# CHAPTER 4 <br> INDIVIDUAL AND MARKET DEMAND 

## EXERCISES

1. The ACME corporation determines that at current prices the demand for its computer chips has a price elasticity of -2 in the short run, while the price elasticity for its disk drives is -1.
a. If the corporation decides to raise the price of both products by 10 percent, what will happen to its sales? To its sales revenue?

We know the formula for the elasticity of demand is:

$$
E_{P}=\frac{\% \Delta Q}{\% \Delta P}
$$

For computer chips, $E_{P}=-2$, so a 10 percent increase in price will reduce the quantity sold by 20 percent. For disk drives, $E_{P}=-1$, so a 10 percent increase in price will reduce sales by 10 percent.
Sales revenue is equal to price times quantity sold. Let $T R_{1}=P_{1} Q_{1}$ be revenue before the price change and $T R_{2}=P_{2} Q_{2}$ be revenue after the price change.

For computer chips:

$$
\begin{gathered}
\Delta T R_{c c}=P_{2} Q_{2}-P_{1} Q_{1} \\
\Delta T R_{c c}=\left(1.1 P_{1}\right)\left(0.8 Q_{1}\right)-P_{1} Q_{1}=-0.12 P_{1} Q_{1}, \text { or a } 12 \text { percent decline. }
\end{gathered}
$$

For disk drives:

$$
\begin{gathered}
\Delta T R_{d d}=P_{2} Q_{2}-P_{1} Q_{1} \\
\Delta T R_{d d}=\left(1.1 P_{1}\right)\left(0.9 Q_{1}\right)-P_{1} Q_{1}=-0.01 P_{1} Q_{1}, \text { or a } 1 \text { percent decline. }
\end{gathered}
$$

Therefore, sales revenue from computer chips decreases substantially, -12 percent, while the sales revenue from disk drives is almost unchanged, -1 percent. Note that at the point on the demand curve where demand is unit elastic, total revenue is maximized.
b. Can you tell from the available information which product will generate the most revenue for the firm? If yes, why? If not, what additional information would you need?

No. Although we know the responsiveness of demand to changes in price, we need to know both quantities and prices of the products to determine total sales revenue.
2. Refer to Example 4.3 on the aggregate demand for wheat in 1998. Consider 1996, at which time the domestic demand curve was $Q_{D D}=1560-60 P$. The export demand curve, however, was about the same as in 1998, i.e., $Q_{D E}=1544-176 P$. Calculate and draw the aggregate demand curve for wheat in 1996.

Given the domestic demand curve for wheat is $Q_{D D}=1560-60 \mathrm{P}$, we find an intercept of 1560 on the quantity axis and an intercept of $\frac{1560}{60}=26$ on the price axis. The export demand curve for wheat, $\mathrm{Q}_{\mathrm{EE}}=1544-176 \mathrm{P}$, has an intercept of 1544 on the quantity axis and an intercept of $\frac{1544}{176}=8.77$ on the price axis. The total demand curve follows the domestic demand curve between the prices of $\$ 26$ and $\$ 8.77$ because the export demand is 0 in this range of prices. At $\$ 8.77$ and a quantity of approximately $1033.7=$ 1560 - (60)(8.77), the total demand curve kinks. As price drops below $\$ 8.77$, total demand is domestic demand plus export demand, which is the horizontal sum of the two
individual demand curves. Between a price of $\$ 26$ and $\$ 8.77$ the equation for total demand is $\mathrm{Q}_{2}=1560-60 \mathrm{P}$ and between a price of $\$ 8.77$ and zero, the equation for total demand is $\mathrm{Q}_{\mathrm{T}}=\mathrm{Q}_{\mathrm{DD}}+\mathrm{Q}_{\mathrm{DE}}=3104-236 \mathrm{P}$. See figure 4.2.


Figure 4.2
5. Suppose you are in charge of a toll bridge that is essentially cost free. The demand for bridge crossings $Q$ is given by $P=12-2 Q$.
a. Draw the demand curve for bridge crossings.

See figure 5.4 a below.
b. How many people would cross the bridge if there were no toll?

At a price of zero, the quantity demanded would be 6 .
c. What is the loss of consumer surplus associated with the charge of a bridge toll of \$6?

The consumer surplus with no toll is equal to $(0.5)(6)(12)=36$. Consumer surplus with a $\$ 6$ toll is equal to $(0.5)(3)(6)=9$, illustrated in Figure 4.4.a. Therefore, the loss of consumer surplus is $\$ 27$.


Figure 5.4.a
6.a. Orange juice and apple juice are known to be perfect substitutes. Draw the appropriate price-consumption (for a variable price of orange juice) and incomeconsumption curves.

We know that the indifference curves for perfect substitutes will be straight lines. In this case, the consumer will always purchase the cheaper of the two goods. If the price of orange juice is less than that of apple juice, the consumer will purchase only orange juice and the price consumption curve will be on the "orange juice axis" of the graph. If apple juice is cheaper, the consumer will purchase only apple juice and the price consumption curve will be on the "apple juice axis." If the two goods have the same price, the consumer will be indifferent between the two; the price consumption curve will coincide with the indifference curve. See Figure 4.6.a.i.


Figure 4.6.a.i

Assuming that the price of orange juice is less than the price of apple juice, the consumer will maximize her utility by consuming only orange juice. As the level of income varies, only the amount of orange juice varies. Thus, the income consumption curve will be the "orange juice axis" in Figure 4.6.a.ii.


Figure 4.6.a.ii

## 5.b. Left shoes and right shoes are perfect complements. Draw the appropriate priceconsumption and income-consumption curves.

For goods that are perfect complements, such as right shoes and left shoes, we know that the indifference curves are $L$-shaped. The point of utility maximization occurs when the budget constraints, $L_{1}$ and $L_{2}$ touch the kink of $U_{1}$ and $U_{2}$. See Figure 4.6.b.i.


Figure 4.6.b.i
In the case of perfect complements, the income consumption curve is a line through the corners of the $L$-shaped indifference curves. See Figure 4.6.b.ii.


Figure 4.6.b.ii
7. Heather's marginal rate of substitution of movie tickets for rental videos is known to be the same no matter how many rental videos she wants. Draw Heather's income consumption curve and her Engel curve for videos.

If we let the price of movie tickets be less than the price of a video rental, the budget constraint, $L$, will be flatter than the indifference curve for the substitute goods, movie tickets and video rentals. The income consumption curve will be on the "video axis," since she only consumes videos. See Figure 4.7.a.


Figure 4.7.a

Heather's Engel curve shows that her consumption of video rentals increases as her income rises, and thus the slope of her Engel curve is equal to the price of a video rental. See Figure 4.7.b.


Figure 4.7.b
8. You are managing a $\$ 300,000$ city budget in which monies are spent on schools and public safety only. You are about to receive aid from the federal government to support a special anti-drug law enforcement program. Two programs that are available are (1) a $\$ 100,000$ grant that must be spent on law enforcement; and (2) a 100 percent matching grant, in which each dollar of local spending on law enforcement is matched by a dollar of federal money. The federal matching program limits its payment to each city to a maximum of $\$ 100,000$.
a. Complete the table below with the amounts available for safety.

## a. $\quad$ See Table 4.8.a.

| SCHOOLS | SAFETY <br> No Govt. <br> Assistance | SAFETY <br> Program (1) | SAFETY <br> Program (2) |
| ---: | :---: | :---: | :---: |
| $\$ 0$ | $\$ 300,000$ | $\$ 400,000$ | $\$ 400,000$ |
| $\$ 50,000$ | $\$ 250,000$ | $\$ 350,000$ | $\$ 350,000$ |
| $\$ 100,000$ | $\$ 200,000$ | $\$ 300,000$ | $\$ 300,000$ |
| $\$ 150,000$ | $\$ 150,000$ | $\$ 250,000$ | $\$ 250,000$ |
| $\$ 200,000$ | $\$ 100,000$ | $\$ 200,000$ | $\$ 200,000$ |
| $\$ 250,000$ | $\$ 50,000$ | $\$ 150,000$ | $\$ 100,000$ |
| $\$ 300,000$ | $\$ 0$ | $\$ 100,000$ | $\$ 0$ |

Table 4.8.a
b. Which program would you (the manager) choose if you wish to maximize the satisfaction of the citizens if you allocate $\mathbf{\$ 5 0 , 0 0 0}$ of the $\mathbf{\$ 3 0 0 , 0 0 0}$ to schools? What about $\mathbf{\$ 2 5 0 , 0 0 0}$ ?
With $\$ 50,000$ to schools and $\$ 250,000$ to law enforcement, both aid programs yield the same amount, $\$ 100,000$, so you are indifferent between the programs. With $\$ 250,000$ to schools and $\$ 50,000$ to law enforcement, program (1) yields $\$ 100,000$ (for a total of $\$ 150,000$ ) and program (2) yields $\$ 50,000$ (for a total of $\$ 100,000$ ), so you prefer program (1).
c. Draw the budget constraints for the three options: no aid, program (1), or program (2).


Figure 4.8.c

With no aid, the budget constraint is the line segment $A B$, from $\$ 300,000$ for schools and nothing for law enforcement to $\$ 300,000$ for law enforcement and nothing for schools. With program (1), the budget constraint, $A C E$, has two line segments, one parallel to the horizontal axis, until expenditures on safety equal $\$ 100,000$, and a second sloping downward until $\$ 400,000$ is spent on safety. With program (2), the budget constraint, $A D E$, has two line segments, one from ( $\$ 0, \$ 300,000$ ) to ( $\$ 200,000, \$ 200,000$ ) and another from $(\$ 200,000, \$ 200,000)$ to $(\$ 400,000, \$ 0)$.
9. By observing an individual's behavior in the situations outlined below, determine the relevant income elasticities of demand for each good (i.e., whether the good is normal or inferior). If you cannot determine the income elasticity, what additional information might you need?
a. Bill spends all his income on books and coffee. He finds $\$ 20$ while rummaging through a used paperback bin at the bookstore. He immediately buys a new hardcover book of poetry.

Books are a normal good since his consumption of books increases with income. Coffee is a normal or neutral good since consumption of coffee did not fall when income increased.
b. Bill loses $\$ 10$ he was going to use to buy a double espresso. He decides to sell his new book at a discount to his friend and use the money to buy coffee.
Coffee is clearly a normal good.
c. Being bohemian becomes the latest teen fad. As a result, coffee and book prices rise by 25 percent. Bill lowers his consumption of both goods by the same percentage.

Books and coffee are both normal goods since his response to a decline in real income is to decrease consumption of both goods.
d. Bill drops out of art school and gets an M.B.A. instead. He stops reading books and drinking coffee. Now he reads The Wall Street Journal and drinks bottled mineral water.

His tastes have changed completely, and we do not know why. We could use more information regarding his level of income, his desire for sleep, and maybe even a change in political affiliation.
10. Suppose the income elasticity of demand for food is 0.5 , and the price elasticity of demand is -1.0. Suppose also that Felicia spends $\$ 10,000$ a year on food, and that the price of food is $\mathbf{\$ 2}$ and her income is $\mathbf{\$ 2 5 , 0 0 0}$.
a. If a $\$ 2$ sales tax on food were to cause the price of food to double, what would happen to her consumption of food? (Hint: Since a large price change is involved, you should assume that the price elasticity measures an arc elasticity, rather than a point elasticity.)
The price of food doubles from $\$ 2$ to $\$ 4$, so arc elasticity should be used:

$$
E_{P}=\left(\frac{\Delta Q}{\Delta P}\right)\left(\frac{\frac{P_{1}+P_{2}}{2}}{\frac{Q_{1}+Q_{2}}{2}}\right)
$$

We know that $E_{P}=-1, P=2, \Delta P=2$, and $\mathrm{Q}=5000$. We also know that Q , the new quantity, is $Q+\Delta Q$. Thus, if there is no change in income, we may solve for $\Delta Q$ :

$$
-1=\left(\frac{\Delta Q}{2}\right)\left(\frac{\frac{2+4}{2}}{\frac{5,000+(5,000+\Delta Q)}{2}}\right)
$$

By cross-multiplying and rearranging terms, we find that $\Delta Q=-2,500$. This means that she decreases her consumption of food from 5,000 to 2,500 units.
b. Suppose that she is given a tax rebate of $\$ 5,000$ to ease the effect of the tax. What would her consumption of food be now?

A tax rebate of $\$ 5,000$ implies an income increase of $\$ 5,000$. To calculate the response of demand to the tax rebate, use the definition of the arc elasticity of income.

$$
E_{I}=\left(\frac{\Delta Q}{\Delta I}\right)\left(\frac{\frac{I_{1}+I_{2}}{2}}{\frac{Q_{1}+Q_{2}}{2}}\right)
$$

We know that $E_{I}=0.5, I=25,000, \Delta I=5,000, Q=2,500$ (from the answer to $10 . \mathrm{a}$ ). Assuming no change in price, we solve for $\Delta Q$.

$$
0.5=\left(\frac{\Delta Q}{5,000}\right)\left(\frac{\frac{25,000+30,000}{2}}{\frac{2,500+(2,500+\Delta Q)}{2}}\right)
$$

By cross-multiplying and rearranging terms, we find that $\Delta Q=238$ (approximately). This means that she increases her consumption of food from 2,500 to 2,738 units.
c. Is she better or worse off when given a rebate equal to the sales tax payments? Discuss.

We want to know if her original indifference curve lies above or below her final indifference curve after the sales tax and after the tax rebate. On her final indifference curve, she chooses to consume 2,738 units of food (for $\$ 10,952$ ) and $\$ 19,048$ of other goods. Was this combination attainable with her original budget? At the original food price of $\$ 2$, this combination would have cost her $(2,738)(\$ 2)+\$ 19,048=\$ 24,524$, thus leaving her an extra $\$ 476$ to spend on either food or other consumption. Therefore, she would have been better off before the sales tax and tax rebate. She could have purchased more of both food and other goods than she could have after the taxes.
11. Suppose that you are the consultant to an agricultural cooperative that is deciding whether members should cut their production of cotton in half next year. The cooperative wants your advice as to whether this will increase the farmers' revenues. Knowing that cotton (C) and watermelons (W) both compete for agricultural land in the South, you estimate the demand for cotton to be:
$\mathbf{C = 3 . 5 - 1 . 0} \mathrm{P}_{\mathrm{C}}+\mathbf{0 . 2 5} \mathrm{P}_{\mathrm{w}}+\mathbf{0 . 5 0 I}$,
where $P_{c}$ is the price of cotton, $P_{w}$ the price of watermelon, and I income. Should you support or oppose the plan? Is there any additional information that would help you to provide a definitive answer?

If production of cotton is cut in half, then the price of cotton will increase, given that we see from the equation above that demand is downward sloping. With price increasing
and quantity demanded decreasing, revenue could go either way. It depends on whether demand is inelastic or elastic at the current price. If demand is inelastic then a decrease in production and an increase in price could increase revenue. If demand is elastic then a decrease in production and an increase in price will clearly decrease revenue. You need to know the current price and/or quantity demanded to figure out the current level of elasticity.

