

## CHAPTER 6 PRODUCTION

### EXERCISES

**1. Suppose a chair manufacturer is producing in the short run when equipment is fixed. The manufacturer knows that as the number of laborers used in the production process increases from 1 to 7, the number of chairs produced changes as follows: 10, 17, 22, 25, 26, 25, 23.**

**a. Calculate the marginal and average product of labor for this production function.**

The average product of labor,  $AP_L$ , is equal to  $\frac{Q}{L}$ . The marginal product of labor,  $MP_L$ , is equal to  $\frac{\Delta Q}{\Delta L}$ , the change in output divided by the change in labor input. For this production process we have:

$L$	$Q$	$AP_L$	$MP_L$
0	0	—	—
1	10	10	10
2	17	8 1/2	7
3	22	7 1/3	5
4	25	6 1/4	3
5	26	5 1/5	1
6	25	4 1/6	-1
7	23	3 2/7	-2

**b. Does this production function exhibit diminishing returns to labor? Explain.**

This production process exhibits diminishing returns to labor. The marginal product of labor, the extra output produced by each additional worker, diminishes as workers are added, and is actually negative for the sixth and seventh workers.

**c. Explain intuitively what might cause the marginal product of labor to become negative.**

Labor's negative marginal product for  $L > 5$  may arise from congestion in the chair manufacturer's factory. Since more laborers are using the same, fixed amount of capital, it is possible that they could get in each other's way, decreasing efficiency and the amount of output.

**2. Fill in the gaps in the table below.**

Quantity of Variable Input	Total Output	Marginal Product of Variable Input	Average Product of Variable Input
0	0	---	---
1	150		
2			200
3		200	
4	760		
5		150	
6			150

Quantity of Variable Input	Total Output	Marginal Product of Variable Input	Average Product of Variable Input
0	0	---	---
1	150	150	150
2	400	250	200
3	600	200	200
4	760	160	190
5	910	150	182
6	900	-10	150

**4. A firm has a production process in which the inputs to production are perfectly substitutable in the long run. Can you tell whether the marginal rate of technical substitution is high or low, or is further information necessary? Discuss.**

The marginal rate of technical substitution,  $MRTS$ , is the absolute value of the slope of an isoquant. If the inputs are perfect substitutes, the isoquants will be linear. To calculate the slope of the isoquant, and hence the  $MRTS$ , we need to know the rate at which one input may be substituted for the other.

**5. The marginal product of labor is known to be greater than the average product of labor at a given level of employment. Is the average product increasing or decreasing? Explain.**

If the marginal product of labor,  $MP_L$ , is greater than the average product of labor,  $AP_L$ , then each additional unit of labor is more productive than the average of the previous units. Therefore, by adding the last unit, the overall average increases. If  $MP_L$  is greater than  $AP_L$ , then  $AP_L$  is increasing. If the  $MP_L$  is lower than the  $AP_L$ , then the last unit reduces the average. The  $AP_L$  is at a maximum when the productivity of the last unit is equal to the average of the previous units (i.e., when  $MP_L = AP_L$ ).

**6. The marginal product of labor in the production of computer chips is 50 chips per hour. The marginal rate of technical substitution of hours of labor for hours of machine-capital is 1/4. What is the marginal product of capital?**

The marginal rate of technical substitution is defined as the ratio of the two marginal products. Here, we are given the marginal product of labor and the marginal rate of technical substitution. To determine the marginal product of capital, substitute the given values for the marginal product of labor and the marginal rate of technical substitution into the following formula:

$$\frac{MP_L}{MP_K} = MRTS, \text{ or } \frac{50}{MP_K} = \frac{1}{4}, \text{ or}$$

$MP_K = 200$  computer chips per hour.

**7. Do the following production functions exhibit decreasing, constant or increasing returns to scale?**

**a.  $Q = 0.5KL$**

Returns to scale refers to the relationship between output and proportional increases in all inputs. This concept may be represented in the following manner, where  $\lambda$  represents a proportional increase in inputs:

$F(\lambda K, \lambda L) > \lambda F(K, L)$  implies increasing returns to scale;

$F(\lambda K, \lambda L) = \lambda F(K, L)$  implies constant returns to scale; and

$F(\lambda K, \lambda L) < \lambda F(K, L)$  implies decreasing returns to scale.

Therefore, we can substitute  $\lambda K$  for  $K$  and  $\lambda L$  for  $L$ , and check the result against an equal increase in  $Q$ .

$$Q^* = 0.5(\lambda K)(\lambda L) = (0.5KL)\lambda^2 = Q\lambda^2 > \lambda Q$$

This production function exhibits increasing returns to scale.

**b.  $Q = 2K + 3L$**

$$Q^* = 2(\lambda K) + 3(\lambda L) = (2K + 3L)\lambda = Q\lambda = \lambda Q.$$

This production function exhibits constant returns to scale.

**8. The production function for the personal computers of DISK, Inc., is given by**

**$Q = 10K^{0.5}L^{0.5}$ , where  $Q$  is the number of computers produced per day,  $K$  is hours of machine time, and  $L$  is hours of labor input. DISK's competitor, FLOPPY, Inc., is using the production function  $Q = 10K^{0.6}L^{0.4}$ .**

**a. If both companies use the same amounts of capital and labor, which will generate more output?**

Let  $Q$  be the output of DISK, Inc.,  $Q_2$  be the output of FLOPPY, Inc., and  $X$  be the same equal amounts of capital and labor for the two firms. Then, according to their production functions,

$$Q = 10X^{0.5}X^{0.5} = 10X^{(0.5 + 0.5)} = 10X$$

and

$$Q_2 = 10X^{0.6}X^{0.4} = 10X^{(0.6 + 0.4)} = 10X.$$

Because  $Q = Q_2$ , both firms generate the same output with the same inputs. Note that if the two firms both used the same amount of capital and the same amount of labor, but the amount of capital was not equal to the amount of labor, then the two firms would not produce the same level of output. In fact, if  $K > L$  then  $Q_2 > Q$ .

**b. Assume that capital is limited to 9 machine hours but labor is unlimited in supply. In which company is the marginal product of labor greater? Explain.**

With capital limited to 9 machine units, the production functions become  $Q = 30L^{0.5}$  and  $Q_2 = 37.37L^{0.4}$ . To determine the production function with the highest marginal productivity of labor, consider the following table:

$L$	$Q$ Firm 1	$MP_L$ Firm 1	$Q$ Firm 2	$MP_L$ Firm 2
0	0.0	---	0.00	---
1	30.00	30.00	37.37	37.37
2	42.43	12.43	49.31	11.94
3	51.96	9.53	58.00	8.69
4	60.00	8.04	65.07	7.07

For each unit of labor above 1, the marginal productivity of labor is greater for the first firm, DISK, Inc.

**9. In Example 6.3, wheat is produced according to the production function**

$$Q = 100(K^{0.8}L^{0.2}).$$

- a. Beginning with a capital input of 4 and a labor input of 49, show that the marginal product of labor and the marginal product of capital are both decreasing.**

For fixed labor and variable capital:

$$K = 4 \Rightarrow Q = (100)(4^{0.8})(49^{0.2}) = 660.21$$

$$K = 5 \Rightarrow Q = (100)(5^{0.8})(49^{0.2}) = 789.25 \Rightarrow MP_K = 129.04$$

$$K = 6 \Rightarrow Q = (100)(6^{0.8})(49^{0.2}) = 913.19 \Rightarrow MP_K = 123.94$$

$$K = 7 \Rightarrow Q = (100)(7^{0.8})(49^{0.2}) = 1,033.04 \Rightarrow MP_K = 119.85.$$

For fixed capital and variable labor:

$$L = 49 \Rightarrow Q = (100)(4^{0.8})(49^{0.2}) = 660.21$$

$$L = 50 \Rightarrow Q = (100)(4^{0.8})(50^{0.2}) = 662.89 \Rightarrow MP_L = 2.68$$

$$L = 51 \Rightarrow Q = (100)(4^{0.8})(51^{0.2}) = 665.52 \Rightarrow MP_L = 2.63$$

$$L = 52 \Rightarrow Q = (100)(4^{0.8})(52^{0.2}) = 668.11 \Rightarrow MP_L = 2.59.$$

Notice that the marginal products of both capital and labor are decreasing as the variable input increases.

- b. Does this production function exhibit increasing, decreasing, or constant returns to scale?**

Constant (increasing, decreasing) returns to scale imply that proportionate increases in inputs lead to the same (more than, less than) proportionate increases in output. If we were to increase labor and capital by the same proportionate amount ( $\lambda$ ) in this production function, output would change by the same proportionate amount:

$$\lambda Q = 100(\lambda K)^{0.8}(\lambda L)^{0.2}, \text{ or}$$

$$\lambda Q = 100K^{0.8}L^{0.2}\lambda^{(0.8+0.2)} = Q\lambda$$

Therefore, this production function exhibits constant returns to scale.