

LONG-HORIZON EXCHANGE RATE PREDICTABILITY?

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Abstract—Several authors have recently investigated the predictability of exchange rates by fitting a sequence of long-horizon error-correction equations. We show by means of a simulation study that, in small to medium samples, inference from this regression procedure depends on the null hypothesis that is used to generate empirical critical values. The standard assumption of a stationary error-correction term between exchange rates and fundamentals biases the results in favor of predictive power. Our results show that evidence of long-horizon predictability weakens when using empirical critical values generated under the more stringent null of no cointegration. Likewise, results are weakened using critical values generated under the null that exchange rates and fundamentals are generated by an unrestricted VAR with no integration restrictions.

I. Introduction

SEVERAL authors have recently reported empirical evidence that monetary fundamentals may contain predictive power for movements in U.S. dollar exchange rates. Mark (1995), Chinn and Meese (1995), and Chen and Mark (1996), among others, apply the long-horizon regression approach to investigate whether economic fundamentals are useful in predicting several leading U.S. dollar spot rates. Although these authors are unable to document short-run exchange rate predictability—in agreement with traditional studies (Meese & Rogoff, 1988; Diebold & Nason, 1990)—they do find evidence of long-horizon predictability. Earlier applications of the long-horizon regression methodology include the study of equity return predictability (Fama & French, 1988; Campbell & Shiller, 1988) and inflation and interest rate predictability (Fama, 1990; Campbell & Shiller, 1991).

In the study of exchange rate predictability, long-horizon regressions contain an error-correction term for spot rates and monetary fundamentals. In this context, it is known that standard asymptotic critical values can be misleading, so small-sample critical values are tabulated by simulation. However, thus far, the null hypothesis used to generate the simulated critical values has assumed the existence of a long-run relationship (cointegration). We show by means of a Monte Carlo study that the small-sample critical values are sensitive to this null hypothesis. In addition, if the long-horizon procedure is carried out when cointegration is not present, several diagnostic statistics falsely suggest evidence of long-horizon predictability. As a result, such evidence cannot be used to distinguish between predictability on the one hand, and lack of cointegration on the other

hand. The presence of similar biases was reported in Stambaugh (1986), Mankiw and Shapiro (1986), and Hodrick (1992), whose Monte Carlo experiments are tailored to equity and bond returns. More recently, Berben and van Dijk (1998) formalized such findings by deriving the asymptotic distribution of least-squares coefficient estimates and associated *t*-statistics under the null that cointegration does not hold. They show that, under the null of no cointegration, the asymptotic distributions of coefficient estimates are increasing in the estimation horizon despite being zero in population.

In the present paper, we obtain empirical critical values for testing predictability by simulating two different null hypotheses. In the first simulation, exchange rates are generated as integrated processes and assumed to be independent of monetary fundamentals. In the second simulation, we relax the integration assumption and fit an unrestricted bivariate VAR to generate the empirical critical values.¹ For both experiments, we tabulate critical values for several in- and out-of-sample statistics considered by Mark (1995). In general, the critical values generated in the two simulation experiments are larger than are the typical critical values generated under the assumption of cointegration between exchange rates and fundamentals. However, the results also indicate that out-of-sample statistics do not display the biases that plague in-sample statistics.

Using either set of our more conservative critical values, we find evidence of predictability for the U.S. dollar/Swiss franc exchange rate but not for the other U.S. dollar exchange rates in our sample: the Canadian dollar, the German mark, and the Japanese yen. These results are in agreement with inference from standard short-run tests of predictability based on systems of equations. Such system methods are also helpful in clarifying what can and cannot be gained from using a sequence of long-horizon regressions. For example, a bivariate error-correction model for spot rates and fundamentals implies that, if the slope coefficient of the one-step-ahead regression is 0 then the slope coefficients must be 0 for all horizons.

The remainder of this paper proceeds as follows. In section II, we discuss the long-horizon regression methodology as commonly applied to monetary models of exchange rate dynamics. In section III, we describe the results of the Monte Carlo experiments. Section IV discusses the results in light of the relationship between long-horizon regressions and the vector error-correction model. Section V discusses the estimation results in the context of our Monte Carlo critical values, and section VI concludes.

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II. Long-Horizon Regressions

The long-horizon regression approach entails estimating K individual equations,

$$\Delta^k s_{t+k} = \alpha_k + \beta_k z_t + \varepsilon_{k,t}, \quad k = 1, \dots, K, \quad (1)$$

where s_t and z_t are observed data,

$$\Delta^k = (1 - L^k), \text{ and}$$

α_k and β_k are the parameters to be estimated.

If the β_k 's, the associated t -statistics, and the regression R_k^2 's are found to increase with k , it is generally concluded that z_t can predict long-run movements in s_t better than short-run changes.

In the context of monetary models of exchange rate dynamics, the following equations are typically estimated:

$$\begin{aligned} \Delta^k s_{t+k} &= \alpha_k + \beta_k (f_t - s_t) + \varepsilon_{k,t}, \\ k &= 1, \dots, K, \end{aligned} \quad (2)$$

where s_t is the log spot rate and f_t is a monetary fundamental. For example, Mark (1995) considers $f_t = (m_t - m_t^*) - (y_t - y_t^*)$, where m_t and y_t denote the log of M1 and of real GDP, respectively, and asterisks represent foreign quantities.

This error-correction representation is motivated by the assumption that exchange rates cannot move independently of macroeconomic fundamentals over long time horizons. According to specification (2), if $\beta_k > 0$ and the error-correction term $(f_t - s_t)$ is, for example, positive, the spot rate is expected to rise in the future. Such predictable movements contrast with the conventional view that floating exchange rates are best characterized by a driftless random walk (Meese & Rogoff, 1988; Diebold & Nason, 1990). In light of this fact, specification (2) can be thought of as a test of whether the inclusion of monetary fundamentals can beat the random-walk forecast.

However, existing empirical evidence is not supportive of the hypothesis of cointegration between exchange rates and the simple monetary fundamentals (Garcéazabal & Regúlez, 1992; MacDonald & Marsh, 1995). Indeed, Mark (1995) and Chinn and Meese (1995) themselves are unable to reject the null hypothesis that $(f_t - s_t)$ is nonstationary for any of the four exchange rates considered.

This persistence in the regressor leads to several difficulties when making inference from long-horizon regressions, especially in presence of small samples and large k 's. As will be detailed below, the least squares (LS) estimate of the slope parameter, $\hat{\beta}_k$, is biased away from zero in small samples. Even a small bias in $\hat{\beta}_k$ may, in turn, lead to a severe bias in the associated t -ratio. This amplification occurs because both the accuracy of the numerator and the denominator of the t -ratio depend on the precision of the estimate of β_k . The empirical distribution of the t -statistics will be shifted away from zero and skewed to the right, regardless of the horizon considered.

To formalize this notion, Berben and van Dijk (1998) show that, if cointegration does not hold, the regression coefficients in a long-horizon regression no longer have a Gaussian asymptotic distribution. Berben and van Dijk show that, under the null hypothesis that $\beta = 0$ and assuming f_t is weakly exogenous, the associated t -statistic converges to a linear combination of Gaussian and Dickey-Fuller distributions. Moreover, they show that the asymptotic distribution of $\hat{\beta}_k$ is k times the distribution of $\hat{\beta}_1$ and that the asymptotic distribution of the associated t -statistics, $t(\hat{\beta}_k)$, also depend on k , although in a more complicated way.

Heuristically, if there is no statistical relationship between s_t and f_t , the long-horizon regressions become close to a classical spurious regression as k increases. To see why the assumption of cointegration is so crucial, note that, if the exchange rate is well approximated by a random walk, then we may write $s_t = \sum_{i=1}^t \eta_i$, where $\eta_i \sim WN(0, \sigma_\eta^2)$. So equation (2) becomes

$$\begin{aligned} \sum_{i=t+1}^{t+k} \eta_i &= \alpha_k + \beta_k (f_t - s_t) + \varepsilon_{k,t}, \\ k &= 1, \dots, K. \end{aligned} \quad (3)$$

For large k , the dependent variable is itself approximately a random walk. If cointegration fails, then $f_t - s_t$ is, of course, integrated of order one, $I(1)$. As a result, when k is large, equation (3) involves regressing an approximately $I(1)$ variable on another $I(1)$ variable. One might reasonably expect to find that this regression behaves like a spurious regression.² This observation casts doubt upon the reliability of inference from the long-horizon regression methodology.³ The usual asymptotic inference associated with long-horizon regressions is thus likely to be misleading, especially with small samples and large k 's.

In applied work, these problems have generally been addressed by tabulating bootstrap (Monte Carlo) critical values. Pseudodata are generated by fitting a restricted vector autoregression for the change in the exchange rate and the error-correction term, $(\Delta s_t, (s_t - f_t))$. This approach conditions on cointegration between s_t and f_t ; thus, it assumes a priori the existence of a long-run relationship between s_t and f_t . In section III, we conduct a Monte Carlo experiment in which s_t and f_t are not cointegrated so that $s_t - f_t$ is not stationary. In practice, this difference in the stationarity status of $s_t - f_t$ results in very different empir-

² Phillips (1986) shows that, for a true spurious regression, conventional t -statistics diverge as the sample size grows (There are no asymptotically correct critical values.) The slope parameter of a bivariate spurious regression is shown to possess a degenerate limiting distribution, and the estimate of the intercept diverges. The R^2 converges to a random variable.

³ Related results are found in Richardson and Stock (1989) who show that, even when cointegration holds, LS estimates of the slope of regression (2) are inconsistent when the forecast horizon grows with the sample size (so that $k/T \rightarrow \delta$, a constant)

ical critical values. Relative to the more conservative null, bootstrapped empirical distributions that assume stationarity will produce critical values that are too small, and will bias results on the side of significance.

Of course, if the true DGP is nonlinear, it is at least possible that long-horizon regressions might detect predictability even though a one-step ahead model does not. Both the theoretical results of Berben and Van Dijk (1998) and the simulation results of the present paper are based on the assumption of linearity.

However, out-of-sample statistics do not rely on any such assumption. That is, Diebold-Mariano statistics should detect predictability if long-horizon regressions capture nonlinear dynamics. If f_t and s_t were independent, the spurious regression problem would bias the $\hat{\beta}_k$ away from zero, and the model forecasts will underperform random-walk forecasts out of sample. On the other hand, if the $\hat{\beta}_k$ are nonzero for large k because of a true nonlinear relationship, then the model outperforms the random-walk forecast.⁴

In the present paper, we do not attempt to model the true data-generating process, as fundamentals and exchange rates are not likely to be literally independent. Rather, our goal is to point out that, if no relationship between f_t and s_t exists, in-sample statistics appear to support long-horizon predictability. True long-horizon predictability and total independence lead to observationally (qualitatively) equivalent in-sample results.

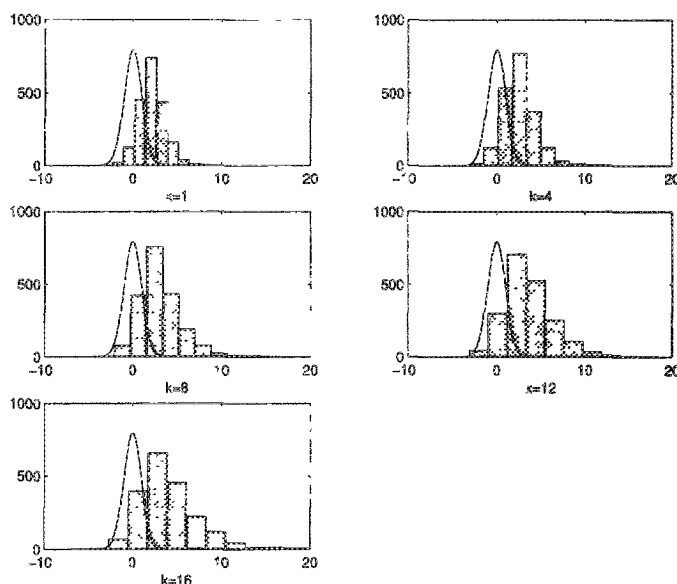
III. Monte Carlo Evidence

Existing empirical evidence is not supportive of the hypothesis of cointegration between exchange rates and the simple monetary fundamentals. Therefore, we emphasize that an equally interesting and more conservative null hypothesis associated with the estimation of equation (2) is of no cointegration between the two time series. To investigate the small sample distribution of conventional diagnostic statistics associated with long-horizon regressions, we construct two simulation experiments. In the first experiment, the two series are statistically unrelated (cointegration does not hold) under the null. In the second, we fit an unrestricted VAR model to s_t and f_t and use this model under the null hypothesis to generate critical values. For each null, we consider the sample statistics considered by Mark (1995) for comparability.

In the first experiment, we begin by generating independent Gaussian random variables, s_t and f_t , with the relative variances of the innovations of the two processes calibrated to quarterly U.S. and German data. We model the exchange rate as a random walk, and the monetary fundamental as an AR(3) with persistence parameters 1.328, -0.159, and -0.223. This choice for the fundamental was made by

⁴ We estimated the Kaplan (1994) test statistic for nonlinearities for each of the four exchange rates (in differences). We were unable to reject the null hypothesis of a linear process for any exchange rate at a 90%-confidence level.

FIGURE 1—MONTE CARLO DISTRIBUTIONS OF NEWKEY-WEST t -STATISTICS UNDER H_0 : INDEPENDENCE



For each horizon of interest, $k = 1, 4, 8, 12,$ and 16 , we plot the histogram of the Newkey-West t -statistic with a bandwidth of 20. The solid line corresponds to the density of a t -distributed random variable with degrees of freedom equal to $85 - (k + 1)$.

fitting ARMA models to the U.S.-German data, with the BIC selecting the lag orders. We generate 2,000 Monte Carlo iterations. For each Monte Carlo data set, we run the long-horizon regressions (equation (2)) and compute the associated diagnostic statistics for $k = 1, 4, 8, 12,$ and 16 . The results are presented in table 1, panel A. Column 3 displays the median, the 90th percentile, and the 95th percentile of the estimated slope coefficients across Monte Carlo trials for each horizon of interest. Despite independence between exchange rates and fundamentals, the median $\hat{\beta}_k$ rises with k to a maximum of 0.635 for $k = 16$. As expected, the associated naive LS t -statistics displayed in column 4 also increase with the horizon. Column 5 and 6 display t -statistics corrected for autocorrelation with a truncation lag of 20 and with Andrews' (1991) rule (labeled $t(20)$ and $t(A)$, respectively). Again, as the horizon increases, so do median values of the slope coefficient's t -statistics. The right shift of the empirical distribution of the t -statistics inflates the empirical critical values. For example, when $k = 16$, the one-sided empirical 95th percentile for $t(A)$ is 9.49 instead of 1.64 (that is, the corresponding asymptotic critical value from a Gaussian distribution). Figure 1 depicts the Monte Carlo distribution of $t(A)$ for each horizon. Clearly, the empirical density becomes more right-skewed and kurtotic as k increases.

Column 8 of table 1, panel A, displays the ratio of root-mean-squared error for out-of-sample regression forecasts over root-mean-squared error implied by the random-walk model. Thus, for values below 1, the regression delivers more-accurate forecasts than does the benchmark random walk. The first out-of-sample forecast is for 1981:1 (the first quarter of 1981), which results in forty one-step

TABLE 1—LONG-HORIZON MONTE CARLO ESTIMATES NULL HYPOTHESIS: INDEPENDENCE
(SPOT RATES FOLLOW A RANDOM WALK AND FUNDAMENTALS AN AR(2) PROCESS)

| k | %-ile | $\hat{\beta}_k$ | $t(\text{LS})$ | $t(20)$ | $t(A)$ | R_k^2 | OUT/RW | DM(20) | DM(A) |
|-----------------------------|-------|-----------------|----------------|---------|--------|---------|--------|--------|--------|
| Panel A: sample size = 85 | | | | | | | | | |
| 1 | 50 | 0.046 | 1.479 | 2.028 | 1.525 | 0.026 | 1.008 | -0.377 | -0.300 |
| | 90 | 0.112 | 2.517 | 3.941 | 2.740 | 0.072 | 0.969 | 2.222 | 1.341 |
| | 95 | 0.143 | 2.827 | 4.502 | 3.132 | 0.088 | 0.957 | 3.197 | 1.758 |
| 4 | 50 | 0.183 | 2.982 | 2.359 | 2.092 | 0.101 | 1.032 | -0.441 | -0.372 |
| | 90 | 0.403 | 5.220 | 4.823 | 4.180 | 0.256 | 0.881 | 2.458 | 1.816 |
| | 95 | 0.495 | 6.012 | 5.666 | 4.947 | 0.314 | 0.840 | 3.430 | 2.400 |
| 8 | 50 | 0.354 | 4.270 | 2.769 | 2.635 | 0.195 | 1.050 | -0.420 | -0.366 |
| | 90 | 0.711 | 7.741 | 6.079 | 5.689 | 0.444 | 0.768 | 2.752 | 2.230 |
| | 95 | 0.839 | 9.004 | 7.442 | 6.813 | 0.519 | 0.699 | 4.058 | 3.015 |
| 12 | 50 | 0.507 | 5.333 | 3.200 | 3.203 | 0.286 | 1.045 | -0.320 | -0.301 |
| | 90 | 0.958 | 9.798 | 7.173 | 6.839 | 0.574 | 0.678 | 3.108 | 2.574 |
| | 95 | 1.067 | 11.267 | 8.928 | 8.112 | 0.641 | 0.605 | 4.552 | 3.524 |
| 16 | 50 | 0.635 | 6.132 | 3.465 | 3.585 | 0.359 | 1.029 | -0.223 | -0.194 |
| | 90 | 1.122 | 11.505 | 8.292 | 7.976 | 0.663 | 0.601 | 3.796 | 2.956 |
| | 95 | 1.244 | 13.360 | 9.790 | 9.488 | 0.727 | 0.516 | 5.089 | 4.122 |
| Panel B: sample size = 1085 | | | | | | | | | |
| 1 | 50 | 0.004 | 1.559 | 1.593 | 1.572 | 0.002 | 0.999 | 0.198 | 0.134 |
| | 90 | 0.011 | 2.565 | 2.734 | 2.595 | 0.006 | 0.991 | 2.495 | 1.559 |
| | 95 | 0.013 | 2.834 | 2.991 | 2.854 | 0.007 | 0.987 | 3.107 | 1.943 |
| 4 | 50 | 0.016 | 3.102 | 1.651 | 1.646 | 0.009 | 0.997 | 0.212 | 0.150 |
| | 90 | 0.041 | 5.145 | 2.805 | 2.853 | 0.024 | 0.962 | 2.562 | 2.014 |
| | 95 | 0.050 | 5.683 | 3.142 | 3.160 | 0.029 | 0.946 | 3.423 | 2.571 |
| 8 | 50 | 0.031 | 4.433 | 1.728 | 1.723 | 0.017 | 0.995 | 0.181 | 0.164 |
| | 90 | 0.082 | 7.385 | 2.976 | 3.053 | 0.048 | 0.920 | 2.908 | 2.452 |
| | 95 | 0.100 | 8.088 | 3.321 | 3.391 | 0.057 | 0.890 | 3.915 | 3.157 |
| 12 | 50 | 0.046 | 5.449 | 1.815 | 1.791 | 0.026 | 0.992 | 0.208 | 0.189 |
| | 90 | 0.121 | 9.098 | 3.146 | 3.191 | 0.072 | 0.874 | 3.509 | 3.007 |
| | 95 | 0.149 | 9.982 | 3.520 | 3.614 | 0.085 | 0.831 | 4.884 | 3.951 |
| 16 | 50 | 0.062 | 6.260 | 1.911 | 1.835 | 0.035 | 0.992 | 0.173 | 0.149 |
| | 90 | 0.158 | 10.505 | 3.292 | 3.374 | 0.093 | 0.826 | 4.433 | 3.807 |
| | 95 | 0.198 | 11.504 | 3.715 | 3.815 | 0.110 | 0.773 | 6.308 | 5.278 |

The table presents estimated slope coefficients, $\hat{\beta}_k$, for equation (2) with the LS t -statistics, heteroskedasticity, and autocorrelation-corrected t -statistics using a Bartlett kernel and a truncation lag of 20 and Andrews' (1991) rule; respectively, $t(\text{LS})$, $t(20)$, and $t(A)$. OUT/RW denotes the ratio of regression mean-squared out-of-sample forecast error to the random-walk, mean-squared, out-of-sample forecast error. DM(20) and DM(A) denote the Diebold-Mariano statistics with a Bartlett kernel and truncation lags of 20 and via Andrews' (1991) rule, respectively.

ahead forecasts and 25 sixteen-step ahead forecasts. As k increases, the finite sample distribution of this statistic becomes more fat-tailed. This is due in good measure to the fact that, as k increases and for a fixed sample size, the number of forecast observations decreases. As a result, empirical critical values decrease dramatically with k .

Column 9 and 10 display the Monte Carlo Diebold-Mariano (1995) predictability statistics, again with either a truncation lag of 20 or using Andrews' (1991) rule (labeled DM(20) and DM(A), respectively). For both truncation rules, the median values of the Diebold-Mariano statistics are negative, implying that the random-walk forecast beats the regression. Again, the empirical distribution of the DM statistics are fat-tailed for large k . The 95th percentile of the DM(A) increases from 1.76 for $k = 1$ to 4.12 for $k = 16$.

These findings result from the combination of problems that arise with long-horizon regressions, when the two series fail to cointegrate. For example, although we argue that explanatory power appears to increase with k (for example, high median R_k^2 's and high empirical critical values of the t -statistics), there are sizeable distortions even for $k = 1$. These biases arise because of the presence of stochastic and highly persistent regressors. To further complicate matters,

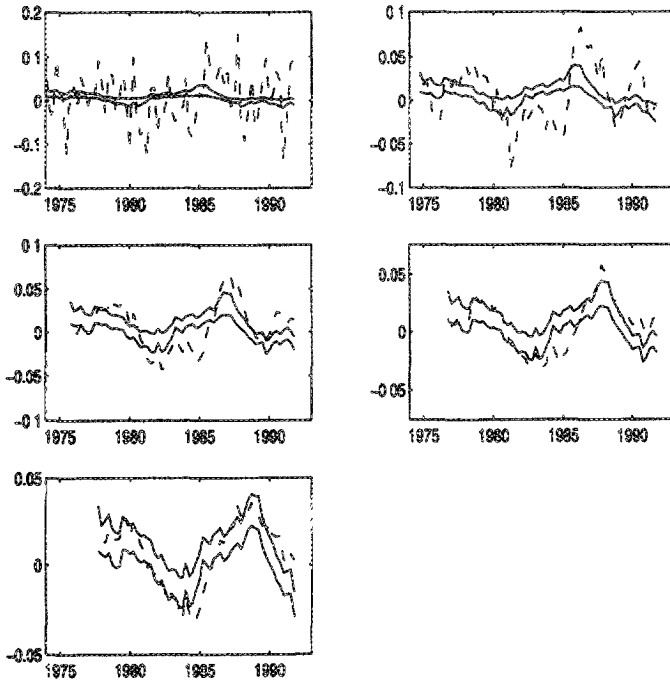
the distribution of the estimated slope coefficient would be different if the regression were estimated without a constant term (as in Dickey and Fuller (1979)).⁵

Such stark results obtain in sample sizes typical of available data. To investigate the behavior in larger samples, we report the results of an identical simulation experiment with a sample size of 1,085 in panel B. Now, the median $\hat{\beta}_k$'s are all lower, but the bias is still sizable for large k ; for example, the median of $\hat{\beta}_k$ is 0.062 for $k = 16$. R_k^2 are low and the ratios of RMSE of regression to random-walk forecasts are very close to 1 for all horizons. However, the presence of small bias in the LS slope parameters introduces large distortions at all horizons in the empirical critical values of the t -statistics, corroborating Berben and Van Dijk's 1998 finding that the asymptotic distribution of $\hat{\beta}_k$ is not Gaussian.

Finally, we suggest that even the graphical evidence of predictability has its pitfalls. In figure 2, we plot the actual k -period changes in the log U.S. dollar/German mark rate (dash-dot lines). Each graph also contains a confidence

⁵ Additional simulation results for the no-intercept case can be obtained from the authors.

FIGURE 2.—CHANGES IN THE LOG U.S. DOLLAR/GERMAN MARK EXCHANGE RATE AND SIMULATED FUNDAMENTALS



For each horizon of interest, $k = 1, 4, 8, 12,$ and 16 , actual k -period changes in the log U.S. dollar/German mark are depicted as dash-dot lines. Solid lines depict 95% error bands. Bands are constructed by estimating 2,000 long-horizon regressions on independent random numbers and taking the fitted values, $(\alpha)2000$ and $(1 - \alpha)2000$ at each date, $\alpha = 0.975$.

tunnel constructed from regression in-sample forecasts with independent simulated fundamental data. To form the tunnel, we generate 2,000 sets of simulated fundamentals, which provides 2,000 sequences of fitted values. We then sort the fitted values (at each quarter) and take the 0.975 and 0.25 percentiles.

As with the actual fundamentals (for example, Mark (1995)), these plots appear to suggest in-sample predictive accuracy for longer horizons. Because in figure 2 the confidence tunnels are built from independent randomly generated fundamentals, we conclude that even the graphical evidence from long-horizon regressions is misleading.

To compare our Monte Carlo results directly with those of Mark (1995), table 2 presents critical values tabulated under the null that cointegration holds. Data are generated by fitting a vector autoregression for the change in the exchange rate and the error-correction term, $(\Delta s_t, (f_t - s_t))$. In panel A, we see that median $\hat{\beta}_k$'s, t -statistics, and R^2 's increase with horizon, but much more modestly than in table 1. For example, at $k = 16$, the median $\hat{\beta}$ is 0.34 rather than 0.64. Similarly, t -statistics are substantially smaller: Newey-West corrected t -statistics have a 90% critical value of approximately 5.5 compared to approximately 8 under our null.

In the second simulation experiment, we fit an unrestricted bivariate VAR to $[f_t, s_t]'$ and use the estimated model to generate critical values. Here, we therefore impose no prior stationarity assumption on $(f_t - s_t)$. This repre-

sents a natural middle ground between Mark's (1995) null of cointegration and the previous experiment's null of independence.

The results are shown in table 3. Again, percentiles of coefficient estimates and in-sample statistics rise as the horizon increases. Indeed under this agnostic null, nearly all critical values are slightly larger than they were under the null of no cointegration.

Taken together, tables 1 through 3 indicate that finite-sample inference from long-horizon regressions should be assessed with great care. Empirical distributions of the statistics simulated under the null of cointegration can yield different inference from the no-cointegration null. As such, we recommend conservative inference—that is, using the null that leads to larger critical values.⁶

IV. Relationship to the Vector Error-Correction Model

The long-horizon approach relies on the assumption that nominal exchange rates and fundamentals cointegrate with cointegrating vector $[1 \quad -1]'$. This implies, by the Granger representation theorem, the following vector error-correction model (VECM):⁷

$$\Delta s_{t+1} = \lambda_1(f_t - s_t) + \omega_{1,t+1} \quad (4)$$

$$\Delta f_{t+1} = \lambda_2(f_t - s_t) + \omega_{2,t+1}.$$

Stationarity of the error correction term, $(f_t - s_t)$, requires that at least one of the loading coefficients, λ_1 or λ_2 , be different from 0 and that $\lambda_2 - \lambda_1 < 0$. In general, the error terms $\omega_{1,t}$ and $\omega_{2,t}$ need not be identically and independently distributed.

Again, letting $z_t = (f_t - s_t)$, it is easy to show that $z_t = \rho z_{t-1} + \omega_t$, where $\rho = (1 + \lambda_2 - \lambda_1)$ and $\omega_t = \omega_{2,t} - \omega_{1,t}$. Exploiting the autoregressive structure of the z -process, we can write

$$z_{t+k} = \rho^k z_t + \xi_{t+k}, \quad \text{with} \quad \xi_{t+k} = \sum_{j=0}^{k-1} \rho^j \omega_{t+k-j}.$$

Hence, the k -change in the log spot rate can be rewritten as:

$$\Delta^k s_{t+k} = \left[\lambda_1 \left(\frac{1 - \rho^k}{1 - \rho} \right) \right] (f_t - s_t) + \sum_{j=0}^{k-1} (\lambda_1 \xi_{t+j} + \omega_{1,t+j}) + \omega_{1,t+k}, \quad (5)$$

$$k = 1, \dots, K.$$

⁶ This approach is discussed by Watson (1994).

⁷ The drift components are omitted from the VECM for expositional simplicity.

TABLE 2.—LONG-HORIZON MONTE CARLO ESTIMATES NULL HYPOTHESIS: COINTEGRATION AS IN MARK (1995)

| k | %-ile | β_k | $t(\text{LS})$ | $t(20)$ | $t(A)$ | R_k^2 | OUT/RW | DM(20) | DM(A) |
|-----------------------------|-------|-----------|----------------|---------|--------|---------|--------|--------|--------|
| Panel A: sample size = 85 | | | | | | | | | |
| 1 | 50 | 0.023 | 0.742 | 1.085 | 0.796 | 0.009 | 1.012 | -0.856 | -0.591 |
| | 90 | 0.091 | 2.038 | 3.048 | 2.181 | 0.049 | 0.983 | 1.737 | 0.895 |
| | 95 | 0.118 | 2.394 | 3.813 | 2.612 | 0.066 | 0.973 | 2.929 | 1.347 |
| 4 | 50 | 0.094 | 1.471 | 1.211 | 1.058 | 0.035 | 1.046 | -0.910 | -0.702 |
| | 90 | 0.326 | 4.073 | 3.587 | 3.062 | 0.177 | 0.942 | 2.072 | 1.236 |
| | 95 | 0.424 | 4.910 | 4.565 | 3.859 | 0.236 | 0.898 | 3.477 | 1.93 |
| 8 | 50 | 0.184 | 2.144 | 1.367 | 1.266 | 0.069 | 1.090 | -1.038 | -0.849 |
| | 90 | 0.571 | 5.734 | 4.449 | 3.983 | 0.310 | 0.890 | 2.643 | 1.642 |
| | 95 | 0.710 | 6.926 | 5.579 | 4.914 | 0.393 | 0.824 | 4.326 | 2.312 |
| 12 | 50 | 0.263 | 2.533 | 1.573 | 1.498 | 0.097 | 1.138 | -1.219 | -0.991 |
| | 90 | 0.733 | 6.941 | 5.027 | 4.597 | 0.405 | 0.838 | 3.922 | 2.085 |
| | 95 | 0.896 | 8.286 | 6.804 | 5.829 | 0.498 | 0.747 | 6.848 | 2.795 |
| 16 | 50 | 0.338 | 2.893 | 1.754 | 1.731 | 0.136 | 1.144 | -1.784 | -1.144 |
| | 90 | 0.867 | 7.730 | 5.568 | 5.257 | 0.476 | 0.795 | 6.165 | 2.428 |
| | 95 | 1.025 | 9.476 | 7.380 | 6.616 | 0.575 | 0.694 | 10.931 | 3.502 |
| Panel B: sample size = 1085 | | | | | | | | | |
| 1 | 50 | 0.001 | 0.179 | 0.192 | 0.185 | 0.000 | 1.000 | -0.428 | -0.273 |
| | 90 | 0.012 | 1.442 | 1.510 | 1.436 | 0.002 | 0.996 | 2.515 | 1.297 |
| | 95 | 0.015 | 1.755 | 1.851 | 1.792 | 0.003 | 0.994 | 4.000 | 1.763 |
| 4 | 50 | 0.005 | 0.364 | 0.202 | 0.203 | 0.000 | 1.001 | -0.463 | -0.319 |
| | 90 | 0.045 | 2.821 | 1.600 | 1.611 | 0.009 | 0.984 | 2.964 | 1.731 |
| | 95 | 0.057 | 3.482 | 1.949 | 1.978 | 0.013 | 0.977 | 4.252 | 2.238 |
| 8 | 50 | 0.011 | 0.532 | 0.221 | 0.224 | 0.003 | 1.001 | -0.334 | -0.249 |
| | 90 | 0.088 | 3.954 | 1.687 | 1.716 | 0.018 | 0.969 | 3.636 | 2.082 |
| | 95 | 0.110 | 4.958 | 2.129 | 2.178 | 0.025 | 0.951 | 5.674 | 2.720 |
| 12 | 50 | 0.018 | 0.702 | 0.251 | 0.255 | 0.005 | 1.001 | -0.296 | -0.178 |
| | 90 | 0.123 | 4.681 | 1.779 | 1.792 | 0.026 | 0.955 | 4.638 | 2.341 |
| | 95 | 0.160 | 6.063 | 2.306 | 2.293 | 0.037 | 0.928 | 8.199 | 3.114 |
| 16 | 50 | 0.024 | 0.793 | 0.264 | 0.254 | 0.006 | 1.001 | -0.381 | -0.211 |
| | 90 | 0.160 | 5.397 | 1.878 | 1.844 | 0.034 | 0.935 | 8.631 | 2.660 |
| | 95 | 0.205 | 6.798 | 2.433 | 2.491 | 0.045 | 0.893 | 14.656 | 3.718 |

The table presents estimated slope coefficients, β_k , for equation (2) with the LS t -statistics, heteroskedasticity, and autocorrelation-corrected t -statistics using a Bartlett kernel and a truncation lag of 20 and Andrews' (1991) rule, respectively, $t(\text{LS})$, $t(20)$, and $t(A)$. OUT/RW denotes the ratio of regression mean-squared out-of-sample forecast error to the random-walk, mean-squared, out-of-sample forecast error DM(20) and DM(A) denote the Diebold-Mariano statistics with a Bartlett kernel and truncation lags of 20 and via Andrews' (1991) rule, respectively

Several considerations emerge by comparing the long-horizon regression (2) with equation (5), the implied long-horizon regression from the VECM.

First, $\beta_1 = \lambda_1$, so that:

$$\beta_k = \beta_1 \frac{1 - \rho^k}{1 - \rho} \quad (6)$$

This implies that, if $\beta_1 = 0$, then $\beta_k = 0$ for all $k > 1$. In the context of a linear model, the lack of one-step-ahead predictability necessarily implies no predictability. If the hypothesis $\beta_1 = 0$ is not rejected, nothing seems to be gained by estimating the long-horizon regressions for $k > 1$ unless unspecified nonlinearities are present.⁸

⁸ The present discussion deliberately ignores the power of the t -test associated with β_k . Under the alternative hypothesis, β_k increases with k ; that is, the alternative hypothesis moves away from the null. Thus, the probability of rejecting a false null increases with k , producing the impression of higher power of the associated t -test for large k 's. However, a proper power comparison requires that the alternative is kept fixed. A priori, it is not clear what the power properties of the t -tests on β_k would be after accounting for the difference in the alternative hypotheses. In the present context, a power comparison would be further complicated by the presence of severe size distortions in the t -tests, a point well documented

Second, the VECM representation (4) clarifies the nature of the null and alternative hypotheses of the testing procedure. Testing the null hypothesis $\beta_k = 0$ for all k 's in equation (2) is equivalent to testing the hypothesis that the spot rate is weakly exogenous for the cointegrating vector in the system (4) (that $\lambda_1 = 0$).⁹ This would imply that, although there is a long-run relationship between s_t and f_t , knowledge of the history of the fundamentals will not be helpful in predicting future values of the spot rate, no matter how long the prediction horizon is.¹⁰

The alternative hypothesis, $\lambda_1 \neq 0$, corresponds to a situation in which the spot rate is not weakly exogenous for the cointegrating vector, so that knowledge of the history of f_t will be helpful in formulating forecasts of s_t . However, to construct valid multistep forecasts of s_t from long-horizon equations, it is necessary that the spot rate does not feed

for large k 's, by the simulation experiments of section III. We are grateful to Hashem Pesaran for these insights.

⁹ For a review of definitions and testing procedures of weak exogeneity in cointegrated systems, see Johansen (1992).

¹⁰ Assuming that the spot rate does not possess, under the null, significant short-run dynamics, that is, assuming $\omega_{1,t} \sim WN$ (the innovation is white noise).

TABLE 3.—LONG-HORIZON MONTE CARLO ESTIMATES NULL HYPOTHESIS. NO PRIOR RESTRICTIONS ON INTEGRATION STATUS (SPOT RATES AND FUNDAMENTALS FOLLOW AN UNRESTRICTED VAR)

| k | %-ile | $\hat{\beta}_k$ | $t(LS)$ | $t(20)$ | $t(A)$ | R_k^2 | OUT/RW | DM(20) | DM(A) |
|-----------------------------|-------|-----------------|---------|---------|--------|---------|--------|--------|-------|
| Panel A: sample size = 85 | | | | | | | | | |
| 1 | 50 | 0.092 | 2.117 | 2.992 | 2.195 | 0.052 | 0.998 | 0.046 | 0.040 |
| | 90 | 0.189 | 3.173 | 4.901 | 3.398 | 0.109 | 0.952 | 2.731 | 1.424 |
| | 95 | 0.224 | 3.478 | 5.684 | 3.748 | 0.128 | 0.939 | 3.545 | 1.884 |
| 4 | 50 | 0.313 | 3.988 | 3.337 | 2.972 | 0.167 | 0.993 | 0.071 | 0.058 |
| | 90 | 0.597 | 6.166 | 6.179 | 5.478 | 0.324 | 0.837 | 2.858 | 2.072 |
| | 95 | 0.692 | 6.955 | 7.329 | 6.371 | 0.379 | 0.801 | 3.795 | 2.700 |
| 8 | 50 | 0.515 | 5.375 | 3.847 | 3.632 | 0.278 | 0.992 | 0.054 | 0.049 |
| | 90 | 0.882 | 8.428 | 7.718 | 7.051 | 0.486 | 0.741 | 2.999 | 2.256 |
| | 95 | 0.993 | 9.376 | 9.253 | 8.345 | 0.539 | 0.667 | 3.871 | 2.905 |
| 12 | 50 | 0.684 | 6.411 | 4.340 | 4.261 | 0.366 | 0.967 | 0.203 | 0.186 |
| | 90 | 1.069 | 10.139 | 8.555 | 7.907 | 0.591 | 0.645 | 3.207 | 2.506 |
| | 95 | 1.174 | 11.571 | 10.257 | 9.386 | 0.653 | 0.577 | 4.264 | 3.110 |
| 16 | 50 | 0.805 | 7.185 | 4.698 | 4.623 | 0.435 | 0.928 | 0.529 | 0.438 |
| | 90 | 1.192 | 11.407 | 9.267 | 8.789 | 0.660 | 0.581 | 3.915 | 2.892 |
| | 95 | 1.304 | 12.935 | 11.342 | 10.677 | 0.714 | 0.498 | 4.971 | 3.617 |
| Panel A: sample size = 1085 | | | | | | | | | |
| 1 | 50 | 0.041 | 5.019 | 5.276 | 5.147 | 0.022 | 0.986 | 1.280 | 0.731 |
| | 90 | 0.054 | 5.822 | 6.351 | 6.046 | 0.030 | 0.972 | 3.597 | 1.939 |
| | 95 | 0.058 | 6.110 | 6.694 | 6.301 | 0.033 | 0.967 | 4.507 | 2.285 |
| 4 | 50 | 0.138 | 9.425 | 5.384 | 5.367 | 0.076 | 0.952 | 1.305 | 0.975 |
| | 90 | 0.183 | 11.056 | 6.547 | 6.588 | 0.101 | 0.902 | 3.546 | 2.468 |
| | 95 | 0.200 | 11.542 | 6.952 | 7.010 | 0.109 | 0.883 | 4.524 | 2.926 |
| 8 | 50 | 0.237 | 12.644 | 5.487 | 5.458 | 0.129 | 0.916 | 1.367 | 1.111 |
| | 90 | 0.308 | 14.828 | 6.800 | 6.989 | 0.169 | 0.811 | 3.725 | 2.711 |
| | 95 | 0.334 | 15.563 | 7.247 | 7.529 | 0.183 | 0.779 | 4.653 | 3.209 |
| 12 | 50 | 0.320 | 15.153 | 5.735 | 5.668 | 0.176 | 0.883 | 1.411 | 1.205 |
| | 90 | 0.411 | 17.823 | 7.140 | 7.476 | 0.228 | 0.727 | 4.017 | 2.928 |
| | 95 | 0.441 | 18.724 | 7.700 | 8.142 | 0.246 | 0.677 | 4.981 | 3.498 |
| 16 | 50 | 0.393 | 17.279 | 6.039 | 5.901 | 0.218 | 0.856 | 1.572 | 1.329 |
| | 90 | 0.500 | 20.440 | 7.582 | 7.958 | 0.281 | 0.640 | 4.580 | 3.255 |
| | 95 | 0.534 | 21.427 | 8.210 | 8.715 | 0.300 | 0.577 | 5.759 | 4.053 |

Table presents the difference between the percentiles shown in table 1 and those generated under Mark's (1995) Monte Carlo simulation which assumes cointegration. Positive $\hat{\beta}_k$'s and t -statistics imply that critical values under H_0 , no cointegration, are more conservative than under H_0 , cointegration. For the out-of-sample statistics, negative values indicate that H_0 , no cointegration yields more conservative inference (OUT/RW, DM(20), DM(A)). See text for details.

back into the equation for the fundamentals in system (4), (that is, that f_t is strongly exogenous). Unfortunately, estimation of the long-horizon regression alone does not reveal anything regarding the exogeneity status of the fundamentals.

Thus far, the discussion has focused on the relationship between a given VECM and long-horizon regressions for a given set of parameters: λ_1 and λ_2 . In reality, these coefficients are unknown and must be estimated. In recent work, Berben and van Dijk (1998) study the asymptotic behavior of least-squares estimates of λ_1 under the assumption that λ_2 is 0.

Berben and van Dijk show that, when $\lambda_1 = 0$ (equivalently, cointegration fails), OLS estimates of λ_1 converge to a mixture of a Gaussian and the Dickey-Fuller distribution. The degree to which the distribution is of the Dickey-Fuller type depends on the long-run correlation between $\omega_{1,t}$ and $\omega_{2,t} - \omega_{1,t}$.¹¹ That is, the Berben and van Dijk results require only that cointegration fails to hold, not that f_t and s_t be statistically independent.

Interestingly, in the special case that f_t and s_t are independent random walks, the long-run correlation collapses to $\sigma_{\omega_{1,t}}/\sigma_{\omega_{2,t}}$, which can be interpreted as an inverse signal-to-noise ratio: the more variation that is inherent to exchange rates, the worse the OLS bias.

Moreover, the OLS estimate of λ_k will be increasing in k . In the absence of cointegration, the asymptotic distribution of $\hat{\beta}_k$ is k times the distribution of $\hat{\beta}_1$ under mild regularity conditions. In addition, the asymptotic distribution of the associated t -statistics, $t(\hat{\beta}_k)$, also depend on k (although in a more complicated way). In general, this is even the case for Newey-West corrected t -statistics. The distribution of Newey-West t -statistics will not depend on k asymptotically only if $\omega_{1,t}$ is not autocorrelated; but, even then, least-squares t -statistics will depend on k even asymptotically. In this sense, Berben and van Dijk formalize the notion that long-horizon regressions are close to a spurious regression as k is allowed to increase.

V. Exchange Rate Predictability

In this section, we revisit estimates of equation (2) with critical values and significance levels calculated under the

¹¹ The role of this correlation is very similar to the condition derived by Elliot and Stock (1994) in a slightly different context.

TABLE 4.—LONG-HORIZON REGRESSION ESTIMATES NULL HYPOTHESIS: NO COINTEGRATION (1973:2–1991:4)

| | k | $\hat{\beta}_k$ | $t(20)$ | p -val | $t(A)$ | p -val | R^2 | OUT/RW | p -val | DM(20) | p -val | DM(A) | p -val |
|-----------------|-----|-----------------|---------|----------|--------|----------|--------|--------|----------|--------|------------|--------|----------|
| Canadian Dollar | 1 | 0.040 | 3.051 | 0.095 | 2.172 | 0.086 | 0.059 | 0.998 | 0.405 | 0.061 | 0.441 | 0.036 | 0.443 |
| | 4 | 0.155 | 2.398 | 0.217 | 2.168 | 0.190 | 0.179 | 1.119 | 0.412 | -1.270 | 0.849 | -0.925 | 0.975 |
| | 8 | 0.349 | 2.539 | 0.225 | 2.527 | 0.212 | 0.351 | 1.145 | 0.712 | -1.036 | 0.958 | -0.890 | 0.901 |
| | 12 | 0.438 | 1.961 | 0.352 | 1.936 | 0.350 | 0.336 | 1.436 | 0.317 | -1.916 | 0.592 | -1.661 | 0.695 |
| | 16 | 0.450 | 1.542 | 0.458 | 1.512 | 0.465 | 0.254 | 1.699 | 0.196 | -2.596 | 0.466 | -1.857 | 0.620 |
| German Mark | 1 | 0.035 | 1.836 | 0.510 | 0.929 | 0.668 | 0.015 | 1.015 | 0.969 | -0.932 | 0.724 | -0.846 | 0.669 |
| | 4 | 0.205 | 2.902 | 0.354 | 2.290 | 0.406 | 0.104 | 1.037 | 0.914 | -1.345 | 0.522 | -0.852 | 0.780 |
| | 8 | 0.554 | 3.487 | 0.354 | 3.558 | 0.313 | 0.265 | 1.002 | 0.809 | -0.027 | 0.814 | -0.020 | 0.814 |
| | 12 | 0.966 | 6.329 | *0.165 | 6.510 | *0.135 | 0.527 | 0.796 | *0.406 | 4.246 | 0.093 | 0.094 | 0.563 |
| | 16 | 1.324 | 9.256 | 0.096 | 9.124 | 0.082 | 0.762 | 0.524 | *0.113 | 8.719 | 0.030 | 8.719 | 0.016 |
| Japanese Yen | 1 | 0.047 | 1.396 | 0.516 | 1.331 | 0.423 | 0.020 | 0.988 | 0.477 | 1.571 | 0.286 | 0.836 | 0.322 |
| | 4 | 0.263 | 2.254 | 0.353 | 2.153 | 0.341 | 0.125 | 0.928 | 0.396 | 2.302 | 0.215 | 1.487 | 0.310 |
| | 8 | 0.575 | 3.516 | 0.228 | 3.496 | 0.227 | 0.301 | 0.819 | 0.304 | 3.096 | 0.172 | 1.803 | 0.309 |
| | 12 | 0.945 | 4.889 | 0.166 | 4.735 | 0.190 | 0.532 | 0.712 | 0.233 | 3.319 | 0.173 | 1.147 | 0.276 |
| | 16 | 1.273 | 4.919 | 0.216 | 4.901 | 0.199 | 0.694 | 0.574 | 0.142 | 5.126 | 0.109 | 3.096 | 0.241 |
| Swiss Franc | 1 | 0.074 | 2.681 | 0.210 | 2.073 | *0.166 | 0.051 | 0.997 | 0.642 | 0.066 | 0.704 | 0.064 | 0.693 |
| | 4 | 0.285 | 3.248 | 0.181 | 3.196 | *0.131 | 0.180 | 0.981 | 0.596 | 0.218 | 0.676 | 0.162 | 0.686 |
| | 8 | 0.568 | 4.770 | 0.095 | 4.696 | 0.081 | 0.336 | 0.917 | 0.458 | 0.703 | 0.621 | 0.560 | 0.635 |
| | 12 | 0.837 | 8.013 | 0.032 | 8.013 | 0.023 | 0.538 | 0.738 | 0.214 | 2.933 | 0.203 | 0.938 | 0.624 |
| | 16 | 1.086 | 17.41 | 0.006 | 12.66 | 0.010 | 0.771 | 0.411 | 0.026 | 9.650 | #0.019 | 1.996 | 0.411 |
| | | | p_1 | | p_2 | | p_3 | | p_4 | | σ^2 | | |
| Canadian Dollar | | | 1.227 | | 0.028 | | -0.045 | | -0.233 | | 0.011 | | |
| German Mark | | | 1.253 | | -0.305 | | | | | | 0.016 | | |
| Japanese Yen | | | 1.267 | | -0.062 | | -0.204 | | | | 0.013 | | |
| Swiss Franc | | | 1.916 | | -1.154 | | 0.236 | | | | 0.014 | | |

Data were kindly supplied by Nelson Mark (originally taken from the OECD *Main Economic Indicators*). For detailed descriptions of the statistics, see the notes to table 1.

* indicates p -values that are no longer significant at a 90% level, but were under the null of cointegration.

indicates the reverse.

The table presents least-squares estimates of equation (2) over horizons of $k = 1, 4, 8, 12,$ and 16 quarters and Monte Carlo p -values, tabulated under the null hypothesis that s_t is a random walk, independent of f_t . The f_t are generated using the following AR models with lag order selected by the BIC criterion.

three null hypotheses studied in the simulation experiments of section III. That is, we consider the null of cointegration, the null of no cointegration, and the null of $\{f_t, s_t\}'$ following an unrestricted VAR. The empirical investigation covers Canada, Germany, Japan, and Switzerland.

A. Does the Null Hypothesis Matter? Mark's 1995 Results Revisited

To assess the importance of the cointegration assumption, we first replicate the estimation results in Mark (1995) with precisely the same data, but with critical values tabulated under our null of no cointegration. The data-generating processes in the simulations are calibrated to the individual exchange rates and monetary fundamentals of the four countries under study.

Table 4, column 2, reports LS estimates of the slope coefficients; column 3 and 5 report Newey-West corrected t -statistics estimated with a truncation lag of 20, or truncation lag estimated with the Andrews' (1991) rule; and column 4 and 6, labeled " p -val," show the associated Monte Carlo p -values. As in the simulation experiment, the slope coefficients rise as the horizon increases. The t -statistics rise as the horizon increases for three of four currencies, as do the R^2 's shown in column 7. However, our Monte Carlo p -values indicate that, for three of the four exchange rates in our sample (the exception being the U.S. dollar/Swiss franc exchange rate), none of the slope coefficients are significantly different from 0 at a 95%-significance level. In

several cases (marked with asterisks), statistics that were significant under Mark's null are no longer significant using our p -values and a 90%-confidence level.

Column 8, labeled "OUT/RW," displays the ratio of root-mean-squared error for out-of-sample regression forecasts over root-mean-squared error implied by the random-walk model over the period 1985:1 to 1991:4. Again, column 9 displays the p -values that are implied by Monte Carlo critical values. In agreement with the t -statistics, only the U.S. dollar/Swiss franc exchange rate, one of four, yields a p -value significant at the 95% level.

Columns 10 through 13 display the Monte Carlo Diebold and Mariano (1995) statistics and p -values, again with either a truncation lag of 20 or using Andrews' rule. As stressed above, the conventional finding is that models underperform the random walk (Meese & Rogoff, 1988). Therefore, it would be more appropriate to base p -values on a two-sided alternative that allows either forecast to dominate. To make this correction, we multiply p -values that are less than 0.50 by 2. When this is done, only one test statistic, at one horizon, is significant for U.S. dollar/German mark rates.

The Diebold-Mariano statistics in table 4 suggest that the model improves out-of-sample forecasts for two of four exchange rates. Indeed, the evidence of U.S. dollar/Swiss franc predictability is strengthened under our null. The reason for this is clear: if cointegration does not hold, the model should grossly underperform the random-walk fore-

TABLE 5.—LONG-HORIZON REGRESSION ESTIMATES—EXTENDED SAMPLE NULL HYPOTHESIS: NO COINTEGRATION (1973:2–1994:4)

| | k | β_k | $t(20)$ | p -val | $t(A)$ | p -val | R_k^2 | OUT/RW | p -val | DM(20) | p -val | DM(A) | p -val |
|-----------------|-----|-----------|----------|----------|----------|----------|----------|--------|----------|--------|------------|--------|----------|
| Canadian Dollar | 1 | 0.035 | 3.013 | *0.136 | 1.979 | *0.182 | 0.041 | 0.994 | 0.428 | 0.169 | 0.540 | 0.176 | 0.512 |
| | 4 | 0.147 | 2.475 | 0.295 | 2.187 | 0.297 | 0.155 | 1.040 | 0.853 | -0.270 | 0.724 | -0.264 | 0.753 |
| | 8 | 0.336 | 2.489 | 0.325 | 2.489 | 0.314 | 0.331 | 1.078 | 0.818 | -0.316 | 0.756 | -0.312 | 0.773 |
| | 12 | 0.430 | 1.799 | 0.505 | 1.696 | 0.531 | 0.334 | 1.280 | 0.416 | -0.842 | 0.967 | -0.855 | 0.995 |
| | 16 | 0.441 | 1.350 | 0.606 | 1.296 | 0.618 | 0.236 | 1.542 | 0.260 | -1.580 | 0.782 | -1.516 | 0.713 |
| German Mark | 1 | 0.038 | 2.269 | 0.431 | 1.274 | 0.614 | 0.021 | 0.998 | 0.715 | 0.117 | 0.713 | 0.132 | 0.687 |
| | 4 | 0.156 | 2.269 | 0.487 | 1.899 | 0.545 | 0.082 | 1.005 | 0.821 | -0.124 | 0.836 | -0.129 | 0.861 |
| | 8 | 0.396 | 2.617 | 0.523 | 2.687 | 0.513 | 0.216 | 1.055 | 0.987 | -0.286 | 0.930 | -0.305 | 0.958 |
| | 12 | 0.697 | 3.250 | 0.474 | 3.460 | 0.443 | 0.393 | 1.133 | 0.886 | -0.340 | *0.961 | -0.352 | 0.978 |
| | 16 | 1.019 | 3.956 | 0.432 | 3.934 | *0.455 | 0.601 | 1.235 | 0.720 | -0.518 | *0.967 | -0.507 | *0.947 |
| Japanese Yen | 1 | 0.032 | 1.079 | 0.745 | 1.081 | 0.662 | 0.012 | 0.976 | 0.242 | 1.818 | 0.267 | 1.800 | 0.093 |
| | 4 | 0.174 | 1.360 | 0.723 | 1.286 | 0.703 | 0.065 | 0.942 | 0.429 | 0.991 | 0.490 | 0.921 | 0.454 |
| | 8 | 0.422 | 2.093 | 0.609 | 2.049 | 0.606 | 0.182 | 0.895 | 0.485 | 0.986 | 0.587 | 0.996 | 0.527 |
| | 12 | 0.719 | 3.027 | 0.485 | 2.968 | 0.499 | 0.364 | 0.932 | 0.715 | 0.338 | 0.820 | 0.332 | 0.811 |
| | 16 | 0.907 | 3.050 | 0.533 | 3.050 | 0.549 | 0.486 | 1.067 | 0.855 | -0.235 | 0.990 | -0.218 | 0.987 |
| Swiss Franc | 1 | 0.080 | 2.559 | 0.304 | 2.087 | 0.244 | 0.052 | 0.949 | *0.035 | 2.195 | 0.180 | 2.177 | *0.037 |
| | 4 | 0.287 | 3.195 | 0.250 | 3.158 | 0.200 | 0.175 | 0.838 | *0.063 | 1.629 | 0.305 | 1.611 | 0.223 |
| | 8 | 0.566 | 4.635 | *0.165 | 4.635 | *0.149 | 0.332 | 0.722 | *0.076 | 1.222 | 0.475 | 1.205 | 0.415 |
| | 12 | 0.848 | 7.942 | 0.066 | 8.079 | 0.050 | 0.538 | 0.455 | *0.015 | 1.428 | 0.505 | 1.355 | 0.470 |
| | 16 | 1.081 | 18.82 | 0.007 | 13.35 | 0.016 | 0.769 | 0.347 | 0.007 | 1.614 | *0.568 | 1.570 | 0.504 |
| | | | ρ_1 | | ρ_2 | | ρ_3 | | ρ_4 | | σ^2 | | |
| Canadian Dollar | | | 1.287 | | -0.071 | | -0.033 | | -0.205 | | 0.011 | | |
| German Mark | | | 1.338 | | -0.159 | | -0.223 | | | | 0.012 | | |
| Japanese Yen | | | 1.273 | | -0.013 | | -0.263 | | | | 0.012 | | |
| Swiss Franc | | | 1.893 | | -1.100 | | 0.202 | | | | 0.014 | | |

Extended sample taken from the OECD *Main Economic Indicators*. For detailed descriptions of the statistics, see the notes to table 1.

* indicates p -values that are no longer significant at a 90% level but were using Mark's (1995) data sample.

indicates the reverse.

The table presents least-squares estimates of equation (2) over horizons of $k = 1, 4, 8, 12,$ and 16 quarters, and Monte Carlo p -values, tabulated under the null hypothesis that s_t is a random walk, independent of f_t . The f_t are generated using the following AR models with lag order selected by the BIC criterion.

cast (which is literally true in our Monte Carlo). However, in the data, the two forecasts are roughly equal, which leads to a rejection of the null.

B. Does the Sample Size Matter?

A natural question arising in this context is to what extent are the results dependent on the sample. The Mark data has only 73 observations. (Three observations are lost through the MA(4) filter that Mark applies to f_t .) Groen (1997), for example, finds that exchange-rate predictability results are extremely sensitive to changes in the sample period.

We therefore repeat the predictability exercise with an extended sample from 1973:2 to 1994:4, also collected from the *Main Economic Indicators* of the OECD. This adds twelve observations, a 16% increase in sample size.

The results are shown in tables 5 and 6. Table 5 reports p -values under the null of no cointegration. P -values marked with an asterisk are not significant at a 90% level but were in Mark's sample. In agreement with Groen, we find substantial fragility with respect to augmented samples. Most strikingly, the out-of-sample results for the U.S. dollar/German mark rate disappear completely. The Diebold-Mariano statistics at $k = 16$ drop from 8.7 to -0.5. In the extended sample, there is no evidence of predictability whatsoever for three out of four exchange rates. Predictability of the Swiss franc rate is robust to the sample, although the OUT/RW statistics indicate predictability at all horizons and the DM(A) at short horizons.

In table 6, we report the extended sample results with p -values tabulated under the null that f_t or s_t follow an unrestricted VAR. Although the p -values are somewhat larger for both in-sample and out-of-sample statistics, they are qualitatively very similar to table 5. Only the Swiss franc rates show signs of predictability. Again, out-of-sample statistics suggest the possibility of predictability at short horizons to a greater extent than over long horizons.

C. Predictability Based on System Estimation Methods

As we have emphasized, given that exogeneity of f_t or s_t cannot in general be ruled out a priori, it seems more appropriate to start a predictability analysis for the spot rate by estimating the full joint model (4), within which the cointegration and exogeneity status of the variables can be easily checked. To illustrate, we present estimates of λ_1 and λ_2 in column 1 and 2 of table 7. The associated Horvath and Watson (1995) test statistics for cointegration with a known cointegrating vector of $[1 \ -1]'$ are displayed in column 3. For this sample, we can reject the joint hypothesis that both $\lambda_1 = 0$ and $\lambda_2 = 0$ only for the U.S. dollar/Swiss franc exchange rate. This is the same exchange rate that displayed evidence of long-horizon predictability out-of-sample. We find agreement between standard in-sample tests for cointegration based on system methods and the long-horizon regression, single-equation approach. Testing the null hypothesis $\beta_k = 0$ for all k 's is equivalent to testing the hypothesis that the spot rate is weakly exogenous in the

TABLE 6.—LONG-HORIZON REGRESSION ESTIMATES NULL HYPOTHESIS: NO PRIOR RESTRICTIONS ON INTEGRATION STATUS (1973:2–1994:4)

| | k | β_k | $t(20)$ | p -val | $t(A)$ | p -val | R_k^2 | OUT/RW | p -val | DM(20) | p -val | DM(A) | p -val |
|-----------------|-----|-----------|---------|----------|--------|----------|---------|--------|----------|--------|----------|--------|----------|
| Canadian Dollar | 1 | 0.035 | 3.013 | 0.426 | 1.979 | 0.506 | 0.041 | 0.994 | 0.557 | 0.169 | 0.597 | 0.176 | 0.572 |
| | 4 | 0.147 | 2.475 | 0.873 | 2.187 | 0.878 | 0.155 | 1.040 | 0.832 | -0.270 | 0.799 | 0.264 | 0.811 |
| | 8 | 0.336 | 2.489 | 0.957 | 2.489 | 0.955 | 0.331 | 1.078 | 0.845 | -0.316 | 0.827 | 0.312 | 0.835 |
| | 12 | 0.451 | 1.799 | 0.989 | 1.696 | 0.990 | 0.334 | 1.284 | 0.907 | -0.842 | 0.877 | 0.855 | 0.893 |
| | 16 | 0.440 | 1.352 | 0.988 | 1.295 | 0.989 | 0.236 | 1.542 | 0.946 | -1.580 | 0.914 | 1.516 | 0.933 |
| German Mark | 1 | 0.038 | 2.269 | 0.740 | 1.274 | 0.903 | 0.021 | 0.998 | 0.497 | 0.117 | 0.483 | 0.132 | 0.469 |
| | 4 | 0.156 | 2.369 | 0.765 | 1.899 | 0.828 | 0.082 | 1.005 | 0.534 | -0.124 | 0.547 | -0.129 | 0.553 |
| | 8 | 0.596 | 2.617 | 0.759 | 2.687 | 0.715 | 0.216 | 1.055 | 0.590 | -0.286 | 0.570 | -0.305 | 0.586 |
| | 12 | 0.697 | 3.255 | 0.691 | 3.462 | 0.645 | 0.393 | 1.133 | 0.661 | -0.340 | 0.585 | 0.352 | 0.595 |
| | 16 | 1.019 | 3.956 | 0.614 | 3.934 | 0.620 | 0.601 | 1.235 | 0.730 | -0.518 | 0.636 | 0.507 | 0.647 |
| Japanese Yen | 1 | 0.032 | 1.079 | 0.986 | 1.081 | 0.968 | 0.012 | 0.976 | 0.359 | 1.818 | 0.214 | 1.800 | 0.056 |
| | 4 | 0.174 | 1.365 | 0.982 | 1.286 | 0.985 | 0.065 | 0.942 | 0.504 | 0.991 | 0.433 | 0.921 | 0.398 |
| | 8 | 0.422 | 2.093 | 0.938 | 2.049 | 0.934 | 0.182 | 0.895 | 0.512 | 0.986 | 0.475 | 0.996 | 0.427 |
| | 12 | 0.719 | 3.027 | 0.855 | 2.968 | 0.846 | 0.364 | 0.932 | 0.583 | 0.338 | 0.604 | 0.332 | 0.593 |
| | 16 | 0.907 | 3.050 | 0.843 | 3.053 | 0.835 | 0.486 | 1.067 | 0.714 | -0.235 | 0.692 | 0.218 | 0.696 |
| Swiss Franc | 1 | 0.080 | 2.559 | 0.513 | 2.087 | 0.440 | 0.052 | 0.949 | 0.062 | 2.195 | 0.150 | 0.177 | 0.034 |
| | 4 | 0.287 | 3.195 | 0.457 | 3.158 | 0.405 | 0.175 | 0.838 | *0.103 | 1.629 | 0.260 | 1.611 | 0.175 |
| | 8 | 0.566 | 4.635 | 0.300 | 4.635 | 0.276 | 0.332 | 0.722 | 0.085 | 1.222 | 0.330 | 1.205 | 0.282 |
| | 12 | 0.848 | 7.942 | 0.090 | 8.079 | 0.069 | 0.538 | 0.455 | 0.010 | 1.428 | 0.301 | 1.355 | 0.266 |
| | 16 | 1.081 | 18.82 | 0.001 | 13.35 | 0.010 | 0.769 | 0.347 | 0.004 | 1.614 | 0.300 | 1.570 | 0.238 |

Extended sample from the OECD *Main Economic Indicators*. The table presents least squares estimates of equation (2) over horizons of $k = 1, 4, 8, 12,$ and 16 quarters and Monte Carlo p -values. For detailed descriptions of the statistics, see the notes to table 1.

* indicates p -values that are not longer significant at a 90% level, but were under the null of no cointegration.

system (4) (that $\lambda_1 = 0$) (again abstracting from unspecified nonlinearities).

Had we assumed cointegration and proceeded to estimate the single equation error-correction model for spot rates, we would have found fairly large t -statistics (displayed in parentheses) for three out of four spot rates. As shown in section III, if cointegration fails, the right-side variable is nonstationary and the correct asymptotic critical values are of the unit root type (and hence higher). However, because the Horvath-Watson statistic fails to reject the null of no cointegration, this t -statistic is likely spurious in the sense of Berben and van Dijk (1998).

By estimating λ_1 and λ_2 and calculating the Horvath-Watson statistics, we are first checking the cointegration status (whether both $\lambda_1 = 0$ and $\lambda_2 = 0$). Only if we reject this null (as we do for the U.S. dollar/Swiss franc) is it then valid to consider the t -statistics for $\lambda_1 = 0$.

It should be noted that Elliot and Stock (1994) show that such a two-step procedure can result in substantial size distortions in typical sample sizes. This occurs when the unit root pretest statistic is correlated with the statistic

calculated in the second step. Instead, Elliot and Stock propose basing inference on a mixture of the relevant $I(0)$ and $I(1)$ distributions in which the weights are given by posterior probabilities that the data is $I(0)$ or $I(1)$ conditional on the pretest. However, Elliot and Stock consider single-equation unit-root pretests, so it is not immediately clear whether their results carry over to the Horvath-Watson test that is considered here.

Although it turns out that the system analysis is in agreement with the long-horizon predictability exercise, we believe that the system approach is the simpler and more natural approach to studying the predictability question. Rather than assuming cointegration or no cointegration and then building up Monte Carlo critical values, one can instead estimate and test λ_1 and λ_2 directly.

VI. Conclusion

Economists have long conjectured that economic fundamentals are important determinants of nominal exchange rates. Unfortunately, empirical evidence has so far proven elusive. We believe that the available evidence on this matter is not conclusive and, in particular, that recent results obtained from long-horizon regressions must be carefully interpreted.

Our skepticism is motivated by several considerations: first, by ignoring an absence of evidence of any short-horizon relationships between fundamentals and exchange rates and focusing on the long horizons, the approach challenges the intuitive result that the long-horizon coefficients on error-correction terms are inherently linked to their short-horizon counterparts. We show that if the slope coefficient from a one-period regression is 0, the coefficients of the long-horizon regressions will also be 0, regardless of the length of the horizon. In this sense, nothing seems to be gained by running

TABLE 7.—ESTIMATES OF VECTOR ERROR-CORRECTION MODELS—EXTENDED SAMPLE (1973:2–1994:4)

| | λ_1 | λ_2 | Wald (HW) | Lag Length |
|-----------------|-----------------|-------------------|-----------|------------|
| Canadian Dollar | 0.031 (1.60) | -0.016 (-1.48) | 5.801 | 2 |
| German Mark | 0.033 (1.11) | 0.001 (0.11) | 1.233 | 2 |
| Japanese Yen | 0.057 (1.49) | -0.002 (-0.25) | 2.442 | 3 |
| Swiss Franc | 0.079 (1.98) | -0.014 (-1.78) | 7.842 | 2 |

The table presents estimated slope coefficients, λ_1 and λ_2 , of the vector error-correction system (3) with t -statistics in parentheses. For each regression, the number of lagged dependent variables is selected by the BIC. Wald(HW) refers to the Horvath and Watson (1995) t -statistic for cointegration with a known cointegrating vector. For the estimated bivariate system, the 90% critical value is 6.61, and the 95% critical value is 8.47.

a sequence of long-horizon regressions, absent evidence of nonlinear dynamics. We also show that, if the estimated one-step-ahead coefficient is nonzero, then estimated coefficients increase as the horizon increases. This implies that the empirical finding of increasing coefficients cannot be taken as evidence of a stronger impact of fundamentals on exchange rates as the horizon increases.

Second, by imposing cointegration between the spot rate and the monetary fundamental a priori and deriving empirical critical values under this assumption, an interpretation of the evidence of a statistical relationship between fundamentals and exchange rates errs on the side of significance.

We demonstrate that Monte Carlo critical values tabulated under the null hypothesis that cointegration does not hold are much higher. We compare out-of-sample forecasts for four U.S. dollar exchange rates generated from long-horizon regressions to those critical values and find weak evidence of predictability at predominantly short horizons. For none of the exchange rates considered do we find long-horizon predictability without short-horizon predictability.

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