

Comments on
“On the Internal Inconsistency of the Black-Scholes Option Pricing Model”

By Jeremy Berkowitz

General assessment:

This paper is quite difficult to assess for the following reasons:

- (i) It is mathematically correct. In particular, it makes a technically valid point about a shortcoming in Black and Scholes initial exposition of their seminal pricing approach.
- (ii) However, not only Black and Scholes pricing has been tremendously successful within the last 30 years but even more importantly, I will argue that it has been successful for good reasons. In other words, I feel that it is worth understanding Black and Scholes pricing in a broader context which can restore its validity.

Therefore, logic can lead us several ways and I cannot forcefully argue in favor or against the publication of this paper in *Econometrica*. It is unfortunately an empirical question to know whether the *Econometrica* readership has still something important to learn from this paper: are *Econometrica* readers already aware of the correct interpretation of Black and Scholes pricing? My prior probability rather leads me to answer “yes” and thus to think that the analysis is not sufficiently innovative to warrant a publication in *Econometrica*. In any case, a publication would at least require a dramatic revision to better take into account the modern ways of understanding Black and Scholes option pricing.

Technical comments:

It is now well known that Black and Scholes option pricing formula is valid as soon as the conditional joint distribution of the log of the terminal payoff of the underlying asset and the log of pricing kernel is normal. A number of examples of this general principle are for instance reviewed in a recent chapter on econometrics of option pricing in the forthcoming handbook of financial econometrics (available on L.P.H. Hansen’s website). This general point of view warrants the validity of Black and Scholes pricing without “internal inconsistency”.

Now, it is true that, as already pointed out by Buraschi and Jackwerth (2001) (RFS, 14, 495-527), the original Black and Scholes setting corresponds to the degenerate case where the log-pricing kernel is an affine function of the log-stock price, described by a Brownian Motion. In that respect, it is true that there is some kind of “internal inconsistency” since it is never explained where the risk in the underlying asset price comes from: it should come from a pricing formula which is itself based on the randomness of the pricing kernel, that is of the underlying asset price, kind of vicious circle.

I wonder however whether it is so important to point this out.

First, this observation is not really new in the literature since Buraschi and Jackwerth (2001) have already pointed out the degeneracy, even though they did not call it “internal inconsistency”.

Second, it is easy to get rid of this degeneracy by imagining any kind of additional risky state variable, like a terminal dividend payment, risky interest rate, conditional heteroskedasticity or anything else. Of course, this means that we slightly extend the original spirit of the 35 years-old Black and Scholes’ paper but I guess that everybody implicitly does that.