(S_s)\right]dJ_s.$$

In this case the forecast error is given by

$$e(h,m,t) = \int_t^{t+h} \phi(\xi_{is},\sigma_s) ds + \left[\phi(\xi_{it},\sigma_t) - \hat{\phi}(\xi_{it},\sigma(Z_t))\right]$$

$$= \left(\int_t^{t+h}\mu_\phi ds + \int_t^{t+h}\Omega\mu_\phi dW_s + \int_t^{t+h}[\phi(S_s+g_s) - \phi(S_s)]dJ_s\right) + \left[\phi(\xi_{it},\sigma_t) - \hat{\phi}(\xi_{it},\sigma(Z_t))\right]. \qquad (28)$$

From assumption 4b the expression in the first parenthesis is $O(h)$ so that forecast error is again

$$O(h) + \left[\phi(\xi_{it},\sigma_t) - \hat{\phi}(\xi_{it},\sigma(Z_t))\right].$$

The remainder of the proof is precisely as in the proof of Theorem 2. ■

Table 1. The *ad hoc* model in the presence of Stochastic Volatility:

Cross-sectional root mean-squared-error

Model	Sample size				
	N =16	N = 25	N = 36	N = 64	N = 81
ABS 1	1.258	1.256	1.254	1.254	1.254
ABS 2	1.247	1.244	1.244	1.242	1.242
ABS 3	1.244	1.241	1.240	1.241	1.240
ABS 4	1.247	1.249	1.249	1.246	1.246

Notes: Data is generated from the bivariate stochastic volatility process (18)-(19). Four versions of the ABS model are used to predict the value of 4 call options with expirations of T = 130 and 160 days, and strikes of K = 40 and K = 40.50. The square root of MSE is calculated over 1000 Monte Carlo simulations.

Table 2. The *ad hoc* model in the presence of Stochastic Volatility:
One-step ahead root mean squared error

Recalibration interval, h	Model	Sample Size $N = 16$	$N = 25$	$N = 36$	$N = 64$	$N = 81$
10 days	ABS 1	1.903	1.900	1.899	1.899	1.900
	ABS 2	1.893	1.890	1.888	1.888	1.888
	ABS 3	1.890	1.887	1.887	1.887	1.888
	ABS 4	1.894	1.895	1.892	1.892	1.893
	Heston	1.065				
5 days	ABS 1	1.563	1.560	1.558	1.559	1.560
	ABS 2	1.552	1.549	1.549	1.547	1.548
	ABS 3	1.549	1.546	1.546	1.547	1.547
	ABS 4	1.553	1.554	1.555	1.555	1.552
	Heston	0.671				
1 day	ABS 1	1.323	1.320	1.318	1.319	1.319
	ABS 2	1.312	1.309	1.308	1.306	1.306
	ABS 3	1.308	1.305	1.304	1.305	1.306
	ABS 4	1.312	1.313	1.314	1.311	1.311
	Heston	0.287				
Half day	ABS 1	1.292	1.289	1.287	1.288	1.288
	ABS 2	1.280	1.277	1.277	1.275	1.276
	ABS 3	1.277	1.274	1.273	1.274	1.275
	ABS 4	1.281	1.282	1.283	1.280	1.280
	Heston	0.198				

Notes: Data is generated from the bivariate stochastic volatility process (18)-(19). Four versions of the ABS model are used to predict the value of 4 call options with expirations of $T = 130$ and 160 days, and strikes of $K = 40$ and $K = 40.50$. The square root of MSE is calculated over 1000 Monte Carlo simulations.

Table 3.

Sample Properties of the S&P 100 Index Options

	Moneyness S/K	Days-to-Expiration <60	60-180	≥180	Subtotal
OTM	<0.9	\$0.12	\$0.40	\$14.89	
		(0.14)	(0.71)	(13.77)	
		{3871}	{4484}	{6991}	{15,346}
	0.9-1.0	\$1.77	\$6.26	\$39.97	
		(3.01)	(6.41)	(21.40)	
		{29014}	{14800}	{10083}	{53,897}
ATM	1.0-1.1	\$30.00	\$38.76	\$74.38	
		(15.89)	(14.59)	22.10	
		{29722}	{13128}	{9425}	{52,275}
	1.1-1.2	\$79.63	\$84.57	\$111.71	
		(15.18)	(15.47)	(20.84)	
		{18680}	{10179}	{7964}	{36,823}
ITM	1.2-1.30	\$122.93	\$125.681	\$145.98	
		(15.13)	(15.63)	(19.95)	
		{7708}	{6955}	{6583}	{21,246}
	≥1.30	\$183.50	\$183.53	\$206.017	
		(36.74)	(35.76)	(37.95)	
		{8143}	{9948}	{13559}	{31,650}
Subtotal		{97163}	{59504}	{54609}	{211276}

The reported numbers are the average bid-ask midpoint price, the standard deviation shown in parentheses and the total number of observations (in braces for each moneyness-maturity category. The sample period extends from January 2, 2005 to December 31, 2007 for a total of 211,276 call options. S denotes the spot S&P 100 index level and K is the exercise price. OTM, ATM and ITM denote out-of-the money, at-the-money and in-the-money, respectively.

Table 4. Valuing S&P 100 Index Options:

Cross-Sectional Pricing Errors from the *ad hoc* Models

	Model	Option 1	Option 2	Option 3	Option 4	Mean
Panel A. Dollar Error						
	Black-Scholes	$2.73	$2.79	$2.65	$2.58	$2.69
	ABS 1	$1.85	$2.12	$1.96	$1.78	$1.93
	ABS 2	$1.91	$2.11	$2.04	$1.94	$2.00
	ABS 3	$1.60	$1.79	$1.67	$1.61	$1.67
	ABS 4	$1.89	$2.01	$1.98	$1.93	$1.95
Panel B. Percentage Error						
	Black-Scholes	0.197	0.226	0.160	0.156	0.185
	ABS 1	0.154	0.172	0.139	0.139	0.149
	ABS 2	0.132	0.142	0.097	0.097	0.117
	ABS 3	0.114	0.124	0.093	0.093	0.105
	ABS 4	0.134	0.131	0.096	0.096	0.115

Table reports RMSE for the Black-Scholes model as well as the 4 versions of the ABS model described in Section 4. Each day the models are estimated from the available set of S&P 100 index call options. The models are then used to predict the value of 4 target options cross-sectionally. The RMSE is then calculated as the square root of the mean squared pricing error across the 4 target options. Panel A reports pricing errors in dollars, Panel B reports the errors as a percentage of the true call option price.

Table 5. Valuing S&P 100 Index Options:
Out-of-Sample Pricing Errors

Panel A. Dollar Pricing Errors

	Model	Option 1	Option 2	Option 3	Option 4	Mean
Recalibration interval, *h=10 days*	Black-Scholes	$6.91	$6.88	$7.04	$7.12	$6.99
	ABS 1	$5.81	$5.48	$5.66	$5.87	$5.71
	ABS 2	$5.56	$5.34	$5.58	$5.54	$5.51
	ABS 3	$5.33	$5.02	$5.30	$5.27	$5.23
	ABS 4	$5.67	$5.39	$5.35	$5.74	$5.58
Recalibration interval, *h=5 days*	Black-Scholes	$5.75	$6.03	$5.95	$5.41	$5.78
	ABS 1	$4.63	$4.92	$4.97	$4.58	$4.78
	ABS 2	$4.40	$4.42	$4.67	$4.40	$4.47
	ABS 3	$4.30	$4.22	$4.44	$4.05	$4.25
	ABS 4	$4.45	$4.62	$4.46	$4.49	$4.50
Recalibration interval, *h=2 days*	Black-Scholes	$4.72	$4.91	$4.71	$4.65	$4.74
	ABS 1	$3.70	$4.11	$4.11	$3.75	$3.92
	ABS 2	$3.40	$3.67	$3.77	$3.62	$3.62
	ABS 3	$3.47	$3.43	$3.41	$3.38	$3.42
	ABS 4	$3.52	$3.48	$3.64	$3.71	$3.58
Recalibration interval, *h=1 day*	Black-Scholes	$4.36	$4.33	$4.04	$4.13	$4.21
	ABS 1	$2.97	$3.56	$3.36	$3.22	$3.27
	ABS 2	$2.99	$3.28	$3.22	$3.21	$3.18
	ABS 3	$2.65	$3.10	$2.97	$2.98	$2.93
	ABS 4	$2.88	$3.32	$3.09	$3.12	$3.10

Panel B. Percentage Pricing Errors

	Model	Option 1	Option 2	Option 3	Option 4	Mean
Recalibration interval, *h=10 days*	Black-Scholes	0.258	0.275	0.218	0.210	0.240
	ABS 1	0.203	0.215	0.206	0.183	0.202
	ABS 2	0.206	0.213	0.193	0.189	0.200
	ABS 3	0.199	0.213	0.192	0.180	0.196
	ABS 4	0.227	0.229	0.196	0.173	0.207
Recalibration interval, *h=5 days*	Black-Scholes	0.224	0.270	0.220	0.191	0.226
	ABS 1	0.186	0.191	0.171	0.165	0.178
	ABS 2	0.196	0.205	0.164	0.167	0.183
	ABS 3	0.183	0.179	0.158	0.157	0.169
	ABS 4	0.198	0.190	0.169	0.169	0.182
Recalibration interval, *h=2 days*	Black-Scholes	0.201	0.255	0.185	0.175	0.204
	ABS 1	0.180	0.183	0.165	0.155	0.171
	ABS 2	0.172	0.163	0.146	0.132	0.153
	ABS 3	0.167	0.146	0.134	0.128	0.144
	ABS 4	0.186	0.147	0.142	0.129	0.151
Recalibration interval, *h=1 day*	Black-Scholes	0.207	0.236	0.174	0.159	0.194
	ABS 1	0.169	0.173	0.156	0.135	0.158
	ABS 2	0.161	0.164	0.123	0.120	0.142
	ABS 3	0.136	0.152	0.121	0.115	0.131
	ABS 4	0.154	0.152	0.121	0.120	0.137

Table reports RMSE for the Black-Scholes model as well as the 4 versions of the ABS model described in Section 5. Each day the models are estimated from the available set of S&P 100 index call options. The models are then used to predict the value of 4 target options over forecast horizons of 10 days, 5 days, 2 days and 1 day. The RMSE is then calculated as the square root of the median squared percentage pricing error across the 4 target options.