

### Problem Set 3

Due Tuesday, October 6, in class.

1. Replicate the  $F_{1,j}$  row of critical values (Table D.3 in Gujarati).
2. Consider the following AR(1) model with Gaussian errors:

$$y_t = \phi y_{t-1} + \varepsilon_t; \quad \varepsilon_t \sim iidN(0, \sigma_\varepsilon^2).$$

We know that if  $|\phi| < 1$ , then as the sample size diverges, the OLS estimator of  $\phi$  becomes Gaussian:

$$\sqrt{T}(\hat{\phi} - \phi) \xrightarrow{d} N(0, 1 - \phi^2).$$

In other words,  $\hat{\phi} \xrightarrow{d} N(\phi, (1 - \phi^2)/T)$

Consider the following ranges for  $\phi$  and  $T$ :

$$\phi = \{0.5, 0.9, 0.95, 0.98, 0.99\}$$

$$T = \{25, 250, 1000\};$$

For each pair of  $\phi$  and  $T$ , generate 1000 artificial time series, and compute the OLS estimator of  $\phi$  in a regression of  $y_t$  on  $y_{t-1}$ . Compare the sample mean and median of  $\hat{\phi}_{OLS}$  with the true value of  $\phi$ . Also compare the mean and median standard error of  $\hat{\phi}_{OLS}$  with its asymptotic standard error. You should find that for smaller values of  $T$ , and larger values of  $\phi$ , that the asymptotic approximation is poor.

### 3. Pooled OLS and Fixed Effects

Consider the following panel regression,

$$\log(\text{rent}_{it}) = \alpha_i + \beta_1 \log(\text{pop}_{it}) + \beta_2 \log(\text{avginc}_{it}) + \beta_3 \log(\text{pctstu}_{it}) + u_{it}$$

Where *rent* is rental price, *pop* is the city population, *avginc* is average income, and *pctstu* is the student population as a percentage of the city population.

(This is a variation of problem 14.6 in Wooldridge's 2<sup>nd</sup> Edition).

Use the data in rental.xls. There are 2 time periods in the panel, *i.e.*  $T = 2$ . A description of the data is in rental.txt. You will need columns A, B, C, E, H, and U.

Do the following in Gauss.

- a. Estimate the regression by Pooled OLS.
- b. Estimate the model by Fixed Effects. Report the  $\beta$  coefficients and standard errors.
- c. Test Pooled OLS vs. Fixed Effects.
- d. Verify that the regression run in first differences is identical to Fixed Effects.