Chapter 8 Mechanical Failure

• How do flaws in a material initiate failure?
• How is fracture resistance quantified; how do different material classes compare?
• How do we estimate the stress to fracture?
• How do loading rate, loading history, and temperature affect the failure stress?

Ship-cyclic loading from waves.

Computer chip-cyclic thermal loading.

Hip implant-cyclic loading from walking.
DUCTILE VS BRITTLE FAILURE

• Classification:

Fracture behavior:  

<table>
<thead>
<tr>
<th>Very Ductile</th>
<th>Moderately Ductile</th>
<th>Brittle</th>
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| %AR or %EL:  

| Large       | Moderate            | Small   |
| Ductile: warning before fracture |                | Brittle: No warning |

• Ductile fracture is desirable!
EX: FAILURE OF A PIPE

• **Ductile** failure:
  -- one piece
  -- large deformation

• **Brittle** failure:
  -- many pieces
  -- small deformation
MODERATELY DUCTILE FAILURE

• Evolution to failure:
  - necking
  - void nucleation
  - void growth and linkage
  - shearing at surface
  - fracture

\[ \sigma \]

• Resulting fracture surfaces (steel)

Particles serve as void nucleation sites.

Fracture surface of tire cord wire loaded in tension.

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BRITTLE FRACTURE SURFACES

• Intergranular (between grains)
  - 304 S. Steel (metal)
  - Polypropylene (polymer)

• Intragranular (within grains)
  - 316 S. Steel (metal)
  - Al Oxide (ceramic)
IDEAL VS REAL MATERIALS

• Stress-strain behavior (Room T):

- \( \sigma \) vs. \( \varepsilon \)
  - E/10 (perfect mat’l-no flaws)
  - E/100 (carefully produced glass fiber)
  - Typical ceramic
  - Typical strengthened metal
  - Typical polymer

- TS\text{engineering} \ll TS\text{perfect materials}

• DaVinci (500 yrs ago!) observed...
  --the longer the wire, the smaller the load to fail it.

• Reasons:
  --flaws cause premature failure.
  --Larger samples are more flawed!
FLAWS ARE STRESS CONCENTRATORS!

• Elliptical hole in a plate:

• Stress distrib. in front of a hole:

• Stress conc. factor: \( K_t = \frac{\sigma_{\text{max}}}{\sigma_o} \)

• Large \( K_t \) promotes failure:

\[
\sigma_{\text{max}} \approx \sigma_o \left(2 \sqrt{\frac{a}{\rho_t}} + 1\right)
\]
ENGINEERING FRACTURE DESIGN

• Avoid sharp corners!

\[ \frac{\sigma_{\text{max}}}{\sigma_o} \]

Stress Conc. Factor, \( K = \) 

- Graph showing the relationship between stress concentration factor, \( K \), and the ratio \( r/h \). 
  - The graph illustrates how increasing \( w/h \) results in a sharper fillet radius, reducing stress concentration.
  - The formula \( K = \frac{\sigma_{\text{max}}}{\sigma_o} \) is used to calculate the stress concentration factor.
WHEN DOES A CRACK PROPAGATE?

• $\rho_t$ at a crack tip is very small!

• Result: crack tip stress is very large.

• Crack propagates when: the tip stress is large enough to make:

$$K \geq K_c$$

$\sigma_{\text{tip}}$ increasing $K$

$\sigma_{\text{tip}} = \frac{K}{\sqrt{2\pi x}}$
FRACTURE TOUGHNESS

• Griffith theory of brittle fracture

\[ \sigma_c = \left( \frac{2 E \gamma_s}{\pi a} \right) \]

Critical stress for crack propagation

Modulus of elasticity
Specific surface energy
One half the length of an internal crack

• Stress intensity factor

\[ K = Y \sigma \sqrt{\pi a} \]

Dimensionless function Y (crack and specimen size and geometries, load…)

• Fracture toughness

\[ K_c = Y \sigma_c \sqrt{\pi a} \]
GEOMETRY, LOAD, & MATERIAL

• Condition for crack propagation:

\[ K \geq K_c \]

Stress Intensity Factor:
--Depends on load & geometry.

Fracture Toughness:
--Depends on the material, temperature, environment, & rate of loading.

• Values of \( K \) for some standard loads & geometries:

\[ K = \sigma \sqrt{\pi a} \]

units of \( K \):
MPa\(\sqrt{m}\)
or ksi\(\sqrt{in}\)

\[ K = 1.1\sigma \sqrt{\pi a} \]
DESIGN AGAINST CRACK GROWTH

- Crack growth condition: $K \geq K_c$

$$Y\sigma\sqrt{\pi a}$$

- Largest, most stressed cracks grow first!

--Result 1: Max flaw size dictates design stress.

$$\sigma_{\text{design}} < \frac{K_c}{Y\sqrt{\pi a_{\text{max}}}}$$

--Result 2: Design stress dictates max. flaw size.

$$a_{\text{max}} < \frac{1}{\pi} \left( \frac{K_c}{Y\sigma_{\text{design}}} \right)^2$$

![Diagram of stress vs. flaw size]

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DESIGN EX: AIRCRAFT WING

• Material has $K_c = 26 \text{ MPa-m}^{0.5}$
• Two designs to consider...

  Design A
  --largest flaw is 9 mm
  --failure stress = 112 MPa

  Design B
  --use same material
  --largest flaw is 4 mm
  --failure stress = ?

• Use... $\sigma_c = \frac{K_c}{Y\sqrt{\pi a_{max}}}$

• Key point: $Y$ and $K_c$ are the same in both designs.
  --Result:

  \[
  \left( \sigma_c \sqrt{a_{max}} \right)_A = \left( \sigma_c \sqrt{a_{max}} \right)_B
  \]

• Reducing flaw size pays off!

Answer: \[
\left( \sigma_c \right)_B = 168 \text{ MPa}
\]
LOADING RATE

• Increased loading rate...
  --increases $\sigma_y$ and TS
  --decreases %EL

• Why? An increased rate gives less time for disl. to move past obstacles.

• Impact loading:
  --severe testing case
  --more brittle
  --smaller toughness
TEMPERATURE

- Increasing temperature...
  --increases %EL and $K_c$
- **Ductile-to-brittle transition temperature (DBTT)**...
DESIGN STRATEGY: STAY ABOVE THE DBTT!

• Pre-WWII: The Titanic


• WWII: Liberty ships


• Problem: Used a type of steel with a DBTT ~ Room temp.
**FATIGUE**

- **Fatigue** = failure under cyclic stress

- Stress varies with time.

- **Key points:** Fatigue...
  - can cause part failure, even though $\sigma_{\text{max}} < \sigma_c$.
  - causes ~ 90% of mechanical engineering failures.
FATIGUE KEY PARAMETERS

• Mean stress

\[ \sigma_m = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2} \]

• Range of stress

\[ \sigma_r = \sigma_{\text{max}} - \sigma_{\text{min}} \]

• Stress amplitude

\[ \sigma_a = \frac{\sigma_r}{2} = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} \]

• Stress ratio

\[ R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} \]
The S-N CURVE

Fatigue strength, fatigue life, and fatigue limit

\[ S = \sigma_a \]
**FATIGUE DESIGN PARAMETERS**

- **Fatigue limit, \( S_{\text{fat}} \):**
  - no fatigue if \( S < S_{\text{fat}} \)

- **Sometimes, the fatigue limit is zero!**

**Diagram:**

- **Case for steel (typ.):**
  - \( S = \text{stress amplitude} \)
  - \( N = \text{Cycles to failure} \)
  - \( S_{\text{fat}} \)
  - unsafe
  - safe

- **Case for Al (typ.):**
  - \( S = \text{stress amplitude} \)
  - \( N = \text{Cycles to failure} \)
  - unsafe
  - safe
FATIGUE MECHANISM

• Crack initiation
• Crack propagation
FATIGUE MECHANISM

• Crack grows *incrementally*

\[
\frac{da}{dN} = (\Delta K)^m \quad \text{typical 1 to 6}
\]

\[
\sim (\Delta \sigma)\sqrt{a} (\Delta K = Y(\sigma_{\text{max}} - \sigma_{\text{min}})\sqrt{\pi a})
\]

increase in crack length per loading cycle

• Failed rotating shaft
  --crack grew even though \(K_{\text{max}} < K_c\)
  --crack grows faster if
    • \(\Delta \sigma\) increases
    • crack gets longer
    • loading frequency increases.

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IMPROVING FATIGUE LIFE

1. Impose a compressive surface stress (to suppress surface cracks from growing)

---Method 1: shot peening

---Method 2: carburizing

2. Remove stress concentrators.

\[ S = \text{stress amplitude} \]
\[ N = \text{Cycles to failure} \]
CREEP

- Occurs at elevated temperature, $T > 0.4 \, T_{\text{melt}}$
- Deformation changes with time.

![Diagram](image)

- Primary
- Secondary
- Tertiary
- Instantaneous deformation
- Creep strain, $\epsilon$
- Time, $t$
- Rupture
CREEP

• Temperature dependence
• Stress dependence
SECONDARY CREEP

• Strain rate is constant at a given $T$, $\sigma$
  --strain hardening is balanced by recovery

\[
\dot{\varepsilon}_s = K_2 \sigma^n \exp\left(-\frac{Q_c}{RT}\right)
\]

• Strain rate increases for larger $T$, $\sigma$
CREEP FAILURE

• Failure: along grain boundaries.

• Time to rupture, $t_r$
  \[ T(20 + \log t_r) = L \]
  
  - Temperature
  - Function of applied stress
  - Time to failure (rupture)

• Estimate rupture time
  S 590 Iron, $T = 800^\circ C$, $\sigma = 20$ ksi

  \[ T(20 + \log t_r) = L \]
  \[ L(10^3 K - \log hr) = 24x10^3 K - \log hr \]

  Ans: $t_r = 233 hr$