The sample predicate argument discussed above ("Cup I") is quite simple. Some predicate arguments, of course, are more complex. Here is an example contained in dialogue from the movie, *Jurassic Park*.[7]

PARK SCIENTIST: *Actually, they can't breed in the wild. Population control is one of our security precautions. There's no unauthorized breeding [among dinosaurs] in Jurassic Park.*

IAN MALCOLM: *How do you know they can't breed?*

PARK SCIENTIST: *Oh, because all of the animals [i.e., dinosaurs] in Jurassic Park are female. We've engineered them that way.*

The park scientist's argument:

> Two dinosaurs can breed only if one is female and one is not.
> All of the dinosaurs in Jurassic Park are female.
> So, the dinosaurs in Jurassic Park can't breed.

If you saw the movie you know that the conclusion of this argument is false (according to the story line), even though the argument is valid. That can happen because the second premise is false. As the theme of the movie puts the matter, *nature will find a way*. One feature of the argument that adds to its complexity is that it involves the relational predicate *breeds*. We will examine the structure of the Jurassic Park argument in Chapters Ten and Eleven.

The good news for students embarking on the study of predicate logic is that predicate logic and propositional logic are intimately connected. Having mastered propositional logic you are already well on your way toward learning predicate logic. For example, all of the symbols of propositional logic appear in the formulas of predicate logic, and all of the propositional inference rules and truth tree rules are also employed in predicate logic. (Obviously it is essential that a person studying this book know the logic of propositions.) In the chapters that follow, I will develop a formal system of predicate logic by grafting new "branches" onto the "trunk" of propositional logic. The symbols of predicate logic will be added to your vocabulary in Chapter Two. Four predicate inference rules are introduced in Chapters Three and Four to accompany the eighteen propositional rules listed on page 278. In Chapter Eight, four predicate truth-tree rules are added to the ten propositional rules listed on page 280. In the last third of the book I extend predicate logic to encompass relational predicates.

[7]Universal Pictures, 1993. Directed by Steven Spielberg. Screenplay by Michael Crichton and David Koepp, based on the book by Michael Crichton.

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2 Basic Symbolization

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This chapter treats sentences of the following five types:

> All watermelons are vegetables.
> No Lutherans are Methodists.
> Some computers are antiques.
> Some numbers are not odd.
> Kelly is ambitious.

The point of the chapter is to teach you how to symbolize such sentences. Many sentences that you encounter (in this book and in your life) exhibit one of these five forms. So, it is well worth knowing how to analyze them. Many other sentences are a lot like these five types of sentences, so learning how to symbolize the basic five will help you grasp the structure of these sentences.

Nearly twenty-four hundred years ago the first systematic logician, Aristotle, studied sentences of the first four types,[1] and today they are still recognized by logicians as being of fundamental importance. Logicians call them *standard categorical propositions*.

[1]Aristotle subsumed sentences of the fifth type under the first kind.

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2.1 Singular Statements

Many English sentences can be viewed as consisting of two parts: an expression that is used to refer to an individual and another expression that is used to ascribe a property to the individual. We will call expressions of the former sort *singular terms* and expressions of the latter kind *predicates*. Sentences composed of singular terms and predicates are known as *singular statements*. Some examples:

Singular Statement	Singular Term	Predicate
George W. Bush is a Republican.	George W. Bush	is a Republican
Stephanie's dog is a rottweiler.	Stephanie's dog	is a rottweiler
He sings poorly.	he	sings poorly
The richest person lives in Seattle.	the richest person	lives in Seattle

The key characteristic of a singular term is that it is customarily used to refer to an individual. I use *individual* broadly, counting not only people but pets, rivers, rocks, roads, bridges, cars, cities, planets, numbers, and so on, as individuals. Singular terms are expressions that function like proper nouns, but the concept is a logical, not a grammatical one. Singular terms may be proper nouns (*Shakespeare*), pronouns (*she*), or noun phrases (*David's right ear, the janitor*). In predicate logic we abbreviate singular terms with lower-case letters of the alphabet from *a* through *v*. (The letters *w, x, y,* and *z* are reserved for another use, which is explained in the next section.) Normally the letter chosen as an abbreviation will be the first letter of a prominent word occurring in the singular term; for example, *the janitor* will typically be abbreviated with a *j*. Let's call the letters that abbreviate singular terms *individual constants*.

A predicate or general term is an expression that may be used to ascribe a property (such as *being a cartoonist*) to an individual or to assert that several individuals stand in some relationship (like *hating*). At present we will concentrate on property predicates, postponing our treatment of relational predicates until Chapter Ten. Predicates may be composed of various parts of speech. Some examples:

Predicate	Part of Speech
sleeps	verb
sleeps poorly	verb + adverb
speaks German	verb + noun
is greedy	copula[2] + adjective
is a Texan	copula + noun phrase

[2]A copula is a word or expression (such as a form of the verb *to be*) that links the subject of a sentence with its grammatical predicate without asserting action.

It will become clear as we proceed that our concept of *predicate* does not correspond exactly to the grammarian's notion. Predicates are abbreviated in our logic by capital letters. The letter selected will usually be the first letter of one of the words comprising the predicate; for example, *is a Texan* will typically be abbreviated by *T*. We call the letters that abbreviate English predicates *predicate letters* (or just *predicates*). A predicate letter in a wff (well-formed formula) will always be followed by one or more lower-case letters. (A capital letter standing alone in a wff is not a predicate letter but the abbreviation of a statement.)

To symbolize an (affirmative) singular statement in the notation of predicate logic, we write the capital that abbreviates the predicate followed by the lower-case letter that serves as the abbreviation of the singular term. S1 is symbolized by F1.

(S1) Stephanie's [d]og is a (r)ottweiler.
(F1) Rd

I use two conventions for designating abbreviations: (1) placing square brackets around a letter in the singular term indicates that it will serve as the individual constant that abbreviates the term, and (2) enclosing in parentheses a letter in the predicate shows that it will serve as the predicate letter. Notice that the predicate-first order displayed by F1 is not the standard pattern of English sentences.

Some singular statements are negative; S2 is an example.

(S2) Thomas [E]dison did not attend (c)ollege.

A negative singular statement is symbolized as the negation of an affirmative one. S2 is symbolized by F2.

(F2) –Ce

This is our first example of the use of a symbol of propositional logic in a predicate-logic wff.

2.2 Existential Statements (*I* and *O*)

Let's define a *singular statement* as a statement containing one or more singular terms. We shall call a statement that has no singular terms a *general statement*. (Does it follow from these definitions that every statement is either singular or general? Do the definitions imply that no statement is both?)[3] A singular

[3]Yes and yes. You will establish the correctness of these answers in an exercise in Chapter Six.

statement mentions a specified individual; a general statement concerns individuals of certain kinds or types but does not name any specific individual. S1 is a sample general statement:

(S1) Some rabbis are Japanese.

S1 concerns rabbis but it refers to no specific rabbi. In this regard S1 contrasts sharply with the singular statement S2.

(S2) Hiroshi Okamoto is a rabbi.

As I mentioned at the beginning of the chapter, logicians have long been interested in general statements (or *categorical propositions*) that exhibit any of four simple forms. These forms are given in the following table.

Standard Categorical Proposition Forms

Form	Example	Code Letter
All $\mathcal{F}$ are $\mathcal{G}$	All pines are conifers.	***A***
No $\mathcal{F}$ are $\mathcal{G}$	No snakes are warm-blooded.	***E***
Some $\mathcal{F}$ are $\mathcal{G}$	Some senators are principled.	***I***
Some $\mathcal{F}$ are not $\mathcal{G}$	Some psychologists are not atheists.	***O***

Obviously the script $\mathcal{F}$ and $\mathcal{G}$ in each statement-form mark gaps that are to be filled by predicates (like *pines* and *conifers*). Code letters have been used for many centuries to identify these four statement-forms. We will call *A* and *E* statements *universal* and *I* and *O* statements *existential.* (The traditional label for *I* and *O* statements is *particular,* but that term would seem to fit singular statements better than *I* and *O* statements.) In the remainder of this section we concentrate on statements of the *I* and *O* types. Section 2.3 treats statements of types *A* and *E*.

"I" Statements. In predicate logic a statement exhibiting one of the four basic general forms is regarded as containing two predicates. Consider again S1.

(S1) Some rabbis are Japanese.

Without altering the content of S1 we can rephrase it as S1′.

(S1′) There exists at least one individual who *is a rabbi* and *is Japanese.*

The two predicates are brought to the surface in S1′; they can be abbreviated with *R* and *J*. Two other symbols of predicate logic are required if we are to symbolize S1′: the *individual variable* (or *variable* for short) and the *existential quantifier.* We employ the lower-case letters *w, x, y,* and *z* as individual variables. If a wff requires more than four variables, we can form additional ones with the prime mark (thus *x*′ or *y*″). Until we reach Chapter Ten, however, we will manage with just one variable: *x*. The variable is a symbolic device for achieving cross-reference. The closest analogue in English to the variable is the pronoun. The existential quantifier consists of an inverted *E* followed by a variable; *∃x* is read *there exists an x such that.*

We now have the machinery required to symbolize S1′ (and thus S1).

There exists at least one individual who	is a rabbi	and	is Japanese
∃x	(Rx	&	Jx)

So, we reach F1 as our symbolization of S1.

(F1) ∃x(Rx & Jx)

F1 may be read *There exists an x such that x is a rabbi and x is Japanese.* Every *I* statement is symbolized in similar fashion. The pair of parentheses in F1 indicate that the scope of the quantifier includes all of "Rx & Jx" (and not just "Rx"). This matter will be explained further in Chapters Five and Thirteen. It is sufficient at present to note that each wff symbolizing an *I* statement (or an *O, A,* or *E*) has a left-hand parenthesis after the quantifier and a right-hand one as concluding symbol.

I statements exhibit both *vagueness* and *ambiguity.* As an example of the former, consider again S1:

(S1) Some rabbis are Japanese.

How many Japanese rabbis must there be if S1 is to express a truth? There is no definite answer to this question, if it is a question about English usage. However, in deciding to paraphrase S1 as S1′, we have stipulated (for the purposes of logic) that *one* Japanese rabbi is sufficient for the truth of S1.

(S1′) There exists at least one individual who is a rabbi and is Japanese.

As an example of the *ambiguity* of sentences of the *I*-type consider S3:

(S3) Some dogs are mammals.

Is S3 true? One who utters S3 may be intending to claim S4, or may be making the stronger claim expressed by S5.

(S4) At *least* some dogs are mammals.
(S5) Some dogs (and at *most* some dogs) are mammals.

Of course, S4 is true and S5 false. In logic, we will regard S3 as equivalent to S4; so, we count S3 a truth.

I statements can be disguised in different English garments. All of the statements in this list are regarded as *I* statements by logicians and are symbolized by F1.

At least one rabbi is Japanese.
Rabbis are sometimes Japanese.
There are rabbis who are Japanese.
Rabbis who are Japanese do exist.

The list is not exhaustive.

"O" Statements. Having learned how to symbolize *I* statements, it is an easy step to the symbolization of *O* statements. S6 serves as an example.

(S6) Some psychologists are not atheists.

S6 may be rephrased as S6′, which in turn is symbolized by F6.

(S6′) There exists an *x* such that *x* is a (p)sychologist and *x* is not an (a)theist.
(F6) ∃x(Px & −Ax)

Every *O* statement may be symbolized in this way. Notice that the dash is properly located after the ampersand. Neither of the following wffs symbolizes S6.

(F7) ∃x−(Px & Ax)
(F8) −∃x(Px & Ax)

F7 and F8 symbolize S7 and S8, respectively, and neither S7 nor S8 is equivalent to S6.

(S7) Someone is not both a psychologist and an atheist.
(S8) It is false that there is a psychologist who is an atheist.

O statements can be expressed in English in various ways, for example:

At least one psychologist is not an atheist.
There are psychologists who aren't atheists.

Each of these sentences is correctly symbolized by F6.

EXERCISES

Note: You can check your symbolizations of exercises with the computer tutorial "PredLogic."

1. (1) Symbolize each statement using the suggested abbreviations. (Each statement is either singular (affirmative or negative), an *I*, or an *O*.) (2) Provide a *dictionary* for the abbreviating symbols. For example, the symbolization and dictionary for (a) are:

   Pt (t = three, Px = x is prime)

   Of course, there is no need to provide a dictionary entry where I have already done so, as I have for *J* in exercise (c).

   (a) [T]hree is (p)rime.
   *(b) (*this text*) "Some (s)ingular statements are (n)egative."
   (c) Some statements that we are (j)ustified in believing are not (t)rue. (Jx = x is a statement that we are justified in believing)
   (d) (*The Koran*) "There are some who (d)ispute about Allah and (s)erve rebellious devils."
   (e) The [h]ighest point in the eastern United States is not in (T)ennessee. (Tx = x is in Tennessee) *Note that the use of "x" in this dictionary entry does not imply that "x" will appear in the symbolization of the sentence.*
   *(f) At least one (h)orse-drawn bus (o)perated in London after 1911. (Ox = x operated in London after 1911)
   (g) Some 1-800 (n)umbers are not (t)oll-free. (Tx = x is toll-free)
   (h) (G)olden (f)aucets exist.
   (i) (*children's book*) "There are . . . (p)eople who cannot (t)ell the difference between an alligator who is smiling and an alligator who is not smiling." (Px = x is a person, Tx = x can tell the difference between an alligator who is smiling and an alligator who is not smiling)
   *(j) (*Pliny the Elder*) "Individuals are occasionally born who belong to both sexes." (Mx = x is male, Fx = x is female)

2. Translate each wff into an English sentence using this dictionary: r = Janet Reno, Px = x is a politician, Ox = x lives in Ohio.

   (a) Pr
   *(b) −Or
   (c) ∃x(Px & Ox)
   (d) ∃x(Ox & Px)
   (e) ∃x(Px & −Ox)
   *(f) ∃x(Ox & −Px)

*Solutions (or partial solutions) to starred problems are provided in Appendix Four.

2.3 Universal Statements (*A* and *E*)

"A" Statements. *A* and *E* statements are called *universal* statements because they allow us to make claims about *all* individuals of a certain kind (that is, individuals possessing a certain property), either the claim that each possesses some further specified property or the claim that each lacks some specified property. The introduction of one more symbol of predicate logic—the *universal quantifier*—will enable us to symbolize *A* and *E* statements. (No additional symbols will be needed until we reach Chapter Ten.) The universal quantifier consists of an inverted *A* followed by a variable; $\forall x$ is read *for any x*. How shall we symbolize S1?

(S1) All pines are conifers.

As a first step we paraphrase S1 as S1′.

(S1′) For any individual, if it is a (p)ine, then it is a (c)onifer.

S1′ is composed exclusively of predicates and expressions that correspond to symbols in our logical vocabulary. We symbolize S1′ (and hence S1) with F1.

(F1) $\forall x(Px \rightarrow Cx)$

F1 is read *For any x, if x is a pine, then x is a conifer.* Any *A* statement may be symbolized in this fashion.

Why employ an arrow rather than an ampersand; why not symbolize S1 with F2?

(F2) $\forall x(Px \& Cx)$

F2 is read *For any x, x is a pine and x is a conifer.* It symbolizes S2, which, of course, does not have the same content as S1.

(S2) Everything is a pine and a conifer.

A statements (or sentences equivalent to such statements) occur very often in natural languages. It is not surprising then that English offers many ways of expressing *A* statements. Some are included in this list.

Every pine is a conifer.	A pine is a conifer.
Each pine is a conifer.	Pines are all conifers.
Each and every pine is a conifer.	Pines are always conifers.
Any pine is a conifer.	Pines are conifers.

Each of these sentences is properly symbolized by F1. Notice that the order of the predicates in an *A* wff is crucial. Switching the predicates of F1 yields F3.

(F3) $\forall x(Cx \rightarrow Px)$

But F3 does not represent S1 or any of the *A* variants listed above. F3 symbolizes (the false) S3.

(S3) All conifers are pines.

How should we symbolize S4?

(S4) Not all (F)orest Service trucks are (g)reen.

We can view S4 as an abbreviation of S5, and accordingly symbolize it with F5:

(S5) It is not the case that all Forest Service trucks are green.
(F5) $-\forall x(Fx \rightarrow Gx)$

Note that S4 has the same content as S6:

(S6) Some Forest Service trucks are not green.
(F6) $\exists x(Fx \& -Gx)$

A statements and *O* statements (having the same grammatical subjects and predicates) are exact opposites. Logicians term them *contradictories.* That *A*'s and *O*'s are contradictories explains the equivalence of S5 (or S4) and S6. Because S4 and S6 are equivalent we can use F6 to symbolize S4. So, we can symbolize S4 with either F5 or F6. Is one of these two symbolizations preferable? I favor F5 because it *tracks* S4 better than F6 does. (A wff *tracks* a sentence when the two have the same or comparable logical structures. The structure of a predicate logic wff is shown by the pattern of quantifiers, variables, connectives, and grouping symbols it contains.) For example, the universal quantifier (coupled with an arrow) in F5 corresponds to the word *all* in S4; the existential quantifier in F6 does not track *all.*

Two *A* variants involving the expressions *only* and *none but* are troublesome and deserve special treatment. S7 and S8 serve as representatives.

(S7) Only males are [Roman Catholic] priests.
(S8) None but males are priests.

Let's begin with S7. S7 has the same content as S9 and S10:

(S9) All (p)riests are (m)ales.
(F9) $\forall x(Px \rightarrow Mx)$

(S10) All non-males are non-priests.
(F10) $\forall x(-Mx \rightarrow -Px)$

Either F9 or F10 will serve as a symbolization of S7. It is hard to say which wff tracks S7 better; I prefer F9 because it is more concise. We can formulate this principle for transforming *only* statements into standard *A* statements:

Only $\mathcal{F}$ are $\mathcal{G}$ = All $\mathcal{G}$ are $\mathcal{F}$

People are often confused by *only* statements. For example, some may believe that S7 has the same content as S11.

(S7) Only males are priests.
(S11) All males are priests.

We can prove conclusively that S7 and S11 are not equivalent. S7 is true while S11 is false. If they were logically equivalent,[4] they would have the same truth value. So, they are not logically equivalent. We can also prove that S7 and S9 are equivalent. Consider these four statements:

(S9) All priests are males.
No priests are non-males.
No non-males are priests.
(S7) Only males are priests.

Each of the first three statements in this list is logically equivalent to the statement directly beneath it. Therefore, S7 is logically equivalent to S9. (This argument is assessed in Chapter Thirteen.)

Having mastered the treatment of *only* statements, we can handle *none but* statements easily. This principle suffices:

None but $\mathcal{F}$ are $\mathcal{G}$ = only $\mathcal{F}$ are $\mathcal{G}$

S8 is equivalent to S7, and therefore to S9.

(S8) None but males are priests.
(S7) Only males are priests.
(S9) All priests are males.

"E" Statements. Of the four standard categorical statement-forms, only the *E* form remains to be treated. S12 is a representative *E* statement:

[4]Two statements are logically equivalent if and only if it is logically impossible in virtue of their forms for one to be true and the other false.

(S12) No catchers are left-handed.[5]

We can paraphrase S12 in a way that employs two predicates and certain other readily symbolized expressions.

(S12′) For any *x*, if *x* is a (c)atcher, then *x* is not (l)eft-handed.

S12′ (and also S12) is symbolized by F12.

(F12) $\forall x(Cx \rightarrow -Lx)$

The dash is properly located after the arrow (as in F12). None of the following wffs symbolizes S12:

$-\forall x(Cx \rightarrow Lx)$
$\forall x-(Cx \rightarrow Lx)$
$\forall x(-Cx \rightarrow Lx)$

There are many ways of expressing *E* statements in English; here are some:

Catchers who are left-handed don't exist.
Catchers aren't left-handed.
There are no catchers who are left-handed.
Catchers are never left-handed.
No one is both a catcher and left-handed.
Catchers are not left-handed.

F12 symbolizes each of these sentences.

An introduction-to-philosophy exam I graded contained this sentence:

(S13) All events are not caused.

Had S13 not been imbedded in an essay, I would not have known whether the student was asserting S14 or S15.

(S14) No events are caused.
(S15) Some events are not caused.

[5]S12 is an overstatement, but *nearly* all catchers are right-handed. The explanation: right-handed catchers have a better lane to throw out runners who are trying to steal second base if the batter is standing to the left of home plate, and most hitters stand on that side of the plate.

Sentences of the form "All *F* are not *G*" are amphibolous.[6] (Distinguish this form from the unambiguous form "Not all *F* are *G*," that was discussed above.) In deciding whether to view a sentence such as S13 as an *E* or an *O* statement, we must pay attention to intonation (for speech) and to the context in which the sentence occurs. A person who dislikes unnecessary ambiguity will avoid formulating sentences of S13's type.

A question on a national high school mathematics examination concerned these statements:

I. All shirts in this store are not on sale.
II. There is some shirt in this store not on sale.
III. No shirt in this store is on sale.
IV. Not all shirts in this store are on sale.

Sentence II is an *O* statement, sentence III is an *E* statement, and sentence IV amounts to the negation of an *A* statement, making it equivalent to an *O* statement. Each of these statements is quite clear. Sentence I, on the other hand, is an amphibolous statement of the type just discussed. It can be read as an *E* or as an *O* statement. (If that is not evident, read the sentence several times stressing different components.) I think the test item is poor because of the ambiguity involved.

In this chapter we have learned how to symbolize singular statements and standard categorical propositions. In the next two chapters we will examine arguments composed of statements of these kinds (and their negations).

EXERCISES

Instructions for exercises 3 and 4: (1) Symbolize each statement using the suggested abbreviations. (Each statement belongs to one of the five types discussed in the chapter.) (2) Provide a dictionary for the abbreviating symbols. There is no need to provide a dictionary entry where I have already done so.

3. (a) (*lyric*) "All who (l)ove are (b)lind."
*(b) (*Alan Dershowitz*) ". . . No (w)ealthy people are ever (e)xecuted."
(c) (*bumper sticker*) "ANY DAY SPENT (A)BOVE GROUND IS A (G)OOD DAY." (Ax = x is a day spent above ground)
(d) Ted [T]urner (f)ounded CNN.
(e) (*ABC News*) "(N)eutrinos have (m)ass."
*(f) (*bumper sticker*) "(W)OMEN WHO SEEK TO BE EQUAL TO MEN LACK (A)MBITION." (Ax = x has ambition)

[6] *Amphiboly* is ambiguity rooted in poor sentence structure.

(g) Every (t)reaty ever made between the U.S. government and native Americans has been (b)roken by the U.S. government.
(h) (*jar lid*) "[S]alsa is (f)at-free." (Fx = x contains fat)
(i) (S)alsa is (f)at-free." (Fx = x contains fat)
*(j) Some royal (p)oincianas have (y)ellow blossoms.
(k) (*Catch-22*) "What's good for M & M (E)nterprises is good for the (c)ountry." (Ex = x is good for M & M Enterprises, Cx = x is good for the country)
(l) (*newspaper*) "There's no (w)all in Washington to (r)emember the thousands of Cherokees who died during the removal [of 1838]." (Rx = x commemorates the Cherokees who died during the removal of 1838)
(m) (*bumper sticker*) "HE WHO D(I)ES WITH THE MOST TOYS IS D(E)AD." (Ix = x dies with the most toys)
*(n) (*Andrew Carnegie*[7]) "The man who dies (r)ich dies (d)isgraced."
(o) Some (w)ater birds lack (o)il glands. (Ox = x has oil glands)

4. (a) (*newspaper*) "All loitering (l)aws are unconstitutional." (Cx = x is constitutional)
*(b) (*conversation*) "(I)nsects all have (a)ntennae."
(c) (*bumper sticker*) "[H]ATRED IS NOT A FAMILY (V)ALUE."
(d) (*anti-smoking crusader*) "Only s(u)ckers s(m)oke."
(e) Whenever it rains our telephone has no dial tone. (Rx = it is raining at time x, Tx = our telephone has a dial tone at time x)
*(f) (*Charlie Brown*) "To (k)now me is to (l)ove me." (Kx = x knows Charlie, Lx = x loves Charlie)
(g) (*Ellen Goodman*) "Not all (p)oliticians today are (r)otten."
(h) (*USFS sign at trailhead*) "If you pack it (i)n, pack it (o)ut." (Ox = you should pack x out)
(i) (*dialogue from* It's a Wonderful Life) "No man is a f(a)ilure who has f(r)iends." (Ax = x is a failure, Rx = x is a person who has friends)
*(j) (*movie title*) *(L)onely are the (B)rave.*
(k) (*Lincoln*) "Those who deny freedom to (o)thers deserve it not for (t)hemselves." (Ox = x denies freedom to others, Tx = x deserves to be free)
(l) (*Samuel Johnson*) "No man but a (l)unatic would be a (s)ailor."
(m) (*Erskine Caldwell*) "You cannot be both a good (s)ocializer and a good (w)riter."
*(n) (*W. C. Fields*) "A thing worth (h)aving is a thing worth (c)heating for." (Hx = x is worth having, Cx = x is worth cheating for)

[7] Steel magnate Andrew Carnegie (1835–1919) gave away $4.6 billion (in dollars adjusted for the year 2000) during his lifetime. Among his many gifts were more than 2,800 public libraries.

5. Translate each wff into a colloquial English sentence using the dictionary provided.

   Nx = x is a newt
   Sx = x is a salamander
   Rx = x is a reptile

   (a) $\forall x(Nx \rightarrow Sx)$
   *(b) $\forall x(Sx \rightarrow -Rx)$
   (c) $\forall x(Rx \rightarrow -Sx)$
   (d) $-\forall x(Sx \rightarrow -Nx)$

6. For each of the following sentences decide whether it is best viewed as a singular statement, an *A* statement, or an *I*.
   (a) A whale is a mammal.
   *(b) A whale has just surfaced off the port side of the ship.
   (c) Manatees are mammals.
   (d) Manatees live in Blue Spring Run in the winter.
   (e) The oldest daughter of Bill and Ruth is a veterinarian.
   *(f) The parent of a parent is a grandparent.

Note: The next two exercises are more difficult than any of the preceding ones; they require ingenuity. If you enjoy a challenge, you will want to tackle them. There are challenging problems in most of the exercise sets in this volume. To distinguish them from the other exercises, I have marked them with the word CHALLENGE.

7. (CHALLENGE) Symbolize these statements using the suggested abbreviations. Each statement is an instance of one of the five types discussed in the chapter, or the negation of such a statement.
   (a) (*Oliver Goldsmith*) "Honour (s)inks where commerce long (p)revails." (Sx = x is a place where honor recedes, Px = x is a place where commerce has long prevailed)
   (b) (*radio commercial*) "(S)moke and (m)ake your doctor rich." (Sx = x smokes, Mx = x makes x's doctor rich)
   (c) (*proverb*) "All that g(l)itters is not g(o)ld."
   (d) (*banker Charles Rice*) "The Harvard Business School never graduated an (M)BA that can't be hornswoggled by the businessmen of the Florida (p)anhandle." (Mx = x is a Harvard MBA, Px = x can be hornswoggled by Florida panhandle businessmen)
   (e) (*newspaper*) "The only senators ever (e)xpelled were those found guilty of (t)reason." (Ex = x is an expelled senator, Tx = x is found guilty of treason) (Note: Distinguish "The only $\mathcal{F}$ are $\mathcal{G}$" from "Only $\mathcal{F}$ are $\mathcal{G}$." Can you formulate a translation principle for statements such as (e)?)
   (f) (*Walt Kelley's Pogo*) "There's nothing but (l)osers in a (w)ar." (Wx = x is involved in a war)

   (g) (*Milton*) "Nothing of all these evils hath (b)efallen me but justly." (Bx = x is an evil that has befallen me, Dx = x is an evil I deserve)
   (h) (*newspaper letter*) "(H)umans are not the only creatures that (m)atter."
   (i) (*J. M. Hilary*) "The perfect (c)ake is the sine qua non of the carefully planned modern (w)edding." (Cx = x includes a perfect cake, Wx = x is a carefully planned modern wedding)
   (j) (*Nigerian general Abubakar*) "There is no (n)ation in the world that hasn't made (m)istakes." (Mx = x has made mistakes)
   (k) (*newspaper*) "The only way to survive an accident on water at that speed [300 mph] is not to have one." (Sx = accident x has survivors, Wx = x is an accident on water at 300 mph)

8. (CHALLENGE) Symbolize these statements, employing the suggested abbreviations. (Neither (e) nor (f) is an instance of one of the five types discussed in the chapter.)
   (a) All (p)arents are (e)ligible.
   (b) Any parent is eligible.
   (c) Not all parents are eligible.
   (d) Not any parent is eligible.
   (e) If all parents are eligible, [R]ita is.
   (f) If any parent is eligible, Rita is.

3

Proofs: ∀O and ∃O

3.1 ∀O

A dying Minnesota teen asked the Make-A-Wish Foundation[1] to send him to Alaska so that he could hunt and kill a Kodiak bear. A lot of controversy ensued when the charity agreed to fund this request. The decision was defended by foundation officials by noting that hunting is a legal activity, suggesting this argument:

> If a request is (l)egal, we'll (f)und it.
> This [t]eenager has requested to do something that is legal.
> So, our foundation will fund this young man's request.

Let's call this argument "Kodiak I." The argument consists of an *A* statement and two singular statements; it is readily symbolized:

$$\forall x(Lx \rightarrow Fx),\ Lt \vdash Ft$$

(Lx = x is a legal request made to the foundation, Fx = the Make-A-Wish Foundation will fund request x, t = the Minnesota teenager's request) Among the many persons who protested this decision was a woman in Asheville, North Carolina, who wrote to a newspaper (in part):

> *Does this mean that Make-A-Wish would fund a young man's wish to experience a prostitute before he dies? After all, prostitution is legal in Nevada.*[2]

[1]This foundation has helped about 40,000 seriously ill children. In 1999 (three years after the controversy described above) the charity's board of directors decided to stop funding requests that involve hunting.

[2]Marcia Zink, "Death wish a cruel puzzle," *Asheville Citizen-Times* (May 28, 1996), p. A5.

Her argument may be viewed as an attack on the first premise of "Kodiak I":

> The request to visit a [p]rostitute is a (l)egal request.
> The foundation would not (f)und that request.
> Therefore, it is false that the foundation will fund every legal request.

We'll call this argument "Kodiak II." This argument consists of two singular statements (one affirmative and the other negative) plus the negation of an *A* statement. It, too, is easily symbolized:

$$Lp,\ -Fp \vdash -\forall x(Lx \rightarrow Fx)$$

(p = the request to visit a prostitute)

Both of these arguments are valid. (Whether the statements that compose them are true is, of course, another matter. In fact, we know that not all six statements can be true because the conclusion of "Kodiak II" is the negation of the first premise of "Kodiak I.")

One of the main concerns of any branch of logic is to develop instruments for establishing the validity of arguments. In this chapter and Chapter Four, I present a device for establishing the validity of the sequents that represent predicate arguments–the method of *formal proof.*[3] Completing a formal proof for a sequent demonstrates that it is valid. In constructing proofs we shall make constant use of the eighteen propositional inference rules listed on page 278. We will also employ four rules permitting us to manipulate universal and existential quantifiers. I present two of these quantifier rules in this chapter and the other two in Chapter Four.

Some definitions prepare the way for these rules. First, we define a *universal quantification* as a wff beginning with a universal quantifier whose scope is the entire wff. The *scope* of a quantifier is the portion of the wff that the quantifier governs; I'll clarify that notion in Chapters Five and Thirteen. Note that while F1 is a universal quantification, F2 and F3 are not.

(F1) $\forall x(Cx \rightarrow Dx)$
(F2) $-\forall x(Cx \rightarrow Dx)$
(F3) $\forall x(Cx \rightarrow Dx)\ \&\ Ef$

Now we define an *instance* of a universal quantification as a wff that results from (a) deleting the quantifier (as well as groupers showing the scope of the quantifier) and (b) replacing each of the remaining occurrences of the variable by the same individual constant. F4 and F5 are instances of F1.

[3]The concept of *formal proof* is discussed in Appendix One.

(F4) Ca → Da
(F5) Cb → Db

F6 and F7 are not instances of F1. Why not?[4]

(F6) Ca → Db
(F7) Cy → Dy

The first predicate inference rule is simplicity itself.

***The Universal Quantifier Out Rule* (∀O): From a universal quantification derive any instance of it.**

(How should "∀O" be pronounced? I suggest "AO.") The ∀O Rule (as well as the other predicate inference rules introduced later) is to be applied to *whole* lines only. To illustrate the use of the rule I will construct a proof for "Kodiak I," which was symbolized:

∀x(Lx → Fx), Lt ⊢ Ft

The proof is short and sweet:

(1) ∀x(Lx → Fx)	A
(2) Lt	A
(3) Lt → Ft	1 ∀O
(4) Ft	3,2 →O

Line 3 is an instance of the universal quantification on line 1; hence the ∀O Rule was correctly applied in deriving line 3. The individual constant I chose to *instantiate* to on line 3 was *t*. I selected *t* because it occurred in the second premise and the conclusion of the sequent. Had I instantiated to any other constant on line 3 I would have been unable to complete the proof. The Arrow Out step on line 4 can be made only if the antecedent of line 3 is identical with line 2; thus line 3 must contain *t*.

Either of two questions may have occurred to you: *Why is the ∀O Rule sound?* and *Why is it needed?* Let's consider these questions in turn. The rule is based on the elementary logical principle that what is true of *every* individual is true of *any named* individual. Thus if it is true of every number that it has a successor, then it is true of 17 that it has a successor. A universal quantification represents a claim about every individual; an instance of that universal quantification represents the same claim applied to a specific (named) individual. The universal quantification on line 1 of the above proof says of every individual that if it is a legal request [made by a dying child], then it will be funded. The instance on line 3 says of the Minnesota teenager's request that if it is legal, then it will be funded.

[4]F6: *a* and *b* are not the same constant. F7: *y* is not a constant but a variable.

Why is the ∀O Rule needed? The rule allows us to derive from a universal quantification a wff that essentially belongs to propositional logic; it allows us to derive a wff to which we can apply the propositional inference rules. Contrast lines 1 and 3 of the above proof. The main symbol in line 1 (that is, the symbol with the greatest scope) is the universal quantifier, not the arrow; line 1 is a quantification, not a conditional. Thus a propositional rule such as Arrow Out cannot be applied correctly to line 1. By contrast, the main symbol in line 3 is the arrow; line 3 is a conditional. Arrow Out can be applied to that line. So, the main purpose of the ∀O Rule is to allow us to derive from universal quantifications wffs that lend themselves to manipulation by the inference rules of propositional logic.

For a second illustration of the use of the ∀O Rule I will construct a proof for "Kodiak II." This inference was symbolized:

Lp, −Fp ⊢ −∀x(Lx → Fx)

A proof for this sequent:

1	(1) Lp	A
2	(2) −Fp	A
3	(3) ∀x(Lx → Fx)	PA
3	(4) Lp → Fp	3 ∀O
1,3	(5) Fp	4,1 →O
1,2,3	(6) Fp & −Fp	5,2 &I
1,2	(7) −∀x(Lx → Fx)	3-6 −I

The fact that the conclusion is a negation suggests that we employ the Dash In strategy; so a provisional assumption of the conclusion less its dash was made on line 3. When a standard contradiction[5] was reached on line 6, an application of Dash In finished the proof. The employment of a provisional assumption in the proof above required the inclusion (on the left) of an assumption-dependence column. This column makes it evident that line 7, the sequent's conclusion, depends only on the original assumptions and not on the provisional assumption. Note that the standard assumption-dependence principle applies to the ∀O Rule (and all the other predicate inference rules to be introduced). That is, the statement derived by ∀O depends on all of the assumptions on which the premise of the step depends.

The proof above is not the only correct proof that can be devised for "Kodiak II." For example, instead of employing Arrow Out, I might have used Modus Tollens. For any valid predicate sequent there are multiple correct proofs. A practical corollary: if a proof you construct for a starred exercise differs from the proof given in Appendix Four, your proof may yet be correct.

[5]A *standard contradiction* is a conjunction whose right conjunct is the negation of the left conjunct.

When Dr. Calvin Shirley, an African-American physician, applied to join the staff at Broward General Hospital in Ft. Lauderdale in the 1950's, he found that he was prevented from doing so by the combination of two rules; one was a hospital rule and the other a rule of the Broward County Medical Society.[6] The situation he faced is summarized by this argument:

> Only members of the (m)edical society are (p)ermitted to join the hospital staff. No (b)lacks are in the medical society. Dr. [S]hirley is black. Therefore, Dr. Shirley is not permitted to join the hospital staff.
>
> $\forall x(Px \rightarrow Mx)$, $\forall x(Bx \rightarrow -Mx)$, $Bs \vdash -Ps$

(Note how the first premise is symbolized.) A proof of the validity of this sequent:

(1)	$\forall x(Px \rightarrow Mx)$	A
(2)	$\forall x(Bx \rightarrow -Mx)$	A
(3)	Bs	A
(4)	$Ps \rightarrow Ms$	1 ∀O
(5)	$Bs \rightarrow -Ms$	2 ∀O
(6)	$-Ms$	5,3 →O
(7)	$-Ps$	4,6 MT

Dr. Shirley won his rightful place on the hospital staff by obtaining a court order forcing the medical society to admit him.

EXERCISES

1. Complete the following proofs. Every assumption has been identified. An assumption-dependence column is required for proof (b).

(a)

(1)	$\forall x(Ax \rightarrow -Bx)$	A
(2)	Bc	A
(3)		1 ∀O
(4)	$Bc \rightarrow -Ac$	
(5)	$-Ac$	

*(b)

1	(1)	De	A
	(2)	$\forall x(Dx \rightarrow Fx)$	A
	(3)	$-Ge$	A
	(4)	$\forall x(Fx \rightarrow Gx)$	PA
	(5)	$De \rightarrow Fe$	
	(6)		
	(7)	Fe	
	(8)		
	(9)	$Ge \& -Ge$	
1,2,3	(10)	$-\forall x(Fx \rightarrow Gx)$	

[6]"Dedicated doctor is still in," *Miami Herald* (February 7, 1998), pp. 1G & 5G.

Note: To practice proof construction without the need for prior symbolization, see the Chapter Three practice problems in the Proofs section of the computer tutorial "PredLogic."

Instructions for exercises 2 through 7: Symbolize each argument on one horizontal line, using the suggested abbreviations. Construct a proof for each sequent. (These exercises are arranged so that the simplest problems occur first. This practice is followed throughout the book.)

2. Air Force Major Robert Lawrence, Jr., was killed in a plane crash during a training exercise in 1967, six months after he was named to the Air Force's manned orbiting laboratory program.[7] By NASA standards, he was an astronaut because he had been selected for astronaut training. NASA's reasoning:

(a) Anyone (s)elected for astronaut training is an (a)stronaut. Major [L]awrence was selected for astronaut training. Hence, he was an astronaut.

By the standards employed by the Air Force in the 1960's, Major Lawrence was *not* an astronaut because he never flew 50 miles above the surface of the earth. The Air Force's argument:

*(b) Only those who have (f)lown 50 miles above the earth are (a)stronauts. Major [L]awrence never flew 50 miles above the earth. Therefore, he was not an astronaut.

Because of the Air Force's stand, Major Lawrence's name was not etched on the Space Mirror, the granite monument at Cape Canaveral that honors astronauts killed in the line of duty, when that monument was erected. Major Lawrence's family worked for years to persuade the directors of the monument foundation to include his name. One of the arguments they employed:

(c) Christa [M]cAuliffe's name is on the (S)pace Mirror.[8] Ms. McAuliffe never (f)lew 50 miles above the earth. So, it is false that only those who have flown 50 miles above the earth have their names inscribed on the mirror.

(m = Christa McAuliffe, Sx = x's name is on the Space Mirror)

Major Lawrence's name was added to the Space Mirror 30 years after his death.

[7]"First black astronaut belatedly recognized," *Miami Herald* (December 8, 1997), p. 4A.
[8]Teacher Christa McAuliffe died in the *Challenger* explosion.

3. C(r)uel and unusual punishment is unconstitutional. So, c[a]pital punishment is unconstitutional because it is cruel and unusual.

   (Ox = x is constitutional)

4. A topless club in New York City discovered that it could evade a law designed to put strip clubs out of business by admitting children as patrons.[9] The lawyer for the club devised this argument (which was accepted by the state supreme court):

   The anti-smut (l)aw applies only to (a)dult establishments. No adult establishment admits (m)inors. [T]en's World Class Cabaret admits minors. It follows that the anti-smut law does not apply to Ten's.

   (Lx = the anti-smut law applies to x)

*5. When Serena Williams announced that she wanted to play on the men's tennis tour, a spokesperson for the men's tour responded,

   To be able to play an ATP tournament, you have to be a member of the Association of Tennis Professionals. Serena Williams cannot become a member of the ATP because she is a woman. That answers the question.[10]

   The argument reformulated:

   Only members of the (A)TP (p)lay in ATP tournaments. Only (m)en are members of the ATP. Therefore, [S]erena Williams will not play in ATP tournaments because she is not a man.

6. Some years ago, when the Rev. Bailey Smith, president of the Southern Baptist Convention at the time, made public his opinion that God does not hear the prayers of Jews, protests came from all quarters. One man wrote *Time* magazine:

   If God cannot hear a Jew's prayers, how could he hear those of Jesus, a Jew?[11]

   The man's reasoning seems to be:

   Jesu[s] is a Je(w). God (h)ears Jesus' prayers. Hence, it is false that God hears the prayers of no Jews.

   (s = Jesus, Wx = x is a Jew, Hx = God hears x's prayers)

   (Rev. Smith's bigoted view deserves to be challenged, but this argument may not do the job. The argument appears to trade on an ambiguity in the term *Jew*: that word can identify ethnicity or religious

[9] "'Adult' club admits kids to remain open (AP)," *(Fort Lauderdale) Sun-Sentinel* (November 7, 1998), p. 3A.

[10] "Sorry, Serena, you can't enter," *Miami Herald* (October 8, 1999), p. 3D.

[11] October 20, 1980, p. 5.

adherence. If our symbolization of the argument is to be acceptable, then the word must be used in the same sense throughout the argument. If it is used to identify ethnicity, the conclusion does not contradict Rev. Smith's position; and if it is used in the other sense, one can argue that the first premise is false. Note that this criticism of the reasoning in the letter is itself an argument.)

7. Officials in Deltona, Florida, informed Deborah Housend that she could no longer keep her pet Vietnamese potbellied pig, Harley, in her backyard.[12] The bureaucrats argued as follows:

   [H]arley is a pot(b)ellied pig. Potbellied pigs are pi(g)s. Pigs are (l)ivestock. Livestock are not permitted in (r)esidential areas. It follows that Harley is not permitted in residential areas.

   (Rx = x is permitted in residential areas)

3.2 ∃O

A newspaper columnist writes:

The predominant idea that only whites owned slaves is not correct. There were also Indian slave holders, particularly among the so-called "Five Civilized Tribes."[13]

The columnist reasons:

Some (I)ndians (o)wned slaves.
No Indians are (w)hites.
So, it is false that only whites owned slaves.

$\exists x(Ix \,\&\, Ox), \forall x(Ix \rightarrow -Wx) \vdash -\forall x(Ox \rightarrow Wx)$

The second premise was assumed, and not stated. This is an obviously valid argument, but we cannot construct a proof for its sequent until we adopt a rule of inference that allows us to make inferences from wffs that begin with existential quantifiers.

***The Existential Quantifier Out Rule* (∃O): From an existential quantification derive any instance of it, *provided that* the individual constant being introduced is a dummy name that is new to the proof.**

(I suggest that "∃O" be pronounced "EO.") I need to define several terms appearing in the ∃O Rule. An *existential quantification* is a wff beginning with an

[12] "Deltona is no hog heaven for Harley the potbellied pig," *Orlando Sentinel* (April 30, 1997), pp. C-1 & C-4.

[13] MariJo Moore, "'Five Civilized Tribes' were owners of African American slaves," *Asheville Citizen-Times* (June 14, 1998), p. A11.

existential quantifier whose scope is the entire wff. An *instance* of an existential quantification is a wff that results from deleting the quantifier (and groupers showing the scope of the quantifier) and replacing each of the remaining occurrences of the variable by the same individual constant. A *dummy name* is an individual constant in a proof that does not appear in the sequent being tested. (We call a constant that *does* appear in the sequent a *genuine name.*) Note that the ∃O Rule incorporates two restrictions (in the clause following *provided that*): (1) the constant must be a dummy name, and (2) it must be new to the proof. The need for these restrictions will be explained below.

We can now construct a proof for (the symbolization of) the argument about slave ownership:

1	(1)	∃x(Ix & Ox)	A
2	(2)	∀x(Ix → –Wx)	A
3	(3)	∀x(Ox → Wx)	PA
1	(4)	Ia & Oa	1 ∃O
2	(5)	Ia → –Wa	2 ∀O
3	(6)	Oa → Wa	3 ∀O
1	(7)	Ia	4 &O
1,2	(8)	–Wa	5,7 →O
1	(9)	Oa	4 &O
1,3	(10)	Wa	6, 9 →O
1,2,3	(11)	Wa & –Wa	10,8 &I
1,2	(12)	–∀x(Ox → Wx)	3-11 –I

Line 4 is an instance of the existential quantification on line 1. Furthermore, the individual constant introduced on line 4 is a dummy name and it is new to the proof (it does not occur on any line above line 4), so the restrictions on the ∃O Rule are satisfied. The individual constant employed on the fourth line was selected at random; any other constant letter would have served as well. However, having chosen *a* for line 4, I had to instantiate to it again on lines 5 and 6 in order to complete the proof. The purpose of the ∃O Rule is to permit us to derive from existential quantifications wffs to which the propositional inference rules apply. Consider the above proof. The Ampersand Out Rule cannot be applied to line 1, because that wff is not a conjunction (its main symbol is not an ampersand), but the rule *can* be applied to line 4 (which was derived from line 1 by the ∃O Rule).

I will construct a second proof employing the ∃O Rule. It was long thought that an email message (as opposed to an *attachment* to an email message) could not harbor a computer virus.[14] The reasoning behind that view:

[14] In 1999 a virus (*Bubbleboy*) was created that could be activated (assisted by a glitch in Microsoft Outlook Express) simply by highlighting its subject line.

Anything that harbors a computer (v)irus is an executable (p)rogram. No (e)mail message is an executable program. Therefore, it is false that some email messages harbor viruses.

∀x(Vx → Px), ∀x(Ex → –Px) ⊢ –∃x(Ex & Vx)

(Vx = x harbors a virus) A proof of validity for this sequent:

1	(1)	∀x(Vx → Px)	A
2	(2)	∀x(Ex → –Px)	A
3	(3)	∃x(Ex & Vx)	PA
3	(4)	Eb & Vb	3 ∃O
1	(5)	Vb → Pb	1 ∀O
2	(6)	Eb → –Pb	2 ∀O
3	(7)	Vb	4 &O
1,3	(8)	Pb	5,7 →O
3	(9)	Eb	4 &O
2,3	(10)	–Pb	6,9 →O
1,2,3	(11)	Pb & –Pb	8,10 &I
1,2	(12)	–∃x(Ex & Vx)	3-11 –I

Notice that the existential quantification on line 3 was instantiated *before* the universal quantifications on lines 1 and 2. This was done to ensure satisfaction of the second restriction on the ∃O Rule–the requirement that the individual constant introduced not appear on any higher line. This procedural principle should be followed:

Wherever possible, employ the ∃O Rule before using the ∀O Rule.

Note that the ∀O rule is not encumbered with either of the restrictions that are imposed on the ∃O Rule. The constant introduced by a step of ∀O may be either a genuine name or a dummy name, and it need not be new to the proof.

I will conclude the section by considering two questions: *Why is the ∃O Rule sound?* and *Why are the two restrictions needed?*

Consider the statement "Some (r)abbis are (J)apanese" and the existential quantification, F1, that symbolizes it.

(F1) ∃x(Rx & Jx)

You certainly couldn't validly infer F2 (where *b* functions as a genuine name, denoting George W. Bush, let's say) from F1.

(F2) Rb & Jb

It is unsound to reason that what is true of some individual (namely, that he or she is a rabbi and also Japanese) must be true of a particular named individual (George W. Bush). F2 may be correctly inferred from F1 when and only when *b* is a dummy name (and also new to the proof containing these wffs). F1 is true iff (if and only if) there is at least one individual to whom the predicates *R* and *J* can be correctly ascribed. Now we probably don't know who that individual is, but we can agree to let *b* function (in the context of the proof) as the name of that individual (or one of those individuals if there are several).[15] If F1 is true, then F2 will also be true. And that is all we need to ask of a rule of inference, that it be *truth-preserving.* It is important that we not attach any additional significance to *b,* and that is why we require that it be a dummy name new to the proof.

We can re-emphasize the necessity of the two restrictions by showing that without them it would be possible to construct "proofs" for invalid sequents. Here is an obviously invalid argument:

> Some (D)emocrats are (p)oliticians. Thus, [E]lizabeth Dole is a Democrat.
>
> ∃x(Dx & Px) ⊢ De

A "proof" for its sequent:

(1)	∃x(Dx & Px)	A
(2)	De & Pe	1 ∃O (*ERROR!*)
(3)	De	2 &O

The move to line 2 violates the first restriction on the ∃O Rule, since the individual constant introduced is a genuine name (it appears in the conclusion), not a dummy name. In symbolizing the argument I chose *e* to refer to Elizabeth Dole. Thus, when I instantiated on line 2, *e* was not functioning as a dummy name.

A second obviously fallacious argument:

> Some Democrats are not politicians. So, it is false that some Democrats are politicians.
>
> ∃x(Dx & −Px) ⊢ −∃x(Dx & Px)

We know that this argument is invalid because it has a true premise and a false conclusion. (The conclusion denies a true assertion.) A "proof" for its sequent:

1	(1)	∃x(Dx & −Px)	A
2	(2)	∃x(Dx & Px)	PA
1	(3)	De & −Pe	1 ∃O
2	(4)	De & Pe	2 ∃O (*ERROR!*)
2	(5)	Pe	4 &O
1	(6)	−Pe	3 &O
1,2	(7)	Pe & −Pe	5,6 &I
1	(8)	−∃x(Dx & Px)	2-7 −I

The derivation of line 4 violates the second restriction on the ∃O Rule, since the individual constant used on line 4 occurs on a preceding line (3) and so is not new to the proof. As *e* was already assigned a use on line 3 (to name some individual that makes line 1 a truth), it was not free on line 4 to function simply as a dummy name in relation to line 2.

Now we should wrap up some loose ends relating to the assignment of meanings to dummy names. (1) When a dummy name is initially introduced into a proof by a step of ∀O (or in a provisional assumption or via ∨I), what significance do we attach to the name? Answer: the name represents an individual selected at random. (2) When a dummy name is brought into a proof for a second (or third, etc.) time by a step of ∀O (or in a provisional assumption or via ∨I), what significance do we attach to the name? Answer: it has the same significance given when the name was first introduced to the proof. (3) In a propositional-logic proof, each line in a proof is correlated with a sequent whose validity is established when that line is reached. The conclusion of the sequent is the wff on the line in question, and the premises are the assumptions on which that wff depends. In a predicate-logic proof this result holds only for lines that do not contain dummy names. Because a dummy name has a reference only in the context of the proof in which it occurs, it cannot be a part of a sequent considered independently of the proof.

[15] But what if there is no such individual? In that case F1 is false, and it really doesn't matter what individual the dummy name denotes; we can let the name designate any arbitrarily chosen individual.

EXERCISES

8. Complete the following proof.

	(1)	∀x(Ax → Bx)	A
	(2)	∃x(Ax & Cx)	A
	(3)	∀x(Cx → −Bx)	PA
	(4)	Ad & Cd	2 ∃O
	(5)		1 ∀O
	(6)		3 ∀O
	(7)		4 &O
	(8)		5,7 →O
	(9)		4 &O
	(10)		6,9 →O
	(11)		8,10 &I
1,2	(12)		3-11 −I

Instructions for exercises 9 through 14: Symbolize each argument on one horizontal line, using the suggested abbreviations. Construct a proof for each sequent.

9. *A* and *O* statements with the same grammatical subjects and predicates are *contradictories*; *E*'s and *I*'s are also. Two statements are contradictories when and only when, by virtue of their forms, they must have opposite truth values. We can begin to establish these results by showing that the arguments below are valid.

   (a) All (o)rthodontists are (d)entists. So, it is false that some orthodontists are not dentists.

   *(b) No (B)uddhists are (A)mish. So, it is false that some Buddhists are Amish.

10. A newspaper story describes Wabash College, one of three remaining all-male colleges in the United States (as of 1999), and its effective honor code.[16] The article advances this argument (paraphrased):

    A (W)abash man is a (g)entleman. No gentleman (c)heats. Thus, it is not the case that there are Wabash men who cheat.

11. Sociologist Amati Etzione, in a newspaper interview:

    Community is . . . not a sufficient condition for a noble social life. There can be a Nazi community. There can be a community that decides to burn books.[17]

    Etzione's reasoning reformulated:

    There are (N)azi (c)ommunities. No Nazi institutions (f)oster a noble social life. Therefore, it is false that all communities foster a noble social life.

    The second premise was taken for granted, and so not made explicit.

12. An appellate court decision includes the following reasoning:

    The plaintiff Pestana contends . . . that the contract herein is a destination contract. . . . He relies for this position on the notation at the bottom of the contract between the parties which provides that the goods were to be sent to Chetumal, Mexico. We cannot agree. A "send to" or "ship to" term is a part of every contract involving the sale of goods where carriage is contemplated and has no significance in determining whether the contract is a shipment or destination contract for risk of loss purposes.[18]

[16] Jon Jeter, "School of Manners, Not Miss Manners," *Washington Post* (March 12, 1999), p. 2-A.

[17] Bob von Sternberg and Martha Sawyer Allen, "Hey, can't we all just get along?" *Minneapolis Star Tribune* (October 14, 1994), p. 22A.

[18] *Pestana* v. *Karinol Corp.* District Court of Appeal of Florida, Third District (1979), 367 So. 2d 1096.

The court's thinking may be expressed:

All sales contracts involving (c)arriage contain the (t)erm "send to." So, since at least some sales contracts involving carriage are not (d)estination contracts, it is false that every contract containing the term "send to" is a destination contract.

The second premise was unstated in the decision.

*13. On an exam in Introduction to Philosophy I asked:

    How would the hard determinist answer the question, "Could a machine have free will?"

    One student answered (in part):

    The hard determinist says that all events are caused. Therefore all machine actions (which are also events) are caused. Therefore a machine could not have free will.

    Formalizing this a bit more we reach:

    All (e)vents are (c)aused. All (m)achine actions are events. Nothing caused is (f)ree. Therefore, it is false that there are free machine actions.

14. A traveling museum show called *Big Bugs* included this sign beside a giant wooden replica of a scorpion:

    Can you tell . . .
    Is a scorpion an insect? Count its legs. Insects have six, while arachnids (spiders and their relatives) have eight.

    The scorpion model had eight legs. Perhaps the naturalist who wrote the sign text was encouraging children to reason like this:

    (I)nsects have si(x) legs. (A)rachnids have (e)ight legs. Nothing with six legs has eight legs. S(c)orpions are arachnids. Thus, it is false that some insects are scorpions.

    (We hope that the naturalist was not also encouraging this reasoning: Arachnids have eight legs. Scorpions have eight legs. Hence, scorpions are arachnids.)

15. (CHALLENGE) Construct a seven-line proof for exercise 9(a).

4

Proofs: ∃I and QE

The inference rules presented in Chapter Three are both "out" rules; they allow us to move *from* quantified wffs, but of course not *to* such wffs. For this reason, the conclusions of the sequents we have proven so far have been either the negations of quantifications (which can be reached by the Dash In Rule) or wffs without quantifiers. In this chapter we extend the proof system so that we can handle sequents whose conclusions are quantifications.

4.1 ∃I

The Existential Quantifier In Rule allows us to infer existential quantifications.

> ***The Existential Quantifier In Rule* (∃I): Derive an existential quantification from any instance of it.**

F2 is an existential quantification and F1 is an instance of it; therefore, we may derive F2 from F1 by ∃I.

(F1) Fa & Ga
(F2) ∃x(Fx & Gx)

Why is this rule sound? Because if some *named* individual bears a property, then surely some individual bears it. (Note that predicate logic assumes that each individual constant names some individual.) This rule works whether the constant in the instance is a genuine or dummy name.

We can illustrate the use of the rule by constructing a proof for the argument "Dead Voters." A newspaper story under the headline, *Dead Men* do *Vote–Miami Probe Widens,* begins:

> *Manuel Yip died in 1993 at age 75. Last week, he voted in the Miami city election.*[1]

The reporters' tongue-in-cheek argument:

> [M]anuel Yip is a (d)ead man. Yip (v)oted in the Miami election. So, some dead man voted in the Miami election.
>
> Dm, Vm ⊢ ∃x(Dx & Vx)

We have here a valid argument with a false conclusion. How can that be? Answer: The second premise is false. Mr. Yip didn't vote; rather, someone voted using his identity. However, our present concern is not with determining the truth or falsity of the statements making up the argument, but with establishing the validity of the argument. The proof couldn't hardly be simpler:

(1) Dm	A
(2) Vm	A
(3) Dm & Vm	1,2 &I
(4) ∃x(Dx & Vx)	3 ∃I

The deduction of line 4 from line 3 is correct because line 3 is an instance of line 4.

For a second sample proof we turn to an argument (call it "False Beliefs") advanced by Plato in the dialogue *Gorgias*[2]:

> All (k)nowledge is (t)rue. Some (b)eliefs are not true. Hence, at least some beliefs are not knowledge.
>
> ∀x(Kx → Tx), ∃x(Bx & −Tx) ⊢ ∃x(Bx & −Kx)

(1) ∀x(Kx → Tx)	A
(2) ∃x(Bx & −Tx)	A
(3) Ba & −Ta	2 ∃O
(4) Ka → Ta	1 ∀O
(5) Ba	3 &O
(6) −Ta	3 &O

[1] Manny Garcia, Frances Robles, and Joseph Tanfani, *Miami Herald* (November 9, 1997), p. 1A.

[2] 454d. *The Collected Dialogues of Plato,* ed. by Edith Hamilton and Huntington Cairns (Princeton, N.J.: Princeton University Press, 1961), p. 238.

(7)	$-Ka$	4,6 MT
(8)	$Ba \& -Ka$	5,7 &I
(9)	$\exists x(Bx \& -Kx)$	8 ∃I

Note that line 8 is an instance of line 9. Also note that the ∃O move was made before the ∀O move in order to avoid violating one of the restrictions on the ∃O Rule.

EXERCISES

1. The *converse* of a categorical proposition is the statement that results when you switch the (logical) predicates. An *I* statement and its converse (S1 and S2, for example) are logically equivalent.

   (S1) Some (M)ethodists are (O)hioans.
   (S2) Some Ohioans are Methodists.

   Complete the proof below to show that S1 entails[3] S2. An isomorphic proof would show that S2 entails S1. Since logical equivalence is mutual entailment, we see that the two statements are equivalent. We can generalize the result and state that *I* statements are (validly) convertible. (There is only one assumption line in this proof.)

(1)	$\exists x(Mx \& Ox)$	A
(2)		
(3)	Mb	
(4)		
(5)		
(6)	$\exists x(Ox \& Mx)$	

Note: To practice proof construction without the need for prior symbolization, see the Chapter Four practice problems in the Proofs section of "PredLogic."

Instructions: Symbolize each argument on one horizontal line, using the suggested abbreviations. Construct a proof for each sequent.

2. Liz Heaston, a place kicker for Willamette University, broke the gender barrier in American college football in 1997,[4] giving us the following argument:

   [H]easton is a (w)oman. She is a college (f)ootball player. Hence, some college football players are women.

[3] One statement *entails* a second iff the argument that has the first statement as sole premise and the second statement as conclusion is valid.

[4] "Kicking down barriers; It's a 1st: Woman plays in college football game," *Miami Herald* (October 21, 1997), pp. 1D & 4D.

*3. Bertrand Russell writes in *A History of Western Philosophy*:

   The syllogism is only one kind of deductive argument. In mathematics, which is wholly deductive, syllogisms hardly ever occur.[5]

   As Russell notes, syllogisms (this term is defined in the next section) are uncommon in mathematics, but they are more common in philosophy—this passage being a case in point. Russell's syllogism:

   Some (d)eductive arguments are not (s)yllogisms. Proof: (M)athematical arguments are all deductive. Some mathematical arguments are not syllogisms.

4. [T]wo is not (o)dd. Two is (p)rime. Therefore, some primes are not odd.

5. Some arguments with (f)alse premises are (v)alid. This proves that some valid arguments do not (e)stablish their conclusions, because no argument that establishes its conclusion has false premises.

   (Fx = x is an argument with false premises, Ex = x is an argument that establishes its conclusion)

6. Since 1992 it has been illegal in Australia to sterilize retarded women without court approval.[6] In the five years following this decision, court permission was granted only 17 times, but at least 1,045 retarded women were sterilized. The reasoning (applied to sterilizations of retarded Australian women) is obvious:

   Sterilizations are (l)egal only if they are done with court (a)pproval. Some (s)terilizations were done without court approval. Hence, some of the sterilizations were illegal.

   (Lx = x is a legal case of sterilization, Ax = x is done with court approval, Sx = x is a case of sterilization)

*7. Philosopher James Rachels advances this argument:

   1. *If an action promotes the (b)est interests of* everyone *concerned and violates* no one's *rights, then that action is (m)orally acceptable.*
   2. *In at least some cases, active (e)uthanasia promotes the best interests of everyone concerned and violates no one's rights.*
   3. *Therefore, in at least some cases active euthanasia is morally acceptable.*[7]

   (Bx = x is an action that promotes the best interests of everyone concerned and violates no one's rights, Mx = x is morally acceptable, Ex = x is an act of active euthanasia)

[5] (New York: Simon and Schuster, 1945), p. 198.

[6] "Panel: Illegal sterilizations in Australia," (AP), *Miami Herald* (December 16, 1997), p. 30A.

[7] "Euthanasia," in *Matters of Life and Death: New Introductory Essays in Moral Philosophy*, 2nd ed., ed. by Tom Regan (New York: Random House, Inc., 1986), p. 52.

8. A naive boy (growing up in a less-sophisticated era) reasoned as follows:

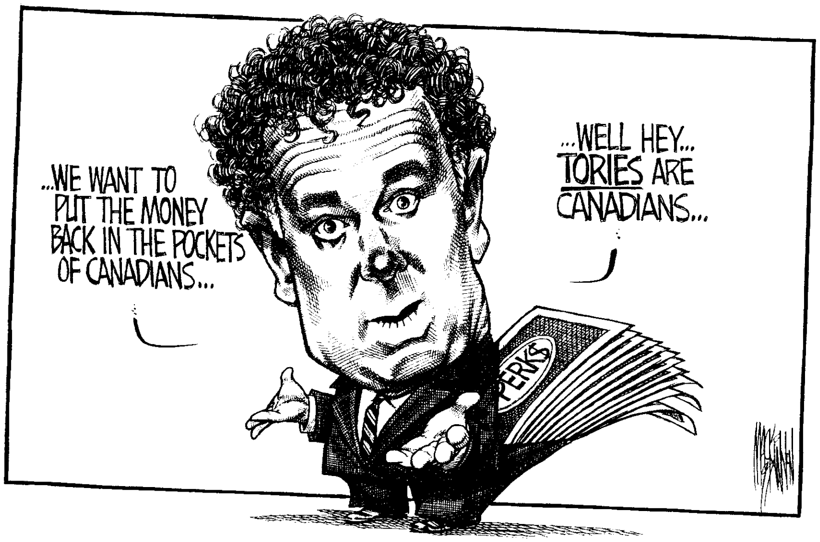
   Whoever engages in (i)ntercourse is (e)vil. (M)inisters are not evil. Some ministers have (c)hildren. Therefore, some who have children have never had intercourse.

   (The reasoning is valid but, of course, most of the statements are false.) An unusual logical feature of this argument is that it contains all of the basic statement forms (*A*, *E*, *I*, and *O*).

9. The political cartoon below skewering Canadian politician Jean Charest advances this argument:

   (M)oney is being put in my [Charest's] pocket. [I] am a (T)ory. Tories are (C)anadians. So, money is being put in the pocket of some Canadians.

   (Mx = money is being put in the pocket of x)



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4.2 QE

Our proof system is still incomplete. We are not yet equipped to deal with sequents whose conclusions are universal quantifications or with sequents having a premise that is the negation of a quantification. We could solve the first problem by adding a Universal Quantifier In Rule (∀I) to our stockpile of rules, and

logicians commonly do this.[8] Adding this rule would provide two advantages: it would enable elegant proofs for sequents whose conclusions are universal quantifications, and it would provide a symmetrical set of rules, with an *In* rule and an *Out* rule for each quantifier. But there are considerations on the other side. The main one is that the set of conditions that must be imposed on an ∀I Rule in order to make it valid are so cumbersome that the benefit of having the rule available for use in proofs is more than neutralized by the difficulty of remembering, and determining the satisfaction of, its conditions. Why must an ∀I Rule be laden with conditions? You can see that, in general, universal quantifications make much more sweeping claims than do existential quantifications. To hold that *every*thing bears some property is to assert a lot more than merely that *some*thing bears that property. So, a move to a universal quantification is an inherently risky move. In order to insure that it is properly done a number of restrictions must be observed. A second consideration is that even if we added an ∀I Rule to our set we would still not be equipped to deal with certain valid sequents, those containing premise wffs that are the negations of quantifications.

My plan is to introduce a fourth inference rule that will solve both problems mentioned above. The addition of this rule will make it unnecessary to include an ∀I Rule. The rule allows us to interchange dashes and quantifiers:

> ***The Quantifier Exchange Rule* (QE): (1) From the negation of a universal quantification derive the wff that results from replacing the quantifier by an existential quantifier and interchanging the dash and the quantifier, and vice versa.**
>
> **(2) From the negation of an existential quantification derive the wff that results from replacing the quantifier by a universal quantifier and interchanging the dash and the quantifier, and vice versa.**

The rule can be expressed more simply as follows:[9]

> **From −∀x*A*x derive ∃x−*A*x, and vice versa.**
> **From −∃x*A*x derive ∀x−*A*x, and vice versa.**

[8]An ∀I Rule may be formulated as follows:
Derive a universal quantification from any instance of it, *provided that* the individual constant is a dummy name that does not occur in:
(1) any line derived by ∃O,
(2) any provisional assumption on which the instance depends, or
(3) the universal quantification itself.

[9]"**−∀x*A*x**" represents the negation of any universal quantification and "**∃x−*A*x**" represents the wff that results from replacing the quantifier by an existential quantifier and interchanging the dash and the quantifier. Analogous comments apply to the second version of the rule.

We can illustrate the use of this rule with a proof that shows that *E* statements are validly convertible. We will show that S3 entails S4. (An isomorphic proof would establish the reverse entailment.)

(S3) No (T)exans are (I)owans.
(S4) No Iowans are Texans.

1	(1)	∀x(Tx → −Ix)	A
2	(2)	−∀x(Ix → −Tx)	PA
2	(3)	∃x−(Ix → −Tx)	2 QE
2	(4)	−(Ic → −Tc)	3 ∃O
1	(5)	Tc → −Ic	1 ∀O
1	(6)	Ic → −Tc	5 CN
1,2	(7)	(Ic → −Tc) & −(Ic → −Tc)	6,4 &I
1	(8)	∀x(Ix → −Tx)	2-7 −O

The last line in the proof is a universal quantification. Because we do not have at our disposal an ∀I Rule, we adopt a Dash Out strategy for reaching line 8. Accordingly we provisionally assume on line 2 the negation of the wff on line 8. Notice that the ∀O Rule cannot be applied to the wff on line 2 because it is not a quantification. (It is not a quantification because it begins with a dash rather than a quantifier.) With the help of the QE Rule we can deduce from line 2 a quantification on line 3; then we can apply the ∃O Rule to that wff. What is true in this proof is true generally: the main purpose of the QE Rule is to transform *the negation of a quantification* into *a quantification*, so that the ∀O or ∃O Rule may then be used.

We can justify the QE Rule by noting the intuitive soundness of English versions of the rule. An English counterpart of the first form of the rule:

From "It is false that everything is *A*" derive "Something is not *A*," and vice versa.

By substituting a specific predicate for the *A* and employing more colloquial English, we reach an instance of the rule that is even more obviously correct:

From "Not all things are physical" derive "Some things aren't physical," and vice versa.

Two English counterparts of the second form of the rule–one abstract and the other concrete:

From "It is false that there is an *A*" derive "Each thing is not *A*," and vice versa.

From "It's false that ghosts exist" derive "Nothing is a ghost," and vice versa.

I will provide a second example of a proof that uses the QE Rule, a proof for this argument (call it "Smoking Collegians"):

Those who knowingly and needlessly (e)ndanger their health act (i)rrationally. College students who (s)moke knowingly and needlessly endanger their health. It follows that college students who smoke act irrationally.

∀x(Ex → Ix), ∀x(Sx → Ex) ⊢ ∀x(Sx → Ix)

(Ex = x knowingly and needlessly endangers x's health, Sx = x is a college student who smokes) A proof of validity:

				proof stages
1	(1)	∀x(Ex → Ix)	A	} 1
2	(2)	∀x(Sx → Ex)	A	
3	(3)	−∀x(Sx → Ix)	PA	2
3	(4)	∃x−(Sx → Ix)	3 QE	3
3	(5)	−(Sa → Ia)	4 ∃O	4
1	(6)	Ea → Ia	1 ∀O	} 5
2	(7)	Sa → Ea	2 ∀O	
1,2	(8)	Sa → Ia	7,6 CH	} 6
1,2,3	(9)	(Sa → Ia) & −(Sa → Ia)	8,5 &I	
1,2	(10)	∀x(Sx → Ix)	3-9 −O	7

(The column on the right is not part of the proof.) Many proofs for sequents whose conclusions are universal quantifications follow the same general pattern, which I call *the seven stages of a Dash Out predicate proof:*

1. The premises of the sequent are assumed.
2. A provisional assumption is made of the negation of the conclusion (in anticipation of stage seven).
3. The QE Rule is applied to assumptions that are negations of quantifications.
4. The ∃O Rule is applied to existential quantifications.
5. The ∀O Rule is applied to universal quantifications.
6. A standard contradiction is derived (by propositional inference rules) from the wffs reached in stages four and five.
7. The conclusion is obtained by Dash Out.

This strategy will work not only for sequents whose conclusions are universal quantifications, but also for sequents with existential quantifications as conclusions. Here is such a proof for "False Beliefs" from the previous section.

1	(1)	∀x(Kx → Tx)	A
2	(2)	∃x(Bx & −Tx)	A

3	(3)	$--\exists x(Bx \ \& \ -Kx)$	PA
3	(4)	$\forall x-(Bx \ \& \ -Kx)$	3 QE
2	(5)	$Bd \ \& \ -Td$	2 ∃O
1	(6)	$Kd \rightarrow Td$	1 ∀O
3	(7)	$-(Bd \ \& \ -Kd)$	4 ∀O
2	(8)	$-Td$	5 &O
1,2	(9)	$-Kd$	6,8 MT
1,2,3	(10)	$-Bd$	7,9 CA
2	(11)	Bd	5 &O
1,2,3	(12)	$Bd \ \& \ -Bd$	11,10 &I
1,2	(13)	$\exists x(Bx \ \& \ -Kx)$	3-12 –O

This proof is four lines longer than the proof displayed in the last section, and this is typical of the difference in length between ∃I and –O proofs for sequents whose conclusions are existential quantifications. Note also that the shorter proof had no provisional assumption and so no assumption-dependence column was needed. For both of these reasons we prefer the ∃I proof. But there is an important theoretical point to make in this connection. The ∃I Rule is superfluous. The other three predicate rules (∀O, ∃O, and QE), when added to the group of eighteen propositional inference rules listed on page 278, form a set that is sufficient for constructing a formal proof for *any* valid sequent that uses the vocabulary presented in Chapter Two. (Of course, it does not follow that a given individual will be able to devise a proof for a particular sequent.) In logicians' terminology, the set of rules is *complete* for this portion of predicate logic. (But it is not complete for relational predicate logic, the subject of Chapters Ten through Thirteen.) The set of inference rules (with or without the addition of ∃I) is also *consistent;* that is, no proof for an invalid sequent can be constructed using just these rules.[10]

Buckle up your seat belts; it's time for a quick lesson in the history of logic. "False Beliefs" and "Smoking Collegians," as well as most of the arguments featured in the exercises of this chapter, are *categorical syllogisms.*

A categorical syllogism is an argument with these features:

(1) It consists of three statements.
(2) Each statement is an *A, E, I,* or *O* statement.
(3) Each (logical) predicate occurs in two statements.

There are 256 forms of categorical syllogisms. Contemporary logicians assess 15 of these forms as valid, and the remaining 241 as invalid.[11] For practical purposes, the 15 valid syllogism forms may be reduced to the six forms listed in this table:[12]

Six Valid Syllogism Forms

All *D* are *E* All *F* are *D* So, all *F* are *E*	No *D* are *E* All *F* are *D* So, no *F* are *E*
All *D* are *E* Some *F* are *D* So, some *F* are *E*	No *D* are *E* Some *F* are *D* So, some *F* are not *E*
Some *D* are not *E* All *D* are *F* So, some *F* are not *E*	All *E* are *D* Some *F* are not *D* So, some *F* are not *E*

[10]For more information on these matters see Appendix Two.

[11]Medieval logicians regarded 24 forms as valid and 232 invalid. The difference between the two positions relates to the issue of *existential import* explored in Chapter Nine.

"False Beliefs" has the form shown in the bottom row, right column, and the form of "Smoking Collegians" is displayed in the top row, left column. Most of the arguments treated in the exercises in this chapter exhibit one or another of these six forms (or variations on them).

The study of categorical syllogisms, called *syllogistic logic* or *Aristotelian logic,* dominated formal logic from the fourth century B.C. through the nineteenth century. Medieval logicians assigned (or in some cases invented) women's names to identify the valid syllogism forms. The vowels in the names indicate the kinds of statements making up the form. The form of "Smoking Collegians," for example, was dubbed *Barbara* because it is composed of three *A* statements. Medieval logicians did not use the same kinds of techniques for determining argument validity that are employed today. They typically devised sets of rules (rules like "No valid syllogism has two negative premises") to be applied to syllogisms. A syllogism is valid iff it adheres to all of the rules in the set.[13] The shortcoming of this approach is that it is restricted to syllogisms. Predicate logic, developed largely in the twentieth century, includes within its scope all syllogisms plus a great many arguments that syllogistic logic is unable to treat.

A *sorites* is a syllogism on steroids. If you don't care for that definition, try this one.

A sorites is an argument with these features:

(1) It consists of four or more statements.

[12]The reduction is accomplished by changing the order of predicates in convertible statements (*E* and *I*).

[13]For a fuller account of syllogistic logic see Chapter Two of Stephen F. Barker, *The Elements of Logic* (5th ed.; New York: McGraw-Hill Book Company, 1989).

(2) **Each statement is an *A, E, I*, or *O* statement.**
(3) **Each predicate occurs in two statements.**

In the 1960s NASA trained 13 women (known as *the Mercury 13*) to be astronauts, but then NASA officials had second thoughts because they feared the public reaction to the death of women in a space accident. So, NASA prevented the women from entering space with the help of a sorites and some convenient regulations:[14]

> Being a jet test (p)ilot is a necessary condition for being an (a)stronaut. Attending jet test-pilot (s)chool is a necessary condition for being a jet test pilot. No (w)omen [are permitted to] attend jet test-pilot school. Thus, no women are astronauts.

$\forall x(Ax \rightarrow Px), \forall x(Px \rightarrow Sx), \forall x(Wx \rightarrow -Sx) \vdash \forall x(Wx \rightarrow -Ax)$

Note the symbolization of the first two premises. The proof for this sequent is very similar to the proof for a syllogism, just a few lines longer.

1	(1)	$\forall x(Ax \rightarrow Px)$	A
2	(2)	$\forall x(Px \rightarrow Sx)$	A
3	(3)	$\forall x(Wx \rightarrow -Sx)$	A
4	(4)	$-\forall x(Wx \rightarrow -Ax)$	PA
4	(5)	$\exists x-(Wx \rightarrow -Ax)$	4 QE
4	(6)	$-(We \rightarrow -Ae)$	5 ∃O
1	(7)	$Ae \rightarrow Pe$	1 ∀O
2	(8)	$Pe \rightarrow Se$	2 ∀O
3	(9)	$We \rightarrow -Se$	3 ∀O
1,2	(10)	$Ae \rightarrow Se$	7,8 CH
1,2	(11)	$-Se \rightarrow -Ae$	10 CN
1,2,3	(12)	$We \rightarrow -Ae$	9,11 CH
1,2,3,4	(13)	$(We \rightarrow -Ae)$ & $-(We \rightarrow -Ae)$	12,6 &I
1,2,3	(14)	$\forall x(Wx \rightarrow -Ax)$	4-13 –O

EXERCISES

10. In exercise 9 of Chapter Three you began the task of showing that *A* and *O* statements with the same grammatical subjects and predicates are *contradictories*, and that the same holds true for *E*'s and *I*'s. Finish the job by completing the proofs for the following arguments. Every assumption has been identified.

[14]"These astronaut trainees never left the earth," *Miami Herald* (October 17, 1998), pp. 1E & 2E.

(a) It is false that some (o)rthodontists are not (d)entists. So, all (o)rthodontists are (d)entists.

1	(1)	$-\exists x(Ox$ & $-Dx)$	A
	(2)		PA
	(3)		1 QE
	(4)		2 QE
	(5)		4 ∃O
	(6)		3 ∀O
	(7)	$Oe \rightarrow De$	6 AR
1,2	(8)		7,5 &I
1	(9)	$\forall x(Ox \rightarrow Dx)$	2-8 –O

*(b) It is false that some (B)uddhists are (A)mish. So, no Buddhists are Amish.

1	(1)	$-\exists x(Bx$ & $Ax)$	A
	(2)	$-\forall x(Bx \rightarrow -Ax)$	PA
	(3)	$\forall x-(Bx$ & $Ax)$	
	(4)	$\exists x-(Bx \rightarrow -Ax)$	
	(5)	$-(Bf \rightarrow -Af)$	
	(6)	$-(Bf$ & $Af)$	
	(7)	$-Bf \vee -Af$	
	(8)	$Bf \rightarrow -Af$	
1,2	(9)	$(Bf \rightarrow -Af)$ & $-(Bf \rightarrow -Af)$	
1	(10)	$\forall x(Bx \rightarrow -Ax)$	

Instructions for exercises 11, 12, and 14 through 16: Symbolize each argument on one horizontal line, using the suggested abbreviations. Construct a proof for each sequent.

11. In the Sixth *Meditation*, Descartes writes:

> *I first take notice here that there is a great difference between the mind and the body, in that the body, from its nature, is always divisible and the mind is completely indivisible.*[15]

He is reasoning syllogistically:

> (B)odies are always (d)ivisible. (M)inds are indivisible. Thus, no minds are bodies.

12. A contestant on the television game show *Who Wants to Be a Millionaire?* faced this question:

Which of these is not a reptile?	
A. Gila Monster	B. Komodo Dragon
C. Salamander	D. Python

[15]Rene Descartes, *Meditations on First Philosophy*, tr. by Laurence J. Lafleur (Indianapolis, Ind.: The Bobbs-Merrill Company, Inc., 1951, 1960), p. 81.

He answered "*D*, python," and exited the game. Host Regis Philbin commented,

He walks away with just one thousand dollars because as you all know, pythons are snakes, and snakes are reptiles, and a salamander is an amphibian.

Regis advanced two syllogisms in one sentence (with some elements left unstated)–quite an achievement! His first syllogism proved that answer *D* is wrong and the second that answer *C* is correct.

*(a) (P)ythons are s(n)akes. Snakes are (r)eptiles. Therefore, pythons are reptiles.

(b) Sa(l)amanders are (a)mphibians. Amphibians are not (r)eptiles. It follows that salamanders are not reptiles.

13. Construct proofs for the arguments in these exercises from the preceding set *without* using ∃I:

(a) 2; *(b) 3; (c) 5; (d) 7.

14. Philosopher John Searle advances this sorites to prove a point about the philosophy of mind:[16]

Computer (p)rograms are defined purely (f)ormally. (M)inds have (i)ntrinsic mental content. Nothing that is defined purely formally has intrinsic mental content. So, minds are not computer programs.

*15. A letter from Samuel Johnson to James Boswell contains a sorites:

Every man who a(t)tacks my belief diminishes in some degree my confidence in it, and therefore makes me uneasy; and I am a(n)gry with him who makes me uneasy.[17]

The first premise precedes the comma. The second premise is "All those who (d)iminish my confidence in my belief make me (u)neasy." The third premise follows the semicolon. Supply the unstated conclusion. (Tx = x attacks my belief, Nx = I am angry with x)

16. Prospector James Kidd vanished in Arizona in the 1950's leaving behind a hand-written will directing that his substantial fortune "go into a research or some scientific proof of a soul of the human body which leaves at death." Many individuals filed for the estate, including Professor Richard Spurney who submitted a file of "evidence" a foot thick. He summarized his main proof as follows:

Death is decomposition. Hence, what cannot decompose cannot die. But decomposition requires divisibility into parts. Thus what is not divisible into parts cannot die. But divisibility into parts requires matter. Hence what has no matter in it is not divisible into parts and so cannot decompose, and so is necessarily immortal.[18]

What Professor Spurney presented as several syllogisms can also be packaged as one sorites:

Everything that d(i)es d(e)composes. Everything that decomposes divides into (p)arts. Only (m)aterial things are divisible into parts. No human (s)ouls are material. Consequently, no human souls die.

This argument leaves a lot to be desired. For one thing, it presupposes just what it is supposed to prove, namely that there are immaterial souls. But at least the sorites is valid.

[16] "Is the Brain a Digital Computer?" *Proceedings and Addresses of the American Philosophical Association,* LXIV, no. 3 (November 1990), p. 21.

[17] Boswell, *The Life of Samuel Johnson* (New York: Random House, Inc., n.d.), p. 618.

[18] "Professor claims miner's jackpot by 'proving' soul" (AP), *Miami Herald* (March 27, 1967), p. 14-A. Professor Spurney's arguments are borrowed from St. Thomas Aquinas.